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Equations of energy exchanges in variable density turbulent flows

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A B S T R A C T

This paper establishes a new formulation of the energy exchanges between the different parts of total energy. The decomposition uses the Reynolds averaging. This leads to a ternary decomposition of kinetic energy into the turbulence kinetic energy, the mean kinetic energy and the mixed kinetic energy, acting as an exchange term between the mean and turbulent motion. The formulation is then extended to distinguish a mean and fluctuating density part of each part of total energy. The formulation thus includes the mean density turbulence kinetic energy, product of the mean density and the half-trace of the velocity fluctuation correlation tensor. Its evolution equation is given in the spectral domain.

1. Introduction

This paper addresses the energy exchanges in turbulent flows with highly variable fluid properties. This covers flows with a high Mach number (high speed flows), such as the flows around a high-speed aircraft, or through a high speed jet or a nozzle [1], and low Mach number flows submitted to a strong temperature gradient, found for instance in heat exchangers, propulsion systems or nuclear or concentrated solar power plants [2–8]. The study of the energy exchanges between the different parts of total energy is a useful tool for both turbulence modeling and the fundamental understanding of turbulence. More detailed information is obtained through the study of the energy exchanges in the spectral domain [9–13,11,14–22]. However, while kinetic energy is fundamental property of any flow, it is not the case of its decomposition into turbulence kinetic energy and mean kinetic energy.

In incompressible flows with constant fluid properties, such decomposition is unique. The averaged kinetic energy is decomposed clearly, unambiguously and straightforwardly into the sum of two contributions: the kinetic energy of the mean motion associated with the mean velocity and the kinetic energy of the turbulent motion associated with the velocity fluctuation [see e.g. 23,24]. In compressible flows with highly variable density, this analysis is hindered by additional density velocity correlations. The decomposition of kinetic energy becomes more complex and arbitrary. It is even more difficult in the spectral domain. The choice ultimately depends on the physical role given to the additional density velocity correlations with respect to what constitutes the mean motion and the turbulent motion [25]. The most popular and successful decomposition extends the incompressible decomposition to the compressible case through the introduction of a density weighted averaging. This decomposition was widely developed by Favre [26,27,28]. Since, it has been used extensively by various authors [29–33]. Another approach, the mixed weighted decomposition, mixes density weighted averaging and Reynolds averaging. It was first introduced by Bauer et al. [34] and further studied by Ha Minh et al. [35,36]. In this formulation, kinetic energy is seen as the product of the velocity and the density weighted velocity. In a third method, kinetic energy is decomposed using a change of variable based on the density square root weighted velocity. This decomposition was first proposed by Yih [37] then adopted by various authors [38–42]. This change of variable allows the study of kinetic energy to be extended easily to the spectral domain. Finally, Chassaing [43] [see also 44–46,25] suggested the decomposition of kinetic energy using the Reynolds averaging. From a modeling perspective, the use of the unweighted averaging may be advantageous in low Mach number flows, in which the energy conservation acts as a constraint on the divergence of the velocity [47]. The square of the fluctuating velocity (without the density) is also encountered for instance in the modeling of two-phase flows [48] or in variable density flows, provided the momentum equation is divided by the density before averaging [49]. In a variable density setting, the use of the Reynolds averaging necessarily leads to the decomposition of kinetic energy into three parts, called

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ternary decomposition. The kinetic energy is thus split into turbulence kinetic energy, mean kinetic energy and a mixed kinetic energy, related to both the mean and turbulent motion. However, we believe the underlying idea behind the ternary decomposition has not been taken to its logical conclusion as no interaction between the mixed kinetic energy and another part of total energy was identified.

This paper aims to establish a new formulation of the energy exchanges between the different parts of total energy in a ternary decomposition that gives to the mixed kinetic energy a full role. The formulation is compared to the formulation of Chassaing [43] and the differences between the two formulations with regard to the physical interpretation of the terms are discussed. We then take the decomposition further and split the density into a mean and fluctuating part. This leads to the definition of the mean density part of total energy and the fluctuating density part of total energy. The mean density turbulence kinetic energy, product of the mean density and the half-trace of the velocity correlation tensor, appears in the mean density part of the decomposition as exchanging energy with the other parts of total energy. This quantity is approximately equal to the turbulence kinetic energy in flows satisfying Morkovin's hypothesis [50].

Once the new formulation of the energy exchanges established, we focus more specifically on the mean density turbulence kinetic energy. We establish its evolution equation in spectral domain, recognizing that the mean density turbulence kinetic energy has with the Reynolds averaging a clear spectral equivalent. The spectral equation extends the spatial equation to the spectral domain, associating to each spatial term a spectral equivalent. To the knowledge of the authors, this has not been achieved in the literature for variable density flows. A purely spectral term that redistributes the energy between scales is identified, as in the work of Lee and Moser [21] and [22]. In order to carry out the Fourier transform, we consider a flow with two homogeneous and periodic directions. This does not lead to a loss of generality as the equations given may easily be adapted to a flow with one or three homogeneous directions.

The complete representation of the energy exchanges between the different parts of total energy is presented in section 2 and the equation of the mean density turbulence kinetic energy in the spectral domain in section 3.

2. Energy exchanges between the different parts of total energy in the ternary decomposition

2.1. General considerations

In this section, we describe a new formulation of the energy exchanges between the different parts of total energy in a ternary decomposition. We will establish the formulation obtained from the decomposition of velocity, but not density, with the Reynolds averaging, referred to as the one-stage formulation in this paper, and from the decomposition of both the velocity and density, referred to as the two-stage formulation in this paper. The two-stage formulation is required to write the spectral equation of the mean density turbulence kinetic energy. We first define here a few useful quantities and give some general remarks on the derivation of the formulation.

The total energy per unit volume $\rho(E+I)$ is a conservative quantity. Its components however are not as they exchange energy among themselves. In the following, the evolution equation of each part of total energy in the ternary representation will be given and we will identify the energy exchanges between these quantities. Many consistent formulations of the energy exchanges can be proposed. The formulation was devised according to the following criteria:

- Each term of the formulation must be either interpreted as a conservative energy transfer or an interaction with exactly one of the other parts of total energy.
- If a term is to be interpreted as a conservative energy transfer, it must be written in a conservative form, that is as a divergence; otherwise, it must be written in a non-conservative form.
- The formulation must be symmetrical, in particular with respect to the manner in which it deals with fluctuations and statistically averaged quantities.
- The formulation must correctly behave when considering a limit case such as laminar, homogeneous or incompressible flows. In particular, a quantity that becomes instantaneously equal to zero must not be associated with any energy exchange.

We consider a non-relativistic compressible flow with highly variable fluid properties under the continuity hypothesis. Without loss of generality, no body forces are taken into account which means gravity is neglected and there is no heat source. The flow is governed by the Navier–Stokes equations under the following form [51]:

- mass conservation

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho U_j}{\partial x_j} = 0, \tag{1}$$

- momentum conservation

$$\frac{\partial \rho U_i}{\partial t} + \frac{\partial \rho U_j U_i}{\partial x_j} = \frac{\partial \Upsilon_{ij}}{\partial x_j}, \tag{2}$$

- energy conservation

$$\frac{\partial \rho I}{\partial t} + \frac{\partial \rho U_j I}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\lambda \frac{\partial T}{\partial x_j}\right) + \Upsilon_{ij}\frac{\partial U_i}{\partial x_j}, \tag{3}$$

with ρ the density, T the temperature, I the internal energy per unit mass, t the time, U_i the i-th component of the velocity, Υ_{ij} the component of the total stress tensor with the i and j indices and x_i the Cartesian coordinate in the i-th direction. Einstein summation convention is used. The total stress tensor Υ_{ij} is given by the contributions of the viscous shear stress tensor and of the pressure stress. We will keep the total stress tensor undissociated throughout this paper because the pressure and viscous contributions are formally similar.

2.2. One-stage formulation

The instantaneous total energy per unit volume $\rho(E+I)$ is the sum of the instantaneous kinetic energy per unit volume ρE and the internal energy per unit volume ρI. In the ternary decomposition, the kinetic energy is decomposed into three parts by splitting the velocity into a mean and fluctuating part [following 52], namely $U_i = \overline{U}_i + u'_i$, where the overline ($\overline{}$) denotes the statistical average and the prime symbol ($'$) the fluctuating part. We use a lowercase u' for the velocity fluctuation for a better visual differentiation but there is no further underlying differences. We obtain [43]

$$\rho E = \frac{1}{2}\rho U_i U_i = \rho \underline{E} + \rho e + \rho \underline{e}, \tag{4}$$

with $\rho \underline{E} = \frac{1}{2}\rho \overline{U}_i\,\overline{U}_i$ the mean kinetic energy, associated with the mean motion, $\rho e = \frac{1}{2}\rho u'_i u'_i$ the turbulence kinetic energy, associated with the turbulent motion, and $\rho \underline{e} = \rho u'_i \overline{U}_i$ the mixed kinetic