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Fidelity based measurement induced nonlocality and its dynamics in quantum noisy channels

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Measurement induced nonlocality (MIN) captures global nonlocal effect of bipartite quantum state due to locally invariant projective measurements. In this paper, we propose a new version of MIN using fidelity induced metric, and the same is calculated for pure and mixed states. For mixed state, the upper bound is obtained from eigenvalues of correlation matrix. Further, dynamics of MIN and fidelity based MIN under various noisy quantum channels show that they are more robust than entanglement.

1. Introduction

One of the most intriguing features of quantum regime is that local measurement on a part of composite system can induce global influence on the system. Such influence, also called as nonlocality, has no analogue in the classical scale. The strange nonclassical phenomenon is attributed to correlation between different parts of the system. Understanding the correlation of simplest composite system, namely bipartite system, is fundamental and relevant for quantum information theory. In this context, many measures of correlation for bipartite system have been proposed in recent years. One notable measure, which goes beyond entanglement, is quantum discord as proposed by Ollivier and Zurek [1]. Though the computation of discord involves complex optimization procedure [2], non-zero discord of separable states reveals that entanglement is not a complete manifestation of nonlocality or quantum correlation.

It is well known for pure states that while separable sates are invariant under von Neumann projective (local) measurements, inseparable states are altered by such measurements. Hence, local measurements may be useful tool for quantifying quantum correlation. On the other hand, notion of geometric quantum discord – distance between an arbitrary state and closest zero discord state, was introduced as a measure of correlation [3]. This notion is conveniently reformulated as the minimized square of Hilbert–Schmidt norm of difference between pre- and post-projective measurement of state under consideration [4]. Further, Luo and Fu presented a new measure of nonlocality for bipartite system, which is also dual to geometric discord, termed as measurement induced nonlocality (MIN) [5]. Both the quantities are significant figure of merit for quantum correlations with wide applications [6–8].

However, both the quantities suffer from the so called local ancilla problem – change may be effected through some trivial and uncorrelated action of the unmeasured party [9]. This problem can be circumvented by replacing density matrix by its square root [10]. Based on this, MIN has also been investigated in terms of relative entropy [11], von Neumann entropy [12], skew information [13] and trace distance [14]. Further, MIN has been investigated for bound entangled states [15], general bipartite system [16] and Heisenberg spin chains [17,18]. The dynamics and monogamy of measurement induced nonlocality has also been studied [19,20].

In this article, we introduce fidelity based measurement induced nonlocality to extract nonlocal effects of two qubit states due to projective measurements. It is shown that this quantity is naturally remedying the local ancilla problem of MIN and also easy to measure. Since fidelity is also experimentally accessible using quantum networks [21], nonlocal measure based on fidelity also enjoys physical relevance. For pure state, we show that the fidelity based MIN is indeed coinciding with other forms of MIN (Hilbert–Schmidt norm, skew information), and geometric discord. Our investigations also provide a closed formula for $2 \times n$ dimensional mixed state and an upper bound for arbitrary $m \times n$ dimensional mixed state. Further, we study the dynamics of MIN and fidelity based MIN under various noisy channel such as amplitude damping, depolarizing and generalized amplitude damping. It is shown

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that the MINs are robust measures of quantum correlation than entanglement against decoherence induced by the noisy channels.

2. MIN based on fidelity

Let us consider a bipartite quantum state ρ in a Hilbert space $\mathcal{H}^a \otimes \mathcal{H}^b$. MIN is defined as the square of Hilbert–Schmidt norm of difference between pre- and post-measurement state [5] i.e.,

$$N(\rho) = {}^{max}_{\Pi^a} \|\rho - \Pi^a(\rho)\|^2 \tag{1}$$

where the maximum is taken over the von Neumann projective measurements on subsystem a. Here $\Pi^a(\rho) = \sum_k (\Pi_k^a \otimes \mathbb{1}^b)\rho(\Pi_k^a \otimes \mathbb{1}^b)$, with $\Pi^a = \{\Pi_k^a\} = \{|k\rangle\langle k|\}$ being the projective measurements on the subsystem a, which do not change the marginal state ρ^a locally i.e., $\Pi^a(\rho^a) = \rho^a$. In fact, the MIN has a closed formula for $2 \times n$ dimensional states.

However, Hilbert–Schmidt norm based MIN could change due to trivial and uncorrelated action on the unmeasured party b. This arises from appending an uncorrelated ancilla c and regarding the state $\rho^{a:bc} = \rho^{ab} \otimes \rho^c$ as a bipartite state with the partition a:bc; then it is easy to verify the following

$$N(\rho^{a:bc}) = N(\rho^{ab}) tr(\rho^c)^2.$$

This relation implies that as long as ρ^c is a mixed state, MIN is altered by the addition of uncorrelated ancilla c – local ancilla problem.

We can resolve local ancilla problem by defining MIN based on fidelity, which is a measure of closeness between two arbitrary states ρ and σ. Defining fidelity as $F(\rho,\sigma) = \left(tr\sqrt{\sqrt{\rho}\sigma\sqrt{\rho}}\right)^2$, one can define a metric $D(\rho,\sigma) = \Phi(F(\rho,\sigma))$, where Φ is a monotonically decreasing function of $F(\rho,\sigma)$ and Φ is required to satisfy all the axioms of distance measure [22]. Due to computational complexity of fidelity, in what follows we employ an alternate definition of fidelity as [23]

$$\mathcal{F}(\rho,\sigma) = \frac{(tr(\rho\sigma))^2}{tr(\rho^2)\, tr(\sigma^2)}$$

which satisfies all the axioms to measure the closeness of two states. Defining MIN in terms of fidelity induced metric (F-MIN) as

$$N_{\mathcal{F}}(\rho) = {}^{max}_{\Pi^a} \mathcal{C}^2(\rho, \Pi^a(\rho)) \tag{2}$$

where $\mathcal{C}(\rho,\sigma) = \sqrt{1-\mathcal{F}(\rho,\sigma)}$ is sine metric. In other words, MIN is defined in terms of the fidelity between pre- and post-measurement state. This quantity can remedy the local ancilla problem of MIN as shown below. After the addition of local ancilla the fidelity between the pre- and post-measurement state is

$$\mathcal{F}\left(\rho^{a:bc}, \Pi^a(\rho^{a:bc})\right) = \mathcal{F}\left(\rho^{ab}\otimes\rho^c, \Pi^a(\rho^{ab})\otimes\rho^c\right).$$

Using multiplicativity property of fidelity,

$$\begin{aligned}\mathcal{F}\left(\rho^{a:bc}, \Pi^a(\rho^{a:bc})\right) &= \mathcal{F}\left(\rho^{ab}, \Pi^a(\rho^{ab})\right)\mathcal{F}(\rho^c,\rho^c)\\ &= \mathcal{F}\left(\rho^{ab}, \Pi^a(\rho^{ab})\right).\end{aligned}$$

Hence $N_{\mathcal{F}}(\rho)$ is a good measure of nonlocality or quantumness in a given system.

3. MIN for pure state

Theorem 1. *For pure bipartite state with Schmidt decomposition $|\Psi\rangle = \sum_i \sqrt{s_i}|\alpha_i\rangle \otimes |\beta_i\rangle$, F-MIN is*

$$N_{\mathcal{F}}(|\Psi\rangle\langle\Psi|) = 1 - \sum_i s_i^2. \tag{3}$$

The proof is as follows. Here, von Neumann projective measurements on party a as $\Pi^a = \{\Pi_k^a\} = \{|\alpha_k\rangle\langle\alpha_k|\}$ do not alter the marginal state ρ^a. Noting that

$$\rho = |\Psi\rangle\langle\Psi| = \sum_{ij}\sqrt{s_i s_j}|\alpha_i\rangle\langle\alpha_j| \otimes |\beta_i\rangle\langle\beta_j|.$$

Since $tr(\rho\Pi^a(\rho)) = tr(\Pi^a(\rho))^2$ the fidelity between pre- and post-measurement state becomes

$$\mathcal{F}(\rho, \Pi^a(\rho)) = \sum_i s_i^2$$

and hence the theorem is proved. Thus F-MIN coincides with Hilbert–Schmidt norm [5] and skew information [13] based MINs and geometric discord [4] for pure states.

4. MIN for mixed state

Let $\{X_i : i = 0,1,2,\cdots,m^2-1\} \in \mathcal{B}(\mathcal{H}^a)$ be a set of orthonormal operators for the state space $\mathcal{H}^a$ with operator inner product $\langle X_i|X_j\rangle = tr(X_i^\dagger X_j)$. Similarly, one can define $\{Y_j : j = 0,1,2,\cdots,n^2-1\} \in \mathcal{B}(\mathcal{H}^b)$ for the state space $\mathcal{H}^b$. The operators X_i and Y_j are satisfying the conditions $tr(X_k^\dagger X_l) = tr(Y_k^\dagger Y_l) = \delta_{kl}$. With this one can construct a set of orthonormal operators $\{X_i \otimes Y_j\} \in \mathcal{B}(\mathcal{H}^a \otimes \mathcal{H}^b)$ for the composite system. Consequently, an arbitrary state of a bipartite composite system can be written as

$$\rho = \sum_{i,j}\gamma_{ij} X_i \otimes Y_j \tag{4}$$

where $\Gamma = (\gamma_{ij} = tr(\rho\ X_i \otimes Y_j))$ is a $m^2 \times n^2$ real matrix.

After a straight forward calculation, the fidelity between pre- and post-measurement state is computed as

$$\mathcal{F}(\rho, \Pi^a(\rho)) = \frac{tr(A\Gamma\Gamma^t A^t)}{\|\Gamma\|^2}$$

where the matrix $A = (a_{ki} = tr(|k\rangle\langle k|X_i))$ is a rectangular matrix of order $m \times m^2$ and superscript t stands for transpose. Then, F-MIN is

$$N_{\mathcal{F}}(\rho) = \frac{1}{\|\Gamma\|^2}\left[\|\Gamma\|^2 - {}^{min}_{A}\, tr(A\Gamma\Gamma^t A^t)\right]. \tag{5}$$

Adapting optimization procedure [4], we already have an upper bound [26]

$$N_{\mathcal{F}}(\rho) \le \frac{1}{\|\Gamma\|^2}\left(\sum_{i=m}^{m^2-1}\mu_i\right),$$

where μ_i are eigenvalues of $\Gamma\Gamma^t$ listed in increasing order.

Theorem 2. *F-MIN has a tight upper bound as*

$$N_{\mathcal{F}}(\rho) \le \frac{1}{\|\Gamma\|^2}\left(\sum_{i=m}^{m^2-1}\lambda_i\right) \tag{6}$$

where λ_i are eigenvalues of matrix $\mathbf{x}\mathbf{x}^t + TT^t$, derived from Γ, listed in increasing order.

Adapting the method [24], we prove the theorem as follows: If $X_0 = \mathbb{1}^a/\sqrt{m}$, $Y_0 = \mathbb{1}^b/\sqrt{n}$, and separating the terms in eq. (4), the state ρ can be written as

$$\begin{aligned}\rho = \frac{1}{\sqrt{mn}}\frac{\mathbb{1}^a}{\sqrt{m}} \otimes \frac{\mathbb{1}^b}{\sqrt{n}} + \sum_{i=1}^{m^2-1} x_i X_i \otimes \frac{\mathbb{1}^b}{\sqrt{n}} + \frac{\mathbb{1}^a}{\sqrt{m}} \otimes \sum_{j=1}^{n^2-1} y_j Y_j\\ + \sum_{i,j\neq 0} t_{ij} X_i \otimes Y_j\end{aligned} \tag{7}$$

where $x_i = tr(\rho\ X_i \otimes \mathbb{1}^b)/\sqrt{n}$, $y_j = tr(\rho\ \mathbb{1}^a \otimes Y_j)/\sqrt{m}$ and $T = (t_{ij} = tr(\rho\ X_i \otimes Y_j))$ is a real correlation matrix of order $(m^2 -$

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