Physics Letters A ••• (••••) •••–•••

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

# Generating ultra wide low-frequency gap for transverse wave isolation via inertial amplification effects

Jingru Li [a], Sheng Li [a,b,*]

[a] State Key Laboratory of Structural Analysis for Industrial Equipment, School of Naval Architecture, Faculty of Vehicle Engineering and Mechanics, Dalian University of Technology, Dalian 116024, PR China
[b] Collaborative Innovation Center for Advanced Ship and Deep-Sea Exploration, Shanghai 200240, PR China

ARTICLE INFO

*Article history:*
Received 19 July 2017
Received in revised form 15 October 2017
Accepted 19 November 2017
Available online xxxx
Communicated by V.A. Markel

*Keywords:*
Transverse wave
Low-frequency gap
Ultra-wide bandwidth
Inertial amplification

ABSTRACT

Low-frequency transverse wave propagation plays a significant role in the out-of-plane vibration control. To efficiently attenuate the propagation of transverse waves at low-frequency range, this letter proposed a new type phononic beam by attaching inertial amplification mechanisms on it. The wave propagation of the beam with enhanced effective inertia is analyzed using the transfer matrix method. It is demonstrated that the low-frequency gap within inertial amplification effects can possess much wider bandwidth than using the local resonance method, thus is more suitable for designing applications to suppress transverse wave propagation.



## 1. Introduction

The band gaps capable of inhibiting wave propagation found in phononic crystals [1–5] or acoustic metamaterials [6–10] have been the focus of extensive research efforts in recent years. The Floquet–Bloch theory allows one to investigate the wave motion of the infinite medium through modeling a representative unit cell. Numerical approaches including the transfer matrix method [11,12], the multi-scattering theory [13,14], the plane wave expansion method [15,16] and the finite element method [17,18], etc., have been widely employed to obtain the band gaps of the unit cell models in literatures.

It is well known that the design of ultra wide low-frequency gap has a significant practical meaning. In previous studies, phononic band gaps in periodic structures are predominantly created by the means of Bragg scattering and local resonance [19–25]. An underlying constraint in the Bragg gaps is that the wavelength should be comparable to the lattice constant. Thus to create low frequency Bragg gaps, high density/low modulus materials or large sized structures are utilized to realize the requirement of low wave speed or large lattice constant [26]. In contrast to Bragg gaps, band gaps due to the addition of local resonators can be achieved at much lower frequencies. However, the bandwidth of the local resonance gap strongly depends on the mass of resonator, which implies that heavy resonator is needed to generate wide low-frequency local resonance gap [27,28]. In order to broaden or shift downward the low-frequency gap, increasing research efforts have been concentrated on the design of local resonant structures [29–32]. An alternative approach for creating band gaps is using the inertial amplification concept, which was proposed by Yilmaz, et al. [28,33]. Stop bands are generated with enhanced effective inertia by amplifying the motion of a small mass. This method has an advantage that one can obtain wide low-frequency gaps without sacrificing the stiffness or increasing the overall mass [34–37]. The provided applications of inertial amplification mechanisms have been used to serve as backbone structural components until Frandsen, et al. [38] proposed to employ them to create band gaps in a continuous structure. In their work, the longitudinal wave characteristics of a continuous rod with a periodic array of inertial amplification mechanisms were investigated. However, to the knowledge of the authors, there are no studies conducted to isolate flexural or transverse wave propagation using the inertial amplification induced gaps.

In this letter, we extend the inertial amplification concept to the design of phononic beams for transverse wave attenuation in continuous structures. The proposed one-dimensional system consisting of the elastic beam and periodically attached inertial amplification mechanisms is expected to show an ultra wide low-frequency gap. The band structure is calculated via a combination of the Bloch theorem and the transfer matrix method in this work. Following the introduction, we present the unit cell model and derive the methodology to analysis the band structures. Then the

* Corresponding author.
*E-mail address:* shengli@dlut.edu.cn (S. Li).

https://doi.org/10.1016/j.physleta.2017.11.023
0375-9601/

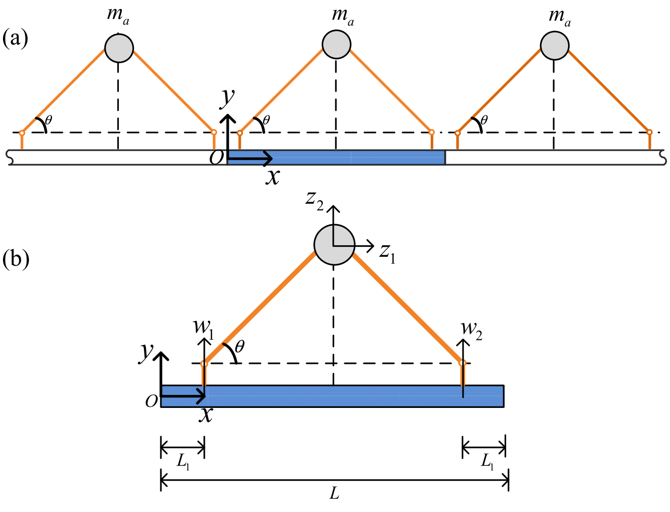


**Fig. 1.** Schematic diagram of the beam with inertial amplification mechanisms: (a) an infinite beam, (b) a unit cell model.

numerical results of inertial amplification induced band gaps are given in Section 3. We further demonstrate that the beam with inertial amplification mechanisms can possess larger bandwidth than placing local resonators on it. At last, the conclusions are drawn in Section 4.

## 2. Descriptions and model formulations

The one-dimensional elastic beam is assumed as an Euler–Bernoulli beam. In previous works the lumped parameter lattices embedded with inertial amplification effects are mostly studied to show the band structure. The inertial amplification mechanism constituting these parametric models is often analyzed to estimate the location of induced band gaps and is simplified in [38] when attached to a continuous rod. This work employs this idealized configuration as the additional inertial amplification unit. Fig. 1 gives the version of an Euler–Bernoulli beam with periodically distributed inertial amplification mechanisms in $x$ direction.

The band structure calculations are performed through modeling a representative unit cell. As seen in Fig. 1(b), the length of the unit cell is assumed as $L$. The vertical and inclined links shown by heavy lines denotes the rigid connections and the corners between them are designed as moment-free hinges in the mechanism. A similar hinge is also used at the top connection [38]. In this way, no moment is transferred through the mechanism. Hence, the connections do not deform but move the amplification mass by rigid-motion. If an out-of-plane excitation is exerted on the beam foundation, the constraint forces between the beam and the mechanism will be produced to restrict the amplification mass $m_a$ to move in $xoy$ plane. The motion of mass $m_a$ quantified by $z_1$ and $z_2$ is governed by the displacements at the attachment points $w_1$, $w_2$ and the amplification angle $\theta$, where $x = L_1$ and $x = L - L_1$. In order to determine these forces, the motion on mass $m_a$ is investigated first. We notice that in-plane extensional vibration is omitted in this study.

The top parts of the static and deformed inertial amplification mechanism are illustrated in Fig. 2. The constraint forces are denoted by $P_1$ and $P_2$. In static condition, the length of the rigid connection is $l$ and the height of mass $m_a$ is $H$. It is easily known that the vertical motions of incline connections are equal to the transverse displacements $w_1$, $w_2$. Using the Lagrange's equations, the governing equations for this mechanism with applied forces $P_1$ and $P_2$ can be derived (see Appendix A for details)

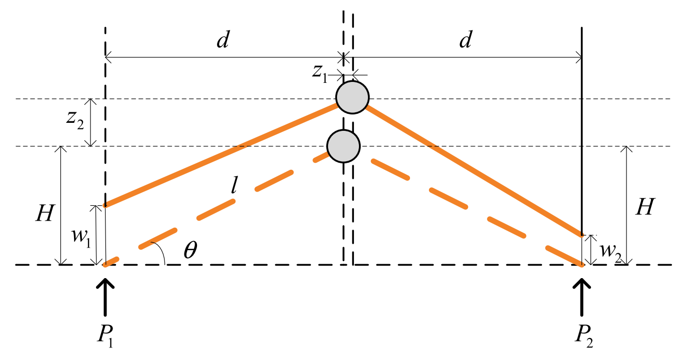


**Fig. 2.** The motion of mass $m_a$ in the inertial amplification mechanism.

$$\begin{aligned} m_{e1}\ddot{w}_1 - m_{e2}\ddot{w}_2 &= P_1 \\ m_{e1}\ddot{w}_2 - m_{e2}\ddot{w}_1 &= P_2 \end{aligned} \tag{1}$$

where $m_{e1} = \frac{m_a}{4}(1+\tan^2(\theta))$, $m_{e2} = \frac{m_a}{4}(\tan^2(\theta)-1)$. From Eq. (1), one can notice that the constraint forces are proportional to the relative acceleration at the attachment points, thus amplifying the effective inertia of the host beam.

The transfer matrix method is adopted to investigate the propagation of transverse waves. The governing equation for the out-of-plane displacement $w(x)$ of the continuous beam with applied forces is

$$EI\frac{\partial^4 w}{\partial x^4} + \rho A\ddot{w} = -P_1\delta(x-L_1) - P_2\delta(x-L+L_1) \tag{2}$$

where $E$ and $\rho$ are the elastic modulus and the density of the beam; the cross-sectional area and the area moment of inertia are denoted by $I$ and $A$. Assuming time-harmonic motion $w = W\exp(-i\omega t)$, $P_j = \tilde{P}_j\exp(-i\omega t)$, the homogeneous solution for a uniform Euler–Bernoulli beam is

$$W(x) = A\exp(ikx) + B\exp(-ikx) + C\exp(-kx) + D\exp(kx) \tag{3}$$

where $k = (\rho A\omega^2/EI)^{1/4}$. We can observe that there are two wave types, transverse and evanescent waves, composing the solution. The terms $\exp(ikx)$ and $\exp(-ikx)$ represent the right- and left-going transverse waves and the terms $\exp(-kx)$ and $\exp(kx)$ represent the right- and left-going evanescent waves. This formulation can also be employed to determine the displacement field in some segments of the unit cell [39–41]. The $n$th unit cell is able to be divided into three segments. The transverse displacements of these segments are

$$\begin{aligned} W_{n_1} &= A_{n_1}\exp(ikx) + B_{n_1}\exp(-ikx) + C_{n_1}\exp(-kx) \\ &\quad + D_{n_1}\exp(kx), \quad 0 \le x \le L_1 \\ W_{n_2} &= A_{n_2}\exp(ikx) + B_{n_2}\exp(-ikx) + C_{n_2}\exp(-kx) \\ &\quad + D_{n_2}\exp(kx), \quad L_1 \le x \le L - L_1 \\ W_{n_3} &= A_{n_3}\exp(ikx) + B_{n_3}\exp(-ikx) + C_{n_3}\exp(-kx) \\ &\quad + D_{n_3}\exp(kx), \quad L - L_1 \le x \le L \end{aligned} \tag{4}$$

It is noted that the transverse wave motion amplitude for $n-1$th cell can also be decomposed of these displacement functions with different coefficients. The continuities of displacement, slope, bending moment, and shear force at the attachment points, i.e., $x = L_1$ and $x = L - L_1$, are given by

Please cite this article in press as: J. Li, S. Li, Generating ultra wide low-frequency gap for transverse wave isolation via inertial amplification effects, Phys. Lett. A (2018), https://doi.org/10.1016/j.physleta.2017.11.023