Physics Letters A 382 (2018) 482–488

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

A new lattice hydrodynamic model based on control method considering the flux change rate and delay feedback signal

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ARTICLE INFO

Article history:
Received 26 October 2017
Received in revised form 14 December 2017
Accepted 15 December 2017
Available online 21 December 2017
Communicated by C.R. Doering

Keywords:
Traffic flow
Lattice hydrodynamic model
Control method
mKDV equation

ABSTRACT

In this paper, a new lattice hydrodynamic model is proposed by taking delay feedback and flux change rate effect into account in a single lane. The linear stability condition of the new model is derived by control theory. By using the nonlinear analysis method, the mKDV equation near the critical point is deduced to describe the traffic congestion. Numerical simulations are carried out to demonstrate the advantage of the new model in suppressing traffic jam with the consideration of flux change rate effect in delay feedback model.

1. Introduction

Due to the increasing of vehicles on the road, traffic congestion which causes a great number of social and economic problems, has aroused wide public concern. In order to deal with the increasing traffic problems, lots of models [1–31], such as lattice hydrodynamic model [9–14], car-following model [15–19] and cellular automaton model [13–15], have been proposed by many experts to explain the essence of traffic flow.

The car-following model describes the relationship between adjacent vehicles at a micro level. In 1961, Newell [15,16] put forward a car following model with differential equation and depicted the optimal velocity (OV) function for the first time. Subsequently in 1965, a classic car-following model was proposed by Bando et al. [26], Zeng et al. [27] called optimal velocity model (OVM). Later, many researchers developed the model with the consideration of many factors. In recent years, the control theory has become more and more important in many fields. From the viewpoint of control method, Zhao et al. [28], Zhou et al. [29] represented coupled-map (CM) car-following model by taking effect of delayed-feedback control [30,31] between the adjacent automobiles into account.

To explain the dynamical phase transitions, Nagatani [32] firstly introduced the lattice hydrodynamic model in single lane incorporating the advantages of macroscopic model [33] and car-following model. In last decades, the lattice hydrodynamic model has widely been used to analyze the traffic flow. In 2012, Peng [34] put forward a new model considering driver's memory model of traffic flow. Later, density difference effect (DDE) model was represented by Wang [35] with the consideration of density difference effect. Although, the application of control method is widespread in car-following model, feedback signal was seldom considered in lattice hydrodynamic model. And most of lattice hydrodynamic models were adopted to describe the traffic flow by density wave. Until 2015, Ge [36] began to take control theory into account in lattice hydrodynamic model with the consideration of flux difference to suppress traffic congestion. Subsequently, a new feedback control signal called delayed-feedback control (DFC) was proposed by Redhu [37]. Recently, Zhu [38] carried out a new control signal called the variation rate of the optimal velocity in lattice hydrodynamic model.

In the actual traffic, drivers adjust their driving behavior with the delay time. By observing the drivers' actual traffic behavior, Herman [39] found that drivers will leave past time information during driving. From Herman's research, we can easily get that delay feedback is necessary in drivers' behavior. But in reality, delay signal usually plays a negative feedback, such as emergency brake. To overcome the disadvantages, a new control signal we called flow change rate effect is taken into account in lattice hydrody-

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https://doi.org/10.1016/j.physleta.2017.12.033

namic model. Based on delay feedback and flux change rate effect, a new lattice hydrodynamic model is proposed to investigate its influence on traffic jam.

The outline of the paper is listed as follows. In section 2, the basic lattice hydrodynamic model is derived, and we introduce the new control signal for lattice hydrodynamic model. In section 3, new control signals will be added into the basic lattice hydrodynamic model and feedback control theory is used to analyze the stability conditions. In section 4, nonlinear analysis is introduced to describe the traffic congestion. In section 5, several numerical simulations are carried out to verify the theoretical result. Conclusions are given in section 6.

2. Basic model

In 1998, Nagatani [32] incorporating the model proposed by Kerner [33] and the idea of microscopic optimal velocity model derived the equations, as follows:

$$\begin{cases} \partial_t \rho + \partial_x(\rho v) = 0 \\ \partial_t(\rho v) = a\rho_0 V(\rho(x+\delta)) - a\rho v, \end{cases} \tag{1}$$

where ρ_0 and ρ represent average density and current density at time t respectively. δ means the average space headway which is the reciprocal of average density: $\rho_0 = 1/\delta$. x is the position of location lattice, so $\rho(x+\delta)$ is the current density on the position of $x+\delta$. a represents the sensitivity coefficient of driver ($a > 0$). Nagatani [32] creatively put forward the dicretization method to transform the equation (1) as follows:

$$\begin{cases} \partial_t \rho_j + \rho_0\left(\rho_j v_j - \rho_{j-1} v_{j-1}\right) = 0 \\ \partial_t(\rho_j v_j) = a\rho_0 V(\rho_{j+1}) - a\rho_j v_j, \end{cases} \tag{2}$$

where j represents the jth lattice in one-dimensional lattice hydrodynamic traffic model. ρ_j and v_j are current density and average velocity on the position of jth lattice at time t respectively. Later, we make $q = \rho v$ called flux, and consider delay feedback and flux change rate control signal in equation (2).

$$\begin{cases} \partial_t \rho_{j+1} + \rho_0\left(q_{j+1} - q_j\right) = 0 \\ \partial_t(q_j) = a\rho_0 V(\rho_{j+1}) - aq_j + u_j, \end{cases} \tag{3}$$

where control signal u_j is listed as follows, and k is the weighting value, which represents delay feedback of drivers ($0 < k < 1$). λ is coefficient of flux change rate:

$$u_j = k(q_{j+1}(t) - q_{j+1}(t-\tau)) + \lambda\partial_t q_{j+1}. \tag{4}$$

Nagatani [25] also proposed the new optimal velocity function about lattice hydrodynamic model which is similar to microscopic model. It is taken as

$$V(\rho) = (v_{max}/2)\,[tanh\,(1/\rho - 1/\rho_c) + tanh\,(1/\rho_c)], \tag{5}$$

where v_{max} represents the maximum velocity of vehicle on this road and ρ_c is safety density.

3. Control scheme

In this section, we investigate the influence of feedback control signal and consider on the traffic flow. Control theory will be applied to analyze the stability condition of the new lattice hydrodynamic model we proposed. We assume the desired density and flux in steady state:

$$[\rho_n(t), q_n(t)]^T = [\rho_n^*, q_n^*]^T, \tag{6}$$

where q_n^* and ρ_n^* represent the theoretical state of flux and density. We apply stability criterion analysis the system considering small perturbation. The equation can be derived by the control method:

$$\begin{cases} \partial_t \delta\rho_{j+1} + \rho_0\left(\delta q_{j+1} - \delta q_j\right) = 0 \\ \partial_t(\delta q_j) = a\rho_0\Lambda_{n+1}\delta\rho_{j+1} - a\delta q_j(s) \\ \qquad + k[\delta q_{j+1}(t) - \delta q_{j+1}(t-\tau)] + \lambda\partial_t\delta q_{j+1}, \end{cases} \tag{7}$$

where $\Lambda_{n+1} = \frac{\partial V(\rho_{n+1})}{\rho_{n+1}}$, $\delta\rho_j = \rho_j - \rho^*$, $\delta q_j = q_j - q^*$.

Taking Laplace transformation on Eq. (3), we can easily get:

$$\begin{cases} sP_{j+1}(s) - \rho_{j+1}(0) + \rho_0[Q_{j+1}(s) - Q_j(s)] = 0 \\ sQ_j(s) - q_j(0) = a\rho_0[\Lambda_{n+1}P_{j+1}(s) - Q_j(s)] \\ \qquad + kQ_{j+1}(1 - e^{-s\tau}) + \lambda s[Q_{j+1} - q_{j+1}(0)], \end{cases} \tag{8}$$

where $L(\rho_{j+1}) = P_{j+1}(s)$, $L(q_{j+1}) = Q_{j+1}(s)$, $L(q_j) = Q_j(s)$. $L(\cdot)$ indicates the Laplace transform function and s represents the transform function variable. Simplifying Eq. (8), we eliminates the variable $P_{j+1}(s)$ as follows:

$$\begin{aligned} &(s^2 + as - a\rho^2\Lambda_{n+1})Q_j(s) \\ &= [ks(1 - e^{\tau s}) + \lambda s^2 - a\rho^2\Lambda_{n+1}]Q_{j+1}(s). \end{aligned} \tag{9}$$

Then the transfer function $G(s)$ can be written by control theory:

$$G(s) = \frac{ks(1 - e^{s\tau}) + \lambda s^2 - a\rho_0^2\Lambda_{n+1}}{s^2 + as - a\rho^2\Lambda_{n+1}}, \tag{10}$$

where the characteristic polynomial $p(s) = s^2 + as - a\rho_0^2\Lambda_{n+1}$.

By the Hurwitz stability criterion, we can easily draw a conclusion that when traffic flow is smooth, characteristic function $p(s)$ is greater than 0. In order to make the system stable, it can be confirmed that $p(s)$ is satisfied by Routh criterion. Inequalities $a + \lambda a > 0$, $-a\rho_0^2\Lambda_{n+1} > 0$, and $G(s)$ must be smaller than 1 for all ω^2 to ensure stability of system by Hurwitz stability criterion. The follow derivation provides a effective measure to work out system stable condition:

$$\begin{cases} \|G(s)\|_\infty = sup_{\omega\in[0,\infty)}\,|G(j\omega)| \le 1 \\ |G(j\omega)| = \sqrt{G(jw)G(-jw)} \\ = \sqrt{\dfrac{2k^2\omega^2(1-cos\omega\tau) + 2(k\lambda\omega^3 + a\rho_0^2 k\Lambda_{n+1}\omega)sin\omega\tau + \lambda^2\omega^4 + a^2\rho^4\Lambda_{n+1}^2 + 2a\omega^2\rho_0^2\Lambda_{n+1}}{\omega^4 + a\omega^2(a + 2\rho_0^2\Lambda_{n+1}) + a^2\rho_0^4\Lambda_{n+1}^2}}. \end{cases} \tag{11}$$

The sufficient condition can be derived as follows:

$$\begin{aligned} &2k^2\omega[1 - cos\omega\tau] + 2(k\lambda\omega^2 + a\rho_0^2 k\Lambda_{n+1})sin\omega\tau + (\lambda^2 - 1)\omega^3 \\ &- (1 + a^2)\omega \le 0 \end{aligned} \tag{12}$$

4. Nonlinear analysis

In this section, the reductive perturbation method is introduced to carry out the mKDV equations. In the unstable region, we assume that the X and T are the slow variables, which can be defined by space variable j and time variable t,

$$X = \varepsilon(j + bt),\ T = \varepsilon^3 t,\ \rho_j = \rho_c + \varepsilon R(X, T), \tag{13}$$

where b is a definite constant and ε is a small positive scaling parameter. Substituting Eqs. (13) into Eq. (3) and carrying the Taylor expansions to the fifth order the fifth order of ε, the equation we get: