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Pair interactions of heavy vortices in quantum fluids

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ABSTRACT

The dynamics of quantum vortex pairs carrying heavy doping matter trapped inside their cores is studied. The nonlinear classical matter field formalism is used to build a universal mathematical model of a heavy vortex applicable to different types of quantum mixtures. It is shown how the usual vortex dynamics typical for undoped pairs qualitatively changes when heavy dopants are used: heavy vortices with opposite topological charges (chiralities) attract each other, while vortices with the same charge are repelled. The force responsible for such behavior appears as a result of superposition of vortices velocity fields in the presence of doping substance and can be considered as a special realization of the Magnus effect. The force is evaluated quantitatively and its inverse proportionality to the distance is demonstrated. The mechanism described in this paper gives an example of how a light nonlinear classical field may realize repulsive and attractive interactions between embedded heavy impurities.

1. Introduction

Quantum vortices appear in many different physical systems, like trapped dilute Bose–Einstein condensates (BEC) [1], liquid helium [2], exciton-polariton condensates [3] and other types of quantum fluids [4]. Motion and interaction of quantized vortex lines is extensively studied for few decades and many remarkable phenomena including vortex lattices [5], vortex lines reconnection [6] and quantum turbulence [7,8] were found. Fascinating discoveries were made by investigating quantum mixtures containing two or more interacting components. A number of exotic composite states of matter like skyrmions, coreless vortices [9,10] and half-quantum vortices [11] was discovered to raise the fundamental question of interaction of quantum vortices with matter. The same question emerges in liquid helium experiments, where various doping substances are used to make vortex cores, which are too thin and optically transparent otherwise, experimentally detectable. For example, in recent experiments vortex lattices in helium nanodroplets [12,13] were doped with xenon atoms which allowed to study them using a femtosecond x-ray coherent diffractive imaging technique. In the experiments of Gordon and colleagues quantum vortices were used to produce long wires of atoms by the ablation of metals in superfluid helium with laser pulses [14,15]. Many questions were raised in this works that require a detailed understanding of the matter-vortex interaction, including, for example, the question of unusual distribution of doped vortices inside the droplet.

It was shown theoretically that the scattering of particles on vortices is inelastic [16]. A certain portion of the particle energy is used to excite Kelvin waves along the vortex core [17]. During the interaction the incident particle may be scattered or remain trapped by the vortex depending on the initial amount of energy it possess. There are certain analogies between this process and the inelastic scattering of electrons on molecules, where an electron may be trapped passing a certain portion of energy to vibrational degrees of freedom of a molecule [18,19]. According to helium experiments [12,15] trapped atoms stick together to produce long cylindrical filaments aligned along the vortex core. The resulting structure is similar to the coreless vortex observed in BEC experiments. A coupled vortex-matter complex, or a doped vortex as it is referred in this paper, is dynamically stable and may demonstrate a distinct behavior, being thus an attractive object for research [20].

To cover underlying characteristics of vortex-matter interaction which appear in physically different quantum systems a universal theoretical approach is required. Classical complex-valued matter field formalism [4] based on the generalized non linear Schrodinger equation (gNLSE) [21] provides the most complete mathematical model of the quantum vortex behavior. It allows to take into account an arbitrary equation of state of a superfluid by incorporating it into the Hamiltonian [22,23] (in the spirit of the density functional theory [24,25]). Nonlocal interactions between the particles can be modeled as well [26]. The simplest local form of the equation containing cubic nonlinearity

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(known as the Gross–Pitaevskii equation) represents a minimalistic model of superfluidity [27]. Being a good model for BEC and condensed exciton-polaritons this equation cannot be directly applied to model strongly interacting liquid helium. It is known that only about 10% of helium is able to condense because of the strong interaction between particles. On the other hand it was demonstrated that NLSE formalism has a certain mathematical parallelism with the Landau two-fluid model and could describe both the condensed phase and the thermal cloud, being thus applicable to model finite temperatures [28,29]. In this sense liquid helium can be modeled phenomenologically assuming that the initial state is prepared appropriately [21]. In its simplest form the classical field equation was used many times in the past to understand qualitatively the behavior of electron bubbles in liquid helium [30,17]. The approach is capable of describing the universal mechanical aspects of matter-vortex interaction in an idealized quantum fluid [31].

It this paper the dynamics of two doped vortices with different chirality combinations is studied. A special attention is payed to the mass of doping particles which is assumed to be larger than the mass of particles forming a vortex. A doped vortex is called a heavy vortex in this case [20]. It is shown here for the first time how the doping matter, being heavy, completely changes the character of a vortex pair dynamics. The obtained results can be checked in BEC experiments. They can also be useful for qualitative understanding of certain aspects of the experiments with liquid helium and exciton-polariton condensates.

2. Description of the model

To study the behavior of doped vortices the following coupled system of nonlinear equations is used [20]

$$\begin{aligned} -i\hbar\psi_t &= \tfrac{\hbar^2}{2m_1}\nabla^2\psi - g_{11}|\psi|^2\psi - g_{12}|\varphi|^2\psi + \mu_1\psi, \\ -i\hbar\varphi_t &= \tfrac{\hbar^2}{2m_2}\nabla^2\varphi - g_{22}|\varphi|^2\varphi - g_{12}|\psi|^2\varphi + \mu_2\varphi, \end{aligned} \tag{1}$$

where the first equation describes the host fluid and the second one defines the behavior of the doping substance. Corresponding complex matter fields ψ and φ are assumed to be normalized as $\int|\psi|^2dV = N_1$ and $\int|\varphi|^2dV = N_2$, where N_1 and N_2 are particle numbers in the fluid and the doping substance respectively and V stands for the volume of the system. Both ψ and φ represent cold bosonic matter. Chemical potentials are denoted as μ_1 and μ_2. The equation coefficients are expressed through masses m_i and scattering lengths l_i of the components of the mixture $(i = 1, 2)$ as follows: $g_{11} = 4\pi l_1\hbar^2/m_1$, $g_{22} = 4\pi l_2\hbar^2/m_2$, $g_{12} = 2\pi l_{12}\hbar^2/m_{12}$, where m_{12} is the reduced mass and $l_{12} = (l_1 + l_2)/2$.

Practically it is convenient to solve the equations using the units where fields are measured in $\psi_\infty = \sqrt{N_1/V}$, distances in $\xi = \hbar/\sqrt{2m_1g_{11}\psi_\infty^2}$ (which is called the healing length) and time in $t_0 = \xi^2 m_1/\hbar$ [32,33]. These units are used throughout the article. It can be shown that the transformed equations contain only relative parameters m_2/m_1, l_2/l_1 and N_2/N_1 and do not depend on the specific type of the host fluid, which makes the results easily scalable to different types of systems [20]. The relative amount of doping matter N_2/N_1 is assumed to be small. Moreover, doping particles are assumed to be larger and heavier than particles of the host fluid, so that $N_2/N_1 = 0.02$, $l_2/l_1 = 2$. Different values of the relative dopant mass m_2/m_1 are considered in the paper.

The equations are solved numerically in 3D using a computational box with a volume $V = (74.0\xi \times 36.5\xi \times 74.0\xi)$ and an elementary cell $\Delta V = (0.5\xi)^3$. At the edges of the computation volume the solid wall boundary condition is applied. The image vortex technique is used to fulfill the condition of zero normal flow at the wall as described in [34]. The selected volume is large enough to study the behavior of a heavy vortex pair and to exclude the influence of the boundary on the dynamics during the time of the numerical experiment. At the same time, the considered volume is assumed to be a small fraction of the typical trap volume in BEC experiments. For this reason the trap potential is not modeled and the vortex-free ground state has a homogeneous density profile. It allows to concentrate purely on the vortex dynamics and exclude possible influence of the density inhomogeneity on the process (while the simulation of the whole volume of the trap including inhomogeneities can be a topic of future works).

In the initial state two vortices are created in the host fluid and the doping matter is distributed along their cores in the form of filaments with a cylindrical symmetry (repeating the symmetry of the vortex) and Gauss-like radial distribution of the density. The initial state is then optimized using the imaginary time propagation technique until the steady state is reached. The convergence is controlled by monitoring the time dependence of the full energy of the system. More details about the initial state preparation can be found in the literature [16,17]. During the real time dynamics the vortex-dopant complex moves as a single object if velocities are not too large. Otherwise the heavy filament may decouple, keeping its original form (in the regimes considered in this paper the doping substance and the host fluid remain phase separated also in the absence of vortices). Such behavior is consistent with the available experimental picture. The limits of coupled dynamics are determined by the vortex-dopant binding energy which depends strongly on the volume of the dopant [16].

To switch from abstract units used in the paper to real-world numbers one should specify masses and scattering lengths for the host fluid and the doping substance. For example, for the mixture of Li and Rb one gets $m_1 = 1.1524 \cdot 10^{-26}$ kg, $m_2 = 1.4192 \cdot 10^{-25}$ kg and the mass ratio $m_2/m_1 \approx 12$. For the evaluation purposes s-wave scattering length parameters could be approximated using the available data about the sizes of atoms (assuming that atoms interact as rigid balls with certain radii in the s-wave approximation). Using the values $l_1 = 2.56$ Å and $l_2 = 4.40$ Å for Li and Rb respectively one obtains the ratio $l_2/l_1 = 1.72$, which is close to the value considered in this paper ($l_2/l_1 = 2$). Using the given numbers, GPE coefficients can be computed: $g_{11} = 0.0194$ eVÅ, $g_{22} = 0.0028$ eVÅ, $g_{12} = 0.0142$ eVÅ. As it is shown above, interspecies interaction coefficient g_{12} is expressed through the interspecies scattering length which, for simplicity reasons (and having in mind the physical meaning of the s-wave), is taken in the form $l_{12} = l_1/2 + l_2/2$. In BEC experiments this parameter can be varied in a wide range using the Feshbach resonance. From the point of view of the theory it can be varied in a certain range too, as long as it provides a good phase separation in the mixture and the usual criterium $g_{11}g_{22} < g_{12}^2$ holds, which is always the case in the paper. For the discussion of possible ranges of parameters in the context of heavy vortices please see [20]. To maximize the effects discussed in this paper it is desirable to consider largest possible mass ratios.

3. Results and discussion

It is known that undoped vortex pairs behave differently, depending on the combination of vortex winding numbers. Two vortices with the same chirality rotate along a circular trajectory. If chiralities are different vortices move parallel to each other. In both cases the distance d between vortices stays unchanged. Quantum vortex velocity field decays with distance r from its center proportionally to $1/r$ and the linear velocity of the pair motion (expressed in units ξ/t_0) is $v = 1/d$ [35].

At the beginning of the numerical experiment two heavy vortices with the same chirality are placed at a distance $d = 14\xi$ from each other (both oriented along y-axis). Then the system is allowed to evolve during the period of time $\Delta t = 220t_0$. The initial