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Matter-wave coherence limit owing to cosmic gravitational wave background

Andrew M. Steane

Department of Atomic and Laser Physics, Clarendon Laboratory, Parks Road, Oxford OX1 3PU, England, United Kingdom

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ABSTRACT

We study matter-wave interferometry in the presence of a stochastic background of gravitational waves. It is shown that if the background has a scale-invariant spectrum over a wide bandwidth (which is expected in a class of inflationary models of Big Bang cosmology), then separated-path interference cannot be observed for a lump of matter of size above a limit which is very insensitive to the strength and bandwidth of the fluctuations, unless the interferometer is servo-controlled or otherwise protected. For ordinary solid matter this limit is of order 1–10 mm. A servo-controlled or cross-correlated device would also exhibit limits to the observation of macroscopic interference, which we estimate for ordinary matter moving at speeds small compared to c.

1. Introduction

In a series of papers, Lamine et al. [1,2], have investigated the effect of a stochastic background of weak gravitational waves on matter-wave interferometers. We use their treatment to obtain the following remarkable observation. If the early universe generated gravitational waves with a scale-invariant spectrum over a wide bandwidth (which is expected in a class of inflationary models of Big Bang cosmology), we show that there is a near-universal cut-off distance scale for the observation of matter wave interference. That is, unless the interferometer (as in Fig. 1a) is shielded or otherwise protected from these primordial waves, the fringe visibility will go rapidly to zero when the radius R of the interfering object exceeds a value which turns out to be of the order of a few millimetres for ordinary solid matter. This is not an absolutely unavoidable decoherence, only a technological difficulty, so it is not possible to settle the quantum measurement problem by this route. However, we also find that cross-correlation or servo-control will not suffice to avoid the decoherence altogether, but merely extend the size or mass limit by an amount which depends weakly on the integration time. To avoid this source of decoherence altogether, it would suffice to shield the interferometer from the fluctuating gravitational background, but this is very hard to do.

2. Calculation

We are interested in a generic interferometer, but for the sake of making precise calculations we consider a Mach–Zehnder interferometer in which de Broglie waves associated with a sphere of proper radius R interfere, and in which the separation of the arms is $2R$ (see Fig. 1a). For such an interferometer, both the interfering object and the separation of the interfering paths becomes of macroscopic size when R is large enough.

Our treatment follows that of Lamine et al. [1–3]. In the presence of a stochastic gravitational background with a spectral density of strain fluctuations $S_h(\omega)$, the variance of the phase of a matter-wave interferometer of the type shown in Fig. 1 is

$$\Delta\phi^2 = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} S_h(\omega) A(\omega) f(\omega) \tag{1}$$

where $A(\omega)$ is the response of the interferometer to a wave of given frequency, and $f(\omega)$ is a high pass filter function. For the case where the sphere's group velocity $v \ll c$, and the wavelength of the gravitational wave is large compared to the interferometer, $kR \ll 1$, one finds [2]

$$A(\omega) = \left(\frac{4mv^2}{\hbar\omega}\right)^2 \sin^2(2\alpha) \sin^4\left(\frac{\omega\tau}{2}\right) \tag{2}$$

where $\tau = R/(v \sin\alpha)$ is the time taken to traverse half of one arm of the interferometer.

E-mail address: a.steane@physics.ox.ac.uk.

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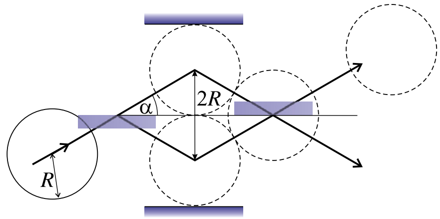

Fig. 1. A sphere of radius R passes through a two-path interferometer whose path separation is $2R$. In practice the mirrors and beam-splitters might, for example, be provided by magnetic or optical forces.

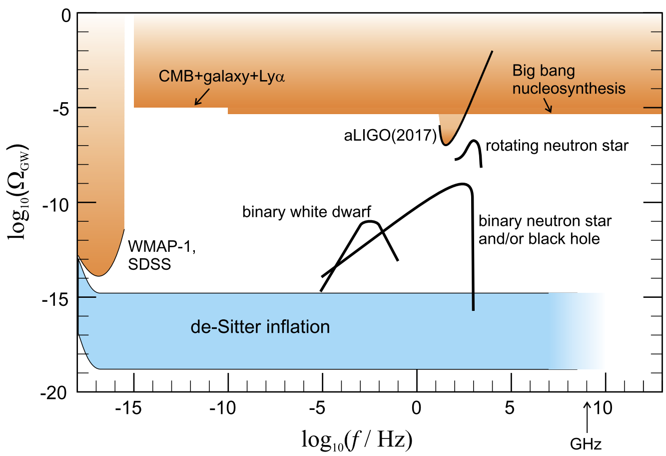

Fig. 2. Gravitational wave sources. The upper shaded regions are ruled out by various studies [4–11]; the lower shaded region shows, approximately, the range and distribution of primordial cosmological waves predicted by one type of inflationary scenario. The curves show the spectra of the main 'ordinary' sources predicted by standard physics and astronomical surveys.

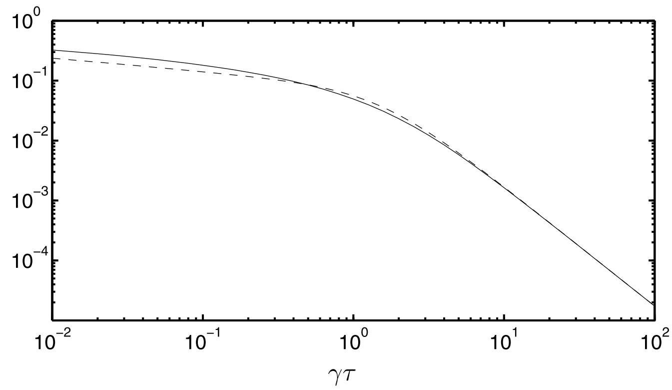

Fig. 3. Full curve: integral in Eq. (3) for $\omega_c\tau \gg \pi$; dashed curve: the function $\sqrt{3}(1+20(\gamma\tau)^{1/4}+10\gamma^2\tau^2)^{-1}$.

The filter function represents the fact that low frequency 'noise' is not noise but signal—if the fringes move slowly enough then their movement can be tracked by the interference experiment. We adopt the filter function $f(\omega)=\omega^2/(\omega^2+\gamma^2)$ where γ is the bandwidth. We will discuss this bandwidth after obtaining an expression for $\Delta\phi^2$ in the presence of stochastic cosmological gravity waves.

Sources of gravitational waves include various 'ordinary' processes such as neutron star binaries, black hole binaries, and rotating neutron stars, and 'exotic' processes that are not yet well established but which are postulated in theoretical descriptions of early universe physics (Fig. 2) [6,5,9,7,8]. We suppose the waves from ordinary sources represent a signal which can be discriminated by its temporal and other signatures, and therefore the noise is only given by the exotic sources. This is outside the present range of confident knowledge, but inflationary models of Big Bang cosmology suggest that there is now, throughout the universe, a low-level gravitational noise of very wide bandwidth. Observations of the cosmic microwave background can in principle detect this gravitational noise at very low frequencies, through the Sachs–Wolfe effect [12,13]. This is not the frequency range we are interested in, but it is useful because it is by far the most sensitive existing experimental observation. The WMAP and Planck measurements currently place an upper bound on the low-frequency gravitational noise close to the level at which inflationary models suggest it is present, thus it remains undetected but the models remain viable [8].

The cosmological gravitational wave spectrum is usually described in terms of the measure $\Omega_{\rm GW}=\nu\tilde{\rho}/(\rho_0 c^2)$, where ρ_0 is the mass density that would close the universe and $\tilde{\rho}$ is the spectral energy density of the gravitational radiation (that is, the energy density per unit frequency range $\Delta\nu$). $\Omega_{\rm GW}$ is dimensionless, and is related to $S_h(\omega)$ by $S_h=3H_0^2\Omega_{\rm GW}/\omega^3$ where $H_0\simeq 2.4\times10^{-18}$ s^{-1} is the Hubble parameter [14]. The important point for our discussion is that inflationary models suggest that $\Omega_{\rm GW}$ is *independent of* ω *over a wide bandwidth*—a property called *scale invariance*. The bandwidth is normally reported in the range 10^7–10^9 Hz. We will model this by adopting a simple cut-off at a frequency ω_c. Hence Eq. (1) reads

$$\Delta\phi^2 = \int_0^{\omega_c} \frac{d\omega}{2\pi}\frac{3H_0^2\Omega_{\rm GW}}{\omega^3}A(\omega)\frac{\omega^2}{\omega^2+\gamma^2}$$

$$= \frac{24H_0^2\Omega_c}{\pi\hbar^2}m^2(\nu\tau)^4\int_0^{\omega_c\tau}\frac{\sin^4(x/2)}{x^3(x^2+\gamma^2\tau^2)}dx \qquad (3)$$

The integrand in (3) falls quickly once $x>2\pi$, with the result that the integral is almost independent of ω_c for $\omega_c\tau>2\pi$. It is then well approximated (except at very low $\gamma\tau$) by the expression

$$\Delta\phi^2 \simeq \frac{24\sqrt{3}H_0^2\Omega_{\rm GW}}{\pi\hbar^2}\frac{m^2(\nu\tau)^4}{1+20(\gamma\tau)^{1/4}+10(\gamma\tau)^n} \qquad (4)$$

where $n=2$ (see Fig. 3). The parameter n allows us to consider the effect of different filter functions. For example, a perfect high-pass filter with a sharp cut-off at $\omega=\gamma$ results in a response given approximately by Eq. (4) with $n=4$.

The above calculation has assumed a point-like model of the interfering sphere, and treats the case where the gravitational wavelength is large compared to the interferometer, i.e. $\omega_c R \ll c$ which implies $R \ll 0.05$ m when $\omega_c = 2\pi\times10^9$ s^{-1}. In order to allow for the extended nature of the interfering sphere, we must consider the fact that, for any given arm of the interferometer, different parts of the sphere may experience different amounts of proper time. Consequently the coordinate associated with the finally measured position of the sphere may become entangled with internal degrees of freedom, leading to decoherence when the latter are averaged over [15]. A symmetric 'bow-tie' design for the interferometer can cancel the static part of this effect, but for a randomly fluctuating background, as here, the result is a loss of coherence, and it is one that cannot be avoided by acquiring data quickly so as to track the fluctuations. Consequently, a reasonable rough model of this is to extend the validity of Eq. (4) to all values of R, but insist that the bandwidth γ of the interferometer must satisfy $\gamma \ll \pi c/R$, so that high-frequency fluctuations always contribute to $\Delta\phi^2$. This is a reasonable approach when the arm separation of the interferometer is similar to the size of the interfering object, as here.