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Large-angle channeling radiation from relativistic electrons in optically transparent crystals

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ABSTRACT

In the work within the frame of quantum electrodynamics are obtained new formulae describing the large-angle photon emission from channeled electrons with taking into account of the dispersion of refractive index. Calculations based on these formulae show that the spectral and angular distributions of large-angle optical and ultraviolet radiation from planar channeled sub-GeV electrons in optically transparent crystal reflect the band structure of transverse energy levels of channeled electrons. Comparison with ordinary Cherenkov radiation spectrum reveals that channeling (depending on the beam energy) leads to sufficient change of the large-angle emission spectrum.

1. Introduction

Radiation and scattering of sub-GeV electrons in the thin crystals (the thickness is less than dechanneling length) was a subject for recent intensive experimental and theoretical studies [1–3]. Our work is dedicated to analysis of large-angle (near the Cherenkov angle) planar channeling radiation (CR) spectral distribution in the optically transparent crystals. The angular distribution of this radiation was recently investigated in [3].

At planar channeling the transverse motion of electrons is governed by the 1D periodic potential being the sum of continuous potentials of crystallographic planes. As a sequence, the transverse energy spectrum has the band structure (see the recent calculations in [3]). The radiative transitions between these energy bands lead to the well known channeling radiation (CR) which has been studied in detail in the case of "forward" or small-angle emission.

The quantum theory of CR was proposed in pioneer works [4] and [5]. The experimental and theoretical CR properties description can be found in Refs. [6] and [7]. Currently, a large number of works are devoted to new types of channeling in the crossed laser fields – see, for example, Refs. [8] and [9].

In a reference frame in which the relativistic channeled particles are at rest, CR angular distribution is the same both in forward and backward directions. In the laboratory system, due to the Doppler effect, the backward radiation becomes negligible in comparison with the forward radiation which is strongly stretched along the channeled particles direction of motion. The energy of CR-photons emitted in forward direction can be from several keV up to hundreds MeV depending on electron beam energy. CR in optical and ultraviolet region occurs under the condition that the crystal refractive index $n > 1$ (see [5]), i.e. at the large angles to the direction of motion of the channeled electrons – near the Cherenkov angle.

The Cherenkov radiation (ChR) can be used, for example, in biomedical research. Unfortunately an interpretation of experimental data meets some difficulties because the theory uses the refractive index $n = const$ [10].

The angular distribution of CR in the optical and ultraviolet region was recently investigated in [3] and reveals very specific properties, if one provides calculations using the realistic dependence of the crystal refractive index on emitted CR photon frequency, i.e. taking into account of dispersion. In our work [3] we showed that angular distributions of optical and ultraviolet radiation from relativistic electrons at planar channeling in optically transparent crystals is characterized by an unusual dependence on the polar and azimuthal angles.

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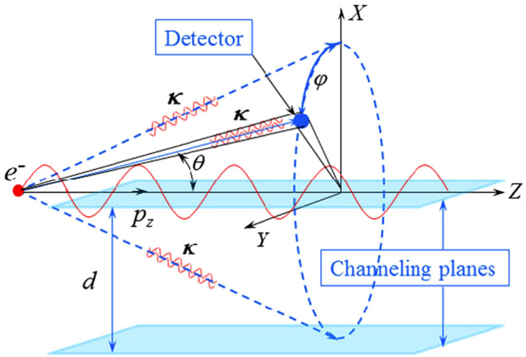

Fig. 1. Coordinate system and channeling planes: X-axis is perpendicular to the planes, Y and Z axes are in the channeling plane, θ is the polar angle between CR photon wave vector $\boldsymbol{\kappa}$ and Z axis, φ is the azimuthal angle of vector $\boldsymbol{\kappa}$ and p_z is the longitudinal electron momentum. For clarity, the classical electron trajectory at planar channeling is shown. The detector arrangement is depicted as (blue) filled circle. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

The spectral distribution of this radiation was not studied up to now and is the subject of our paper.

2. Large angle CR neglecting dispersion

As in [3], the planar channeled electrons are entering into the crystal so that longitudinal motion with momentum p_z occurs along Z axis, while the transverse motion occurs along X axis, i.e. $p_y = 0$. The coordinate system is shown in Fig. 1, where θ is the angle between CR photon wave vector $\boldsymbol{\kappa}$ and Z axis and φ is the azimuthal angle of $\boldsymbol{\kappa}$ vector in XY plane.

In the coordinate system chosen, the spectral distribution of CR arising due to spontaneous transitions between initial i and final f transverse energy levels, when the refractive index does not depend on the photon frequency, $n = const$, in the dipole approximation is described by the well known formula obtained first by Beloshitskii and Kumakhov [5]:

$$\frac{dI_{if}}{d\omega} = \frac{e^2 x_{if}^2 \Omega_{if}^2}{2c^3\beta^3 n^2} P_i(\theta_0)\omega\Theta(W_+)\Theta(W_-) \times \\ \times \left[1 - \frac{2\omega}{\Omega_{if}^m} + \frac{\omega^2}{\Omega_{if}^{m\,2}} + n^2\beta^2\right], \tag{1}$$

$$\Omega_{if}^m = \frac{\Omega_{if}}{1 - n^2\beta^2}\,. \tag{2}$$

Here $\hbar\Omega_{if} = \varepsilon_i - \varepsilon_f$, ε_i and ε_f are the energies of the i-th and f-th transverse quantum states of channeled electron, x_{if} the matrix element of transition probability, $\beta = v/c$ and v the longitudinal electron velocity. In Eq. (1) $P_i(\theta_0)$ stands for initial population of the i-th transverse quantum state depending on the angle of incidence $\theta_0 = \arctan[p_x/p_z]$ with respect to the crystallographic planes, and the arguments of the Heaviside's functions are:

$$W_+ = \omega - \frac{\Omega_{if}}{1 + n\beta}\,, \\ W_- = \frac{\Omega_{if}}{1 - n\beta} - \omega\,. \tag{3}$$

As we already mentioned, in Eqs. (1)–(3) the refractive index does not depend on the photon frequency, $n = const < 1$, that means these equations are valid for emission of very hard photons. In the optical and ultraviolet regions the refractive index of optically transparent crystals $n > 1$ and depends on the photon frequency, $n = n(\omega)$. That means, to calculate emission characteristics of CR large-angle "soft" photons when the dispersion can play sufficient role, the Eqs. (1)–(3) should be modified.

3. Large angle CR in the presence of dispersion – theory

To modify Eq. (1) when the dispersion presents, we shall use the expression for CR emission probability dw_{if} of the photon of energy $\hbar\omega$ by channeled electron obtained in [3] in the first order perturbation theory following the standard procedure described in [11,12]:

$$dw_{if} = \frac{e^2}{2\pi\hbar} P_i(\theta_0) \sum_\tau \left|\mathbf{e}_\tau \boldsymbol{\alpha}_{if}\right|^2 \delta(\kappa_{if} - \kappa)\frac{d^3\boldsymbol{\kappa}}{\kappa}\,. \tag{4}$$

In Eq. (4), $\tau = (\sigma, \pi)$ is an index denoting the components of polarization vector $\boldsymbol{\epsilon}_\tau$ of CR photon, $c^*\hbar\kappa_{if} = (E_i - E_f)$, $c^* = c/n(\omega)$ the phase velocity of CR photon in a crystal, $\kappa = |\boldsymbol{\kappa}|$ and $E_{i(f)} \simeq \varepsilon_{i(f)} + c(p_{i(f)z}^2 + m^2c^2)^{1/2}$ total relativistic energy of channeled electron in initial i and final f quantum transverse states.

The components of the vector $\boldsymbol{\alpha}_{if}$ for our case of planar channeling are [3]:

$$(\alpha_{if})_x = -\mathbf{i}\langle x\rangle_{if}\frac{\Omega_{if}}{c},\ \langle x\rangle_{if} = \langle f \mid x e^{-\mathbf{i}\kappa_x x} \mid i\rangle, \\ (\alpha_{if})_y = 0, \\ (\alpha_{if})_z = -\mathbf{i}\langle F\rangle_{if}\beta\,,\ \langle F\rangle_{if} = \mathbf{i}\langle f \mid e^{-\mathbf{i}\kappa_x x} \mid i\rangle. \tag{5}$$

Here, $\kappa_x = |\boldsymbol{\kappa}|\sin\theta\cos\varphi$. If the argument of an exponential term in Eq. (5) is small, both the matrix element $\langle x\rangle_{if}$ defining x-component and the form-factor $\langle F\rangle_{if}$ defining z-component of $\boldsymbol{\alpha}_{if}$ can be expanded into a series:

$$\langle x\rangle_{if} = \langle f \mid x \mid i\rangle + \kappa_x\langle f \mid x^2 \mid i\rangle + \dots\,, \\ \langle F\rangle_{if} = \mathbf{i}\langle f \mid i\rangle + \kappa_x\langle f \mid x \mid i\rangle + \dots\,. \tag{6}$$

Even from (5) and more clear from (6) one can see, that in contrast to dipole approximation (1) at $i = f$ there arise the contributions to both components (x and z) of the vector $\boldsymbol{\alpha}_{ii}$: smaller – connected with the matrix element $\langle x\rangle_{ii}$ and more substantial – connected with the form-factor $\langle F\rangle_{ii}$. Apparently, if the transverse state during radiation is the same, we deal with Cherenkov radiation from channeled particle (see below section 4). This case will be studied in detail in a separate paper.

With these comments, let proceed further with CR probability dw_{if}, if the dispersion law $n = n(\omega)$ is known. In this case, $\kappa = \omega n(\omega)/c$, and one has to replace integration over $\boldsymbol{\kappa}$ in (4) by integration over ω

$$d^3\boldsymbol{\kappa} = J(\omega)d\omega do\,. \tag{7}$$

Here, $do = \sin\theta d\theta d\varphi$ is the elementary solid angle and $J(\omega)$ – Jacobian of this transformation:

$$J(\omega) = \left(n(\omega)\frac{\omega}{c}\right)^2\left(n(\omega) + \omega\frac{\partial n(\omega)}{\partial\omega}\right)\,. \tag{8}$$

Using (5) and (8) we find the formula for intensity $dI_{if} = \hbar\omega dw_{if}$ of CR emitted into the solid angle Δo:

$$dI_{if} = \frac{e^2}{2\pi c^4}P_i(\theta_0)x_{if}^2 \times \\ \times \int_{\Delta o} f_{if}(\omega,\theta)n\left(n + \omega\frac{\partial n}{\partial\omega}\right)\omega^2\delta(\kappa_{if} - \kappa)do d\omega\,. \tag{9}$$

Here, $\Delta o = \Delta\theta\Delta\varphi$ is the angular size of the photon detector and