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A dynamical approach to identify vertices' centrality in complex networks

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A B S T R A C T

In this paper, we proposed a dynamical approach to assess vertices' centrality according to the synchronization process of the Kuramoto model. In our approach, the vertices' dynamical centrality is calculated based on the Difference of vertices' Synchronization Abilities (DSA), which are different from traditional centrality measurements that are related to the topological properties. Through applying our approach to complex networks with a clear community structure, we have calculated all vertices' dynamical centrality and found that vertices at the end of weak links have higher dynamical centrality. Meanwhile, we analyzed the robustness and efficiency of our dynamical approach through testing the probabilities that some known vital vertices were recognized. Finally, we applied our dynamical approach to identify community due to its satisfactory performance in assessing overlapping vertices. Our present work provides a new perspective and tools to understand the crucial role of heterogeneity in revealing the interplay between the dynamics and structure of complex networks.

1. Introduction

Recent decades have witnessed a vigorous development of the network science, which is an interdisciplinary academic field to understand the behavior of natural, social and technical systems under the fundamental framework of complex networks [1]. Empirical analysis shows that many real complex networks, where vertices represent the elementary units of a given system and links describe the interactions between units [2,3], exhibit some nontrivial properties, such as the heterogeneous nature of vertices (e.g., the power-law distribution of vertex's connectivity) and the community structure (also called clustering or module). Those properties demonstrate that heterogeneity means difference, which indicates that to identify the vital vertices has its remarkable role in analyzing the structure and dynamics of complex networks [4]. For example, in complex network with a clear community structure, vital vertices with higher betweenness centrality, which are called the overlapping vertices, belong to more than one community commonly. To identify the overlapping vertices effectively benefits the community detection [5–8]. Meanwhile, the influential vertices can be quantified by various indexes. Take degree centrality for example, vertices with larger degree have an ability to influence more other vertices. To monitor those influential vertices is helpful for the prediction and control of spreading dynamics [9,10].

The vital vertices, which can be identified using the concept of centrality, are largely affected and reflected by the topological structure and dynamical pattern of the network to which they belong. A tremendous number of methods for centrality have been proposed and well studied mainly based on the local or global topological structure [4,11], such as the degree centrality, K-core decomposition, betweenness centrality and eigenvector centrality. And those methods neglect the critical role of dynamical processes in identifying vertices' importance. Although some dynamics-based centrality has been studied [4,9,12–14], to identify the vital vertices effectively and efficiently from the dynamical perspective also remains a big challenge. Since the structure and dynamics are tightly coupled in complex networks, dynamics is fundamental in assessing the impact of vertices in global performance [15–18]. For example, the Kuramoto model shares different synchronization behaviors on the homogeneous and heterogeneous complex networks [19].

As is well known, synchronization is a collective phenomenon occurring in systems of mutual interaction between units and

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is ubiquitous in nature, society and technology [20]. And the emergence of synchronization patterns in these systems has been shown to be closely related to the underlying topology of interactions [21–23]. For example, community (a set of oscillators which are placed at the vertices) with highly density of interactions synchronizes more easily than that with sparse connections, which leads to the application of the Kuramoto model in detecting community and overlapping in complex networks [22–24]. In this paper, we focus on the analysis of the difference of the local synchronization paces and propose a dynamical approach to identify the importance of vertices in a complex network. The fundamental idea of our dynamical approach is that vertices' centrality is calculated based on the DSA. We analyze the robustness and efficiency of our dynamical approach through applying it to identify community in the PPIN, the DSN and the benchmark network. We find that our approach has powerful advantages of timely, robustness and efficiency to identify vital vertices, for instance the overlapping community, in complex networks.

2. Methodology and analysis

One of the simple paradigm to understand the synchronization phenomenon is the Kuramoto model [25,26], which has been applied to study the synchronization patterns and to identify community in complex networks. Consider a network of N vertices joined in pairs by E links, which represent the interaction between vertices, for example, the acquaintance or collaborations between individual in social networks. In graph description, the network can be represented by means of a $N \times N$ connectivity matrix A, where $A_{ij} = 1$ when vertices i and j are linked, and $A_{ij} = 0$, otherwise. Each vertex, says i, is encoded a phase oscillator with the natural frequency ω_i and phase θ_i. The phase θ_i evolves in time according to the Kuramoto model:

$$\frac{d\theta_i(t)}{dt} = \omega_i + \frac{\lambda}{k_i} \sum_{j\in\Gamma_i} sin(\theta_j(t) - \theta_i(t)) \tag{1}$$

where, Γ_i is the subgraph of vertex i's nearest neighbors, k_i is the connectivity of vertex i and λ is the positive coupling strength between vertices. Note that vertices are represented by their corresponding oscillators in the following part of our paper. Many previous literatures show that there exists a critical coupling λ_c, above which synchronization emerges spontaneously [25,26]. We here realize the Kuramoto model on complex networks using the Runge–Kutta methods. Without lack of generality, the coupling strength $\lambda = 2(>\lambda_c)$ is fixed, and the natural frequencies ω_is and the initial phase θ_is are chosen from a uniform distribution $g(x)$ with mean $\langle x\rangle = 0$ in the interval $(-0.5, 0.5)$ and $(-\pi, \pi)$, respectively.

The macroscopic complex order parameter, which describes the synchronized behavior of the whole system, is defined as:

$$R(t)e^{i\Phi(t)} = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j(t)} \tag{2}$$

where $R(t)$ $(0 \le R(t) \le 1)$ quantifies the extent of synchronization in a system of N oscillators, and $\Phi(t)$ is the average phase of the system at time t. The larger the $R(t)$ is, the more oscillators tend to synchronize to a common phase. In the special case of $R(t) = 1$, all oscillators share the same phase and the system reaches the coherence state. However, $R(t)$ does not give any further information about the synchronization behavior in terms of local cluster, which plays a crucial importance to understand the structural and dynamical role of heterogeneity in complex networks. For this reason,

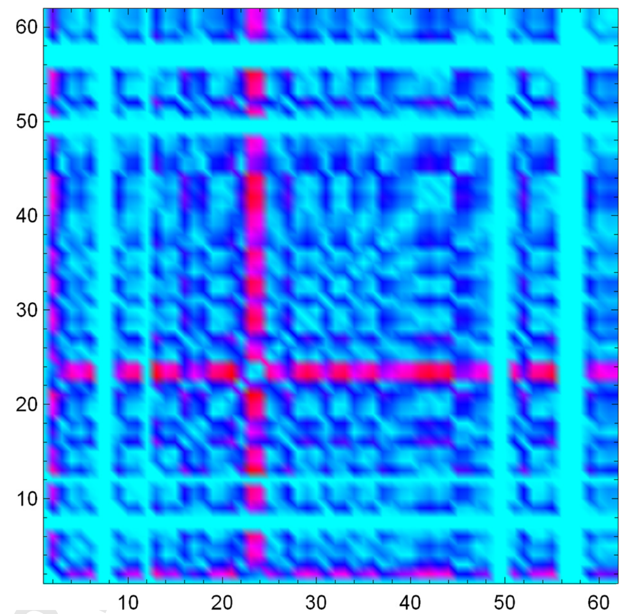

Fig. 1. (Color online.) The re-scaled DSA between pairs of oscillators in the DSN at the dynamical time $t = 300$. The colors are a gradation between cyan (0) and the red (1).

instead of considering the global order parameter, the real-valued local order parameter $r_j(t)$ at time t is defined as follows:

$$r_j(t)e^{i\Phi_j(t)} = \frac{1}{k_j+1}\sum_{l\in\Gamma'_j} e^{i\theta_l(t)} \tag{3}$$

where, Γ'_j is a subgraph including oscillator j and its nearest neighbors, $\Phi_j(t)$ describes the average phase of oscillators in the given subgraph Γ'_j, and $r_j(t)$ describes the extent of synchronization of the set of oscillators surrounding oscillator j at time t. Further, the local order parameter $r_j(t)$, which represents the dynamical function of oscillator j, can be used to quantify the synchronization ability of oscillator j at time t. Take a network with a clear community structure for example, oscillators belonging to the same community tend to share the same synchronization ability, and not necessarily equal for all communities [23]. While some oscillators often belong to more than one community and can be called the overlapping oscillators. Those overlapping oscillators will have a weak synchronization ability due to the limited from different communities. We here focus on the DSA among oscillators and do not consider the similar correlation between oscillators [22].

The DSA between oscillators, says i and j, is written as,

$$\Delta r_{ij}(t) = |r_i(t) - r_j(t)| \tag{4}$$

And we operate a simple algebraic calculation to re-scale all those $\Delta r_{ij}(t)$ to fall in the range of $[0, 1]$,

$$\Delta' r_{ij}(t) = \frac{r_{ij}(t) - r_{min}(t)}{r_{max}(t) - r_{min}(t)} \tag{5}$$

where, $r_{min}(t) = Min\{r_{ij}(t), \forall i, j\}$ and $r_{max}(t) = Max\{r_{ij}(t), \forall i, j\}$. Take a complex network with clear community structure for example, it is obvious that oscillators, says i and j, in the same community have the similar synchronized ability with the smaller $\Delta' r_{ij}(t)$ at time t.

In Fig. 1, we represent $\Delta' r_{ij}(t)$ at $t = 300$ for the DSN [27]. We can identify that the oscillators with No. 22 and 23 have a large gap about the re-scaled DSA with other oscillators, which clearly indicates that the two oscillators have a large probability to be

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