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Entanglement swapping via three-step quantum walk-like protocol

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ABSTRACT

We present an entanglement swapping process for unknown nonmaximally entangled photonic states, where the standard Bell-state measurement is replaced by a three-step quantum walk-like state discrimination process, i.e., the practically nontrivial coupling element of two photons is replaced by manipulating their trajectories, which will greatly enrich the dynamics of the coupling between photons in realizing quantum computation, and reduce the integration complexity of optical quantum processing. In addition, the output state can be maximally entangled, which allows for entanglement concentration as well.

1. Introduction

In quantum communication and quantum computation, quantum entanglement finds many significant applications, including quantum teleportation [1], quantum superdense coding [2], quantum cryptography [3], etc. Typically, only maximally entangled states (MES) can lead to the perfect implementation of the above protocols. But in real experiments, unavoidable decoherence of quantum system is a serious hindrance to the realization of quantum information processing and quantum computation. In general, it is inevitable that the degree of entanglement decreases with the channel length, leading to an effective non-maximally entangled state. Undoubtedly, the use of non-MES could lead to severe decrease in the efficiency and fidelity of a quantum communication protocol. Therefore, creation of a MES from non-MESs attracts considerable attention in the community. To circumvent this problem, Schmidt projection scheme and Procrustean scheme have been proposed [4]. Although entanglement swapping scheme was proposed for entangling two remote qubits without direct interaction between them [5,6], it can be regarded as an entanglement concentration method too [7].

In the standard entanglement swapping process, a Bell state measurement constitutes the main swapping mechanism [5]. But, the realization of a Bell-state measurement (BSM) is not an easy task in experiment, so efforts have been made to design the entanglement swapping schemes without BSM. For instance, several implementation schemes of the entanglement swapping without BSM have been proposed both in cavity QED systems [8,9] and in quantum dot systems [10]. Essentially, the entanglement swapping schemes with or without BSM both require the coupling interactions between two qubits at the intermediate location. The coupling interactions in the swapping scheme with BSM can lead to a full discrimination of the four Bell states, meanwhile the coupling interactions in the swapping scheme without BSM can only lead to a partial discrimination of the four Bell states.

Recently, it was shown that quantum walk (QW) [11] can be used to implement a generalized measurement, i.e. a positive operator value measure (POVM) [12], and furthermore, a generalized measurement has been realized in discriminating non-orthogonal quantum states by executing a properly engineered QW [13]. But, in these advances, only the non-orthogonal quantum state discrimination of a single qubit has been studied and realized via QW. If this QW based state discrimination process can be generalized to the two-qubit case, it can be used to implement entanglement swapping too. In this paper, we present a three-step QW-like state discrimination scheme for four non-orthogonal two-qubit states, and thus the entanglement swapping scheme for two unknown non-maximally entangled states. The output state of the swapping process is maximally entangled, which allows for entanglement concentration as well.

In addition, the coupling between two qubits is the core part of the entanglement swapping schemes. But the current existing cou-

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pling mechanisms for photonic qubits and matter qubits are not suitable for integration. The coupling mechanism for two photons in our entanglement swapping process is realized by manipulating the trajectories of the photons. Thus, it is very easy to implement and integrate, and the versatile site-dependent operations and the intersite trajectory manipulations will greatly enrich the dynamics that this process can produce. Since our protocol can formally be described as a two-particle three-step QW, this opens new possibilities for quantum computation using the existing optical implementations of QWs.

This paper is organized as follows. In Sec. 2 we briefly introduce the concepts of entanglement swapping for non-maximally entangled states. In Sec. 3 we present our entanglement swapping scheme. Sec. 4 summarizes our results.

2. Entanglement swapping for unknown non-maximally entangled states

Suppose there are two pairs of polarization-entangled photons (1, 2) and (3, 4) shared by three remote users Alice, Bob and Clare:

$$|\psi\rangle_{12} = a\,|HH\rangle_{12} + b\,|VV\rangle_{12}\,, \tag{1}$$

$$|\psi\rangle_{34} = a\,|HH\rangle_{34} + b\,|VV\rangle_{34}\,, \tag{2}$$

where a, b satisfy the normalization condition $|a|^2 + |b|^2 = 1$. Photons (1, 2) belong to Alice and Clare, respectively, and photons (3, 4) belong to Clare and Bob. Here, $|H\rangle$ ($|V\rangle$) denotes the horizontal (vertical) polarization state of the photons. Without loss of generality, we can assume that the superposition coefficients a and b are all real numbers. Initially, the state of the two photon pairs is in a product form, which can be written as

$$\begin{aligned}|\psi\rangle_{1234} &= |\psi\rangle_{12} \otimes |\psi\rangle_{34}\\ &= \sqrt{\frac{a^4+b^4}{2}}(|\psi\rangle^{+}_{14}|\psi\rangle^{1}_{23} + |\psi\rangle^{-}_{14}|\psi\rangle^{2}_{23})\\ &\quad + ab(|\varphi\rangle^{+}_{14}|\psi\rangle^{3}_{23} + |\varphi\rangle^{-}_{14}|\psi\rangle^{4}_{23}),\end{aligned} \tag{3}$$

where

$$|\psi\rangle^{\pm}_{14} = \frac{1}{\sqrt{2}}(|HH\rangle_{14} \pm |VV\rangle)_{14}, \tag{4}$$

$$|\varphi\rangle^{\pm}_{14} = \frac{1}{\sqrt{2}}(|HV\rangle_{14} \pm |VH\rangle_{14}), \tag{5}$$

$$|\psi\rangle^{1}_{23} = \frac{1}{\sqrt{a^4+b^4}}(a^2\,|HH\rangle_{23} + b^2\,|VV\rangle_{23}), \tag{6}$$

$$|\psi\rangle^{2}_{23} = \frac{1}{\sqrt{a^4+b^4}}(a^2\,|HH\rangle_{23} - b^2\,|VV\rangle_{23}), \tag{7}$$

$$|\psi\rangle^{3}_{23} = \frac{1}{\sqrt{2}}(|HV\rangle_{23} + |VH\rangle_{23}), \tag{8}$$

$$|\psi\rangle^{4}_{23} = \frac{1}{\sqrt{2}}(|HV\rangle_{23} - |VH\rangle_{23}). \tag{9}$$

From Eq. (3), we can see that, as long as Clare, who has access to photons 2 and 3 (as depicted in Fig. 1), can discriminate the four states in Eqs. (6)–(9), four maximally entangled states in Eqs. (4), (5) can be generated among the two remote users Alice and Bob. But the four states in Eqs. (6)–(9) are not orthogonal to each other, and they cannot be distinguished with unit probability. So generalized measurements (POVMs) must be introduced to discriminate these non-orthogonal states [14,15]. Because the

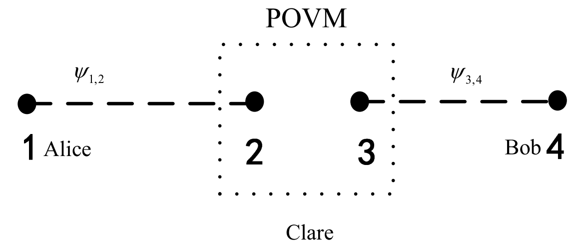

Fig. 1. The schematic diagram illustrating the procedure of entanglement swapping for unknown non-maximally entangled states.

states to be swapped are unknown,[1] the states in Eqs. (6), (7) are unknown for us too, and thus these two states cannot be distinguished. Nevertheless, the states in Eqs. (8), (9) are totally known for us, so, in the following section, we will design a three-step QW-like process to discriminate these two states among the four non-orthogonal quantum states in Eqs. (6)–(9).

3. Quantum walk-like swapping mechanism

In this section, we are going to design a three-step QW-like scheme to distinguish the two states in Eqs. (8), (9) from the four non-orthogonal quantum states in Eqs. (6)–(9), where the polarization degrees of photons 2, 3 are regarded as coin degrees of the three-step QW-like evolution, and the final position measurements on these two one-dimensional (1D) QW-like processes after three appropriately designed steps will tell us whether the discrimination succeeds or not.

Because our three-step scheme is a QW-like one, the state evolutions of the scheme are in similar forms as in QW systems, it is helpful for us to briefly review a standard 1D discrete-time QW [11]. The total Hilbert space of a walker consists of coin and position degrees of freedom, and is given by the tensor product $H \equiv H_{\mathcal{C}} \otimes H_{\mathcal{P}}$ of two subspaces spanned by $\{|H\rangle_{\mathcal{C}}, |V\rangle_{\mathcal{C}}\}$ and $\{|n\rangle_{\mathcal{P}}, n \in \mathbb{Z}\}$, respectively. Here, the subindex $\mathcal{C}$ denotes the coin degree, $\mathcal{P}$ denotes the position degree, and from now on they will be omitted for simplicity. One-step evolution of the system involves the coin flipping and conditional position shift based on the outcome of the coin flipping, and the corresponding unitary operation U is

$$U = S(C \otimes I), \tag{10}$$

where $C \in U(2)$ is the coin flipping operator, I is the identity operator in the position space, and the conditional position shift operator S takes the form $S = \sum_x (|x+1\rangle\langle x| \otimes |H\rangle\langle H| + |x-1\rangle\langle x| \otimes |V\rangle\langle V|)$. Without loss of generality, we assume the walker is at the position $x = 0$ initially, and the initial state of the coin is a superposition of $|H\rangle$ and $|V\rangle$ states. If the walk starts with the initial state $|\Psi(0)\rangle$, the final state of the system after t steps becomes

$$|\Psi(t)\rangle = U^t|\Psi(0)\rangle. \tag{11}$$

In our scheme, the states to be distinguished are two-photon (photons 2 and 3 at Clare's location) joint states in Eqs. (6)–(9) rather than the single-photon states, so the state evolutions for realizing this discrimination process are similar with the case of two walkers on two different lines. The joint Hilbert space of the two photons 2 and 3, on "line 2" and "line 3", respectively, is given by

$$H_{23} \equiv H_2 \otimes H_3 \equiv (H_{\mathcal{C}2} \otimes H_{\mathcal{P}2}) \otimes (H_{\mathcal{C}3} \otimes H_{\mathcal{P}3}). \tag{12}$$

[1] They are unknown in a sense that coefficients a and b are unknown, but the type of the states (superposition of both photon polarisations being either horizontal or vertical) is known.

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