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Scattering characteristics of relativistically moving concentrically layered spheres

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ABSTRACT

The energy extinction cross section of a concentrically layered sphere varies with velocity as the Doppler shift moves the spectral content of the incident signal in the sphere's co-moving inertial reference frame toward or away from resonances of the sphere. Computations for hollow gold nanospheres show that the energy extinction cross section is high when the Doppler shift moves the incident signal's spectral content in the co-moving frame near the wavelength of the sphere's localized surface plasmon resonance. The energy extinction cross section of a three-layer sphere consisting of an olivine-silicate core surrounded by a porous and a magnetite layer, which is used to explain extinction caused by interstellar dust, also depends strongly on velocity. For this sphere, computations show that the energy extinction cross section is high when the Doppler shift moves the spectral content of the incident signal near either of olivine-silicate's two localized surface phonon resonances at 9.7 μm and 18 μm.

1. Introduction

Researchers have investigated planewave scattering by electrically small objects in uniform translatory motion at relativistic speeds. Typically, the investigations are done with a frame-hopping technique in which two inertial reference frames, one affixed to the laboratory and the other to the moving object, are used [1–3]. The frequency-domain constitutive relations of the material of the object as well as the object's dimensions are specified in the co-moving frame. The electrically small object is modeled as an assembly of two co-located dipoles, one electric and the other magnetic. The Lorentz transformation is used to transform the incident and the scattered fields from one reference frame to the other.

An object may be electrically small only if the wavelength of the incident plane wave in the co-moving frame exceeds a threshold value [4]. At shorter wavelengths, the same object may even be electrically large. Due to the Doppler effect, an object that is electrically small in the laboratory frame when at rest may be electrically large in its co-moving frame when it is in motion. Furthermore, with modern instrumentation, spectroscopic information about an object is often obtained by illuminating it with a pulse. A pulse has finite duration and can have a large bandwidth, in contrast to a plane wave which is supposed to be eternal and monochromatic. Therefore, the time-domain scattering response of any object, moving or not, is desirable.

In this Letter, we use the frame-hopping technique to investigate the effects of uniform translational motion on the energy extinction, energy absorption, and total energy scattering cross sections [5] of inhomogeneous objects. We chose concentrically layered spheres as suitable examples because an analytical solution for planewave scattering by these objects exists [6,7] in the co-moving frame, thereby avoiding inaccuracies arising from the implementation of numerical techniques such as the finite-difference time-domain method [8,9]. We examined the energy cross sections of a hollow gold sphere as functions of velocity and inner diameter, the outer diameter being fixed. We also computed the energy cross sections as functions of velocity of a three-layer sphere proposed by Voshchinnikov et al. [10] to explain observations of infrared extinction due to interstellar dust. Energy cross sections are appropriate for pulse scattering just as power cross sections [11, Sec. 3.4] are for planewave scattering.

2. Method

We used the frame-hopping technique [12] to compute the electric and magnetic fields of the scattered signals arising from the illumination of a uniformly translating concentrically layered sphere by a signal of finite duration. The technique has four steps: (i) define the electric and magnetic fields of the incident signal

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in the laboratory inertial reference frame K', (ii) use the Lorentz transformation to express the fields of the incident signal in the co-moving inertial reference frame K, (iii) compute the electric and magnetic fields of the scattered signals in K, and (iv) transform the fields of the scattered signals to K' with the inverse of the Lorentz transformation used in Step (ii).

The incident signal was chosen to be a plane wave with its amplitude modulated by a Gaussian pulse. The signal travels in the $+z'$ direction with its electric field in the x' direction. The electric and magnetic fields of the incident signal in K' are

$$\mathcal{E}'_{inc}\left(\mathbf{r}', t'\right) = \hat{\mathbf{x}}' \cos(\omega'_c \tau') \exp\left(-\frac{\tau'^2}{2\sigma'^2}\right), \tag{1a}$$

$$\mathcal{B}'_{inc}\left(\mathbf{r}', t'\right) = \frac{\hat{\mathbf{z}}' \times \mathcal{E}'_{inc}\left(\mathbf{r}', t'\right)}{c}, \tag{1b}$$

where $\mathbf{r}'$ is the position vector, t' is time, $\tau' = t' - (\hat{\mathbf{z}}' \cdot \mathbf{r}')/c$, σ' is the width parameter of the Gaussian pulse, ω'_c is the angular carrier frequency, c is the speed of light in free space, and every unit vector is decorated by a caret.

We converted the electromagnetic fields of the incident signal to the frequency domain by using a discrete Fourier transform and computed the scattered field phasors using an analytical technique [6,7] based on the Lorenz–Mie series solution [11] in Step (ii). Details of the transformations between time and frequency domains are available elsewhere [5,12]. As with a solid, homogeneous sphere, the power scattering, power absorption, and power extinction cross sections in K were computed using the coefficients of the scattered field phasors in the Lorenz–Mie solution [11, Eqs. (4.61, 62)]. To find these coefficients, we used (i) expressions from Bohren and Huffman [11, Eq. (8.2)] for hollow gold spheres and (ii) expressions from Shore [13] for three-layer spheres. The energy absorption and energy extinction cross sections were determined from the power absorption and power extinction cross sections [5, Eqs. (34), (40)], respectively. We computed the total energy scattering cross section by determining the scattered signals in all scattering directions and numerically integrating the scattered power with respect to time to get the scattered energy density (with units of energy per solid angle) in all directions. The total scattered energy was calculated by numerically integrating the scattered energy density using 41-point Gauss–Kronrod quadrature [14], [15, pp. 153–155] over θ' and 32-point rectangular integration over ϕ' [15], as described elsewhere [5, Sec. IIA].

We used measured, wavelength-dependent constitutive parameters for bulk gold [16], olivine silicate ($MgFeSiO_4$) [17], and magnetite (Fe_3O_4) [18]. The parameters for gold were obtained from the online database at http://www.refractiveindex.info and those for olivine silicate and magnetite from the University of Jena website, http://www.astro.uni-jena.de/Laboratory/OCDB/index.html.

The *normalized* total energy scattering, absorption, and extinction cross sections in K' are defined as [5]

$$Q'_{\text{sca}} = \frac{W'_{\text{sca}}}{U'_{inc} A}, \tag{2a}$$

$$Q'_{\text{abs}} = \frac{W'_{\text{abs}}}{U'_{inc} A}, \tag{2b}$$

$$Q'_{\text{ext}} = \frac{W'_{\text{ext}}}{U'_{inc} A}, \tag{2c}$$

where U'_{inc} is the energy density of the incident signal with units of energy per area; A is the physical cross-sectional area of the sphere in K'; and W'_{sca}, W'_{abs}, and W'_{ext} are the total energies scattered by the object, absorbed by the object, and removed from the incident signal by the object, respectively. Analogous normalized energy cross sections may also be defined in K.

We also define the standard normalized power extinction cross section in K as [5]

$$\tilde{Q}_{\text{ext}}(\lambda) = \frac{\tilde{P}_{\text{ext}}(\lambda)}{\tilde{U}_{inc}(\lambda) A}, \tag{3}$$

where $\tilde{P}_{\text{ext}}(\lambda)$ is the time-averaged power removed from a plane wave of free-space wavelength λ and $\tilde{U}_{inc}(\lambda)$ is the power density of the incident plane wave.

3. Numerical results

3.1. Hollow gold nanophere

Fig. 1 shows the normalized energy extinction, energy absorption, and total energy scattering cross sections of a hollow gold sphere as functions of velocity and inner diameter (in K), when the outer diameter in K is fixed at 50 nm. In K', the carrier wavelength of the incident signal is $\lambda'_c = 550$ nm, and the width parameter of the Gaussian function is $\sigma' = 1.833$ fs.

All three cross sections, Q'_{ext}, Q'_{abs}, and Q'_{sca}, are high when the sphere advances toward the source of the incident signal at speeds approaching c. This occurs because the Doppler shift increases the electrical size of the sphere in K as the sphere advances toward the source at speeds approaching c. Q'_{ext}, Q'_{abs}, and Q'_{sca} are near zero when the sphere recedes from the source at speeds approaching c, because the electrical size of the sphere in K goes to zero. Q'_{ext} and Q'_{abs} also have high values when the inner diameter ranges from about 34 to 48 nm and the sphere recedes from the source of the incident signal at velocities ranging from $\mathbf{0}$ to $0.3c\hat{\mathbf{z}}'$. The cause of this high-Q'_{ext} regime may be understood by considering the normalized power extinction cross section $\tilde{Q}_{\text{ext}}(\lambda)$ in K, as shown in Fig. 2. As the inner diameter increases, the strength of the localized surface plasmon resonance [19, Sec. 2.4.1] increases and its wavelength shifts from about 515 nm to about 955 nm. Q'_{ext} has a maximum when the Doppler shift causes the spectral content of the incident signal in K to coincide with the localized surface plasmon resonance.

Bulk permittivity becomes inapplicable to a metal shell whose thickness h is less than the mean free path of electrons in the bulk metal. Therefore, we also computed the normalized energy cross sections of a hollow gold sphere using a modified permittivity function for small spheres given by Kreibig and Fragstein [20]. In a particle whose linear dimensions are on the order of the mean free path, free electrons may collide with the edges as well as the lattice defects in the material [19, pp. 78–85]. The collisions with the boundary decrease the mean free path and increase the collision frequency in the Drude model of the permittivity.

We assumed that the frequency-dependent relative permittivity of bulk gold given by Hagemann et al. [16] is a combination of a Drude component [22, Eq. (2)] and a bound-electron component; thus,

$$\begin{aligned}\epsilon_{\text{bulk}}(\omega) &= \epsilon_{\text{bound}}(\omega) + \epsilon_{\text{Drude,bulk}}(\omega) \\ &= \epsilon_{\text{bound}}(\omega) - \frac{\omega_p^2}{\omega^2 + i\omega\gamma_{\text{bulk}}}\end{aligned} \tag{4}$$

with angular plasma frequency $\omega_p = 1.36 \times 10^{16}$ rad s^{-1} and angular collision frequency $\gamma_{\text{bulk}} = 4.07 \times 10^{13}$ rad s^{-1} [22, Table 1]. In a solid, homogeneous sphere of radius R, the collision frequency becomes [20,21]

$$\gamma(R) = \gamma_{\text{bulk}} + \frac{v_F}{R}, \tag{5}$$