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Non linear analysis of obliquely propagating spin electron acoustic wave in a partially spin polarized degenerate plasma

Z. Iqbal, G. Murtaza

Salam Chair, Department of Physics, G. C. University Lahore, Katchery Road, Lahore 54000, Pakistan

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ABSTRACT

By employing the separated spin evolution quantum hydrodynamic model, non-linear evolution of obliquely propagating spin electron acoustic wave (SEAW) is presented. The solitary structures of SEAW is investigated through the Korteweg–de Vries (KdV) equation derived using reductive perturbation method. From the first order perturbations we derive the dispersion relation of SEAW and find that both the spin polarization and the propagation angle reduce the phase velocity while the electron streaming enhances it. Using small amplitude approximation, the solitary structure of SEAW is analyzed and the effects of spin polarization, propagation angle and electron streaming on the SEA soliton are studied. Our numerical results demonstrate that the spin polarization and the propagation angle play a balancing act on the soliton structures. The possible applications of our investigation to the astrophysical environments like white dwarfs is also discussed.

1. Introduction

The acoustic waves like the electron acoustic and the ion acoustic waves are the fundamental modes of plasma. Linear and non linear properties of both the electron acoustic and the ion acoustic waves have been extensively studied in classical electron–ion plasma [1–8]. Washimi and Taniuti [9] showed that such waves, in a weakly nonlinear regime, can be mathematically modeled by the well known Korteweg–de Vries (KdV) equations. The study of these waves also gained its importance in quantum plasmas in order to understand the electrostatic wave propagation at the microscopic level. In this regard, the quantum hydrodynamic (QHD) model has been used to investigate the quantum effects on the linear and nonlinear properties of the ion acoustic wave in an unmagnetized and magnetized electron–ion plasma [10–12] and was found that the quantum effects significantly modify the linear and non linear properties of this wave. Linear and non linear propagation of ion acoustic wave has also been studied for electron–positron–ion (e–i–p) plasma in the Refs. [13–17].

Recently, another electron spin dependent acoustic type of mode called spin electron acoustic wave (SEAW) has been reported in Ref. [26] by considering the separate spin evolution (SSE) of spin-up and spin-down electrons in a degenerate magnetized plasma. It is demonstrated that in the presence of ambient magnetic field, the equilibrium concentration of spin-up and spin-down electrons are different ($n_u \neq n_d$) which in turn is responsible for the difference of Fermi pressures of the spin-up and spin-down electrons. This difference of Fermi pressures gives birth to SEAW. Separate spin evolution-quantum hydrodynamic model (SSE-QHD) is an extension of QHD model which was presented in [18–20]. The spin-1/2 quantum plasmas in which the spin properties of electrons are taken into account, have received great attention due to the occurrence of new wave phenomena. [21–28] Subsequently, the quantum kinetics for spin-1/2 quantum plasmas has been developed and applied to study the dispersion properties of various plasma waves. [23,29–34] A pair of SEAW and positron acoustic wave was found by considering oblique propagation of longitudinal waves with SSE-QHD equations in electron–ion (e–i) and electron–positron–ion (e–p–i) plasmas [27,28]. This novel type of spin dependent wave has also been found by applying the SSE-QHD equations to the two dimensional electron gas in plane samples and nanotubes embedded in an external magnetic field in Refs. [35]. SEAWs have been studied on the surface degenerate spin polarized electron gas of magnetically ordered materials and their linear interaction with the surface Langmuir wave has been investigated [37]. SEAW also explains the mechanism of high temperature superconductivity. It has been reported that the quanta of SEAW (spelnon) interact with electrons which gives a mechanism of the Cooper pairs formation [38] which in turn explains the high temperature superconductivity [38]. Existence of the extraordinary spin electron acoustic wave (SEAW) demonstrated has been

E-mail address: abbasiravian@yahoo.com (Z. Iqbal).

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demonstrated in Ref. [39]. It is found that shifting in the spectrum of extraordinary SEAW is due to the variation in ratio of cyclotron frequency to Langmuir frequency.

The above mentioned applications show that electron acoustic and ion acoustic waves are well studied in both the classical and quantum plasmas but in the present work we focus on the new spin dependent acoustic mode. From above cited literature about SEAW one may notice that most of the properties and applications of SEAW have been studied in the linear regime. Only a little attention has been paid in Refs. [40,41] to study the non linear properties of SEAW but these authors considered only parallel propagation of non linear SEAW. To the best of our knowledge, no one has investigated the obliquely propagating non linear structures of SEAW. The above mentioned interesting features of SSE-QHD model motivate us to generalize our calculations in magnetized degenerate plasma for oblique propagation of electrostatic waves including streaming effects and study the soliton evolution of SEAW.

2. Governing equations

We study the obliquely propagating electrostatic waves in collisionless electron ion plasma embedded in an external magnetic field B_0 that is directed along the z direction and the wave propagates in the (x, z) plane. The ions are considered immobile and satisfy the neutrality condition. In order to govern the dynamics of electrons, we consider the SSE-QHD equations which were recently developed in [26,27]. In the SSE-QHD equations, under the action of external magnetic field each species with spin-up and spin-down is considered as independent fluid. Therefore, the continuity equation with spin projection of each species is presented as

$$\partial_t n_{es} + \nabla(n_{es}\mathbf{v}_{es}) = (-1)^{i_s} T_z, \tag{1}$$

where $s = u, d$ for the spin-up and spin-down conditions of particles, n_{es} and $\mathbf{v}_{es}$ are the concentration and velocity field of electrons being in the spin state s, $T_{ez} = \frac{\gamma_e}{\hbar}(B_x S_{ey} - B_y S_{ex})$ is the z-projection of spin torque, $|\gamma_e| = \mu_B$, μ_B is the Bohr magneton, i_s: $i_u = 2$, $i_d = 1$, with the spin density projections S_{ex} and S_{ey}, each of them simultaneously describe evolution of the spin-up and spin-down particles of each species. Therefore, the functions S_{ex} and S_{ey} do not bear subindexes u and d. In this model the z-projection of the spin density S_{ez} is not an independent variable, it is a combination of concentrations $S_{ez} = n_{eu} - n_{ed}$. The momentum equation for electrons is given as [26]

$$\begin{aligned} &m_e n_{es}(\partial_t + \mathbf{v}_{es}.\nabla)\mathbf{v}_{es} + \nabla P_{es} \\ &= -e n_{es}\left(\mathbf{E} + \frac{1}{c}[\mathbf{v}_{es}, \mathbf{B}]\right) + (-1)^{i_s}\gamma_e n_{es}\nabla B_z \\ &\quad + \frac{\gamma_e}{2}(S_{ex}\nabla B_x + S_{ey}\nabla B_y) + (-1)^{i_s} m(\widetilde{\mathbf{T}}_{ez} - \mathbf{v}_{es}T_{ez}), \end{aligned} \tag{2}$$

with $P_{es} = (6\pi^2)^{\frac{2}{3}} n_{es}^{\frac{5}{3}}\hbar^2/5m$, $\widetilde{\mathbf{T}}_{ez} = \frac{\gamma_a}{\hbar}(\mathbf{J}_{(M)ex}B_y - \mathbf{J}_{(M)ey}B_x)$, which is the torque current, where $\mathbf{J}_{(M)ex} = (\mathbf{v}_{eu} + \mathbf{v}_{ed})S_{ex}/2$, and $\mathbf{J}_{(M)ey} = (\mathbf{v}_{eu} + \mathbf{v}_{ed})S_{ey}/2$ are the convective parts of the spin current tensor. $\mathbf{E} = E_0 + E_1$ and $B = B_0 + B_1$, here E_0 and B_0 are equilibrium electric and magnetic fields and E_1 and B_1 are perturbed electric and magnetic fields. In this paper we consider oblique propagation of longitudinal waves such that wave vector $\mathbf{k}$ is parallel to electric field vector $\mathbf{E}$. Therefore the perturbation of magnetic field is zero ($B_1 = 0$). So the above Eqs. (1) and (2) can be written in a simplified form as

$$\partial_t n_{es} + \nabla.(n_{es}\mathbf{v}_{es}) = 0 \tag{3}$$

$$m_e n_{es}(\partial_t + \mathbf{v}_{es}.\nabla)\mathbf{v}_{es} + \nabla P_{es} = -e n_{es}\left(\mathbf{E} + \frac{1}{c}[\mathbf{v}_{es}, \mathbf{B}]\right), \tag{4}$$

and the Poisson equation can be written as

$$\nabla.\mathbf{E} = -4\pi e(n_{eu} + n_{ed}). \tag{5}$$

To investigate the nonlinear dynamics of the spin-1/2 quantum degenerate magnetized plasma with SSE, we employ the reductive perturbation method [9]. We study the oblique propagation of non linear longitudinal waves in $x - z$ plane. For oblique propagation of waves unit vector k in (x, z) plane requires $\widehat{k}.\overrightarrow{r} = x\sin\theta + z\cos\theta$. Therefore, the space coordinate x or z generally used in the case of one dimensional propagation should be replaced with $x\sin\theta + z\cos\theta$. We suppose the scaling of independent variables through the following stretched coordinates

$$\xi = \epsilon^{\frac{1}{2}}(x\sin\theta + z\cos\theta - ut), \tau = \epsilon^{\frac{3}{2}}t \tag{6}$$

where ϵ is a small parameter measuring the strength of perturbation or weakness of nonlinearity, u is phase velocity of waves. The perturbed quantities n_s, v_s, and ϕ are expanded about their equilibrium positions in the form of small parameter ϵ as follows:

$$n_s = n_{0s} + \epsilon n_{1s} + \epsilon^2 n_{1s} + \ldots. \tag{7}$$

$$v_{sx} = \epsilon^{3/2} v_{sx1} + \epsilon^2 v_{sx2} + \ldots. \tag{8}$$

$$v_{sy} = \epsilon^{3/2} v_{sy1} + \epsilon^2 v_{sy2} + \ldots. \tag{9}$$

$$v_{sz} = v_0 + \epsilon v_{sz1} + \epsilon^2 v_{sz2} + \ldots. \tag{10}$$

$$\phi = \epsilon\phi_1 + \epsilon^3\phi_2 + \ldots. \tag{11}$$

Here s is for spin up and spin down electrons and v_0 is the streaming velocity of electrons which is same for spin up and spin down electron. Since in the presence of external magnetic field, action of the Lorentz force in the perpendicular direction of magnetic field is different from that in the parallel direction. Therefore, in Eq. (8)–(11) we introduce anisotropy in the expansion of velocity fields in the direction transverse and parallel to the external magnetic field. By using $E = -\nabla\phi$ in the governing Eqs (3), (4) and (5) and substituting the Eqs. (7)–(11) into these equations and collecting terms of lowest order in ϵ from the continuity equations and equations of motion, we obtain the perturbed number densities and velocities of electrons in term of electric potential ϕ_1, for electrons

$$n_{1es} = \sum_{s=u,d} \frac{n_{0es}\cos\theta}{u - v_0} v_{z1} \tag{12}$$

$$v_{z1} = \sum_{s=u,d} \frac{(-u + v_0)\frac{e}{m}\cos\theta}{\left((-u + v_0)^2 - V_{Fes}^2\cos^2\theta\right)}\phi_1 \tag{13}$$

where $V_{Fu}^2 = 1/3v_{Fe}^2(1-\eta)^{\frac{2}{3}}$ and V_{Fd}^2 $1/3v_{Fe}^2(1+\eta)^{\frac{2}{3}}$, $\eta = 3\mu_B B_0/2\varepsilon_{F_e}$ where ε_{F_e} is the Fermi energy and

$$v_{Fe} = (3\pi^2 n_{0e})^{1/3}\hbar/m_e$$

is the Fermi velocity of electrons. Using Eqs. (14)–(16) in the Poisson equation in the first order of ϵ which is given as $n_{eu1} + n_{eu1} = 0$, we obtain dispersion relation of phase velocity of SEAW as

$$\begin{aligned} &\frac{(1-\eta)}{\left(\left(w - \sqrt{3}\frac{v_0}{v_{Fe}}\right)^2 - (1-\eta)^{\frac{2}{3}}\cos^2\theta\right)} \\ &+ \frac{(1+\eta)}{\left(\left(w - \sqrt{3}\frac{v_0}{v_{Fe}}\right)^2 - (1+\eta)^{\frac{2}{3}}\cos^2\theta\right)} = 0. \end{aligned} \tag{14}$$

In Eq. (14), we have defined the wave phase velocity u in term of electron Fermi velocity as $u^2 = w^2 v_{Fe}^2/3$ so that w represents the dimensionless phase velocity. Eq. (14) shows that the phase

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