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Photon exchange and entanglement formation during transmission through a rectangular quantum barrier

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ABSTRACT

When a quantum particle traverses a rectangular potential created by a quantum field both photon exchange and entanglement between particle and field take place. We present the full analytic solution of the Schrödinger equation of the composite particle–field system allowing investigation of these phenomena in detail and comparison to the results of a classical field treatment. Besides entanglement formation, remarkable differences also appear with respect to the symmetry between energy emission and absorption, resonance effects and if the field initially occupies the vacuum state.

1. Introduction

The behavior of a quantum particle exposed to an oscillating rectangular potential has been studied by several authors under different aspects involving, for example, tunneling time [1,2], chaotic signatures [3,4], appearance of Fano resonances [5], Floquet scattering for strong fields [6] and its absence for non-Hermitian potentials [7], chiral tunneling [8], charge pumping [9] and other photon assisted quantum transport phenomena in theory [10–12] and experiment [13–18], recently realized particularly in quantum dots [19–22].

In these works, though the potential is treated as a classical quantity, the change of the particle's energy is explicitly attributed to a photon emission or absorption process. Here, we introduce the photon concept in a formally correct way by describing the field generating the potential as quantized. Hence, we pursue the ideas which we started to elaborate in our previous publication [23]. There, we only arrived at an algebraic expression for the photon transition amplitudes whereas we now are able to present analytic results for all important initial field states enabling advanced investigations on photon exchange processes and entanglement formation.

In order to compare semiclassical and fully-quantized treatment in our physical scenario, we recapitulate the results of the calculation for a classical field (Section 2). Then, we turn to the quantized field treatment (Section 3). After presenting the general algebraic solution, we will explicitly evaluate the photon exchange probabilities for an incoming plane wave and for a field being initially in an arbitrary Fock state, a thermal state or a coherent state. The special cases of no initial photons (vacuum state) and of high initial photon numbers will be treated in particular.

2. Classical treatment of the field

The potential created by a classical field is a real-valued function of space and time in the particle's Hamiltonian. Our considered potential oscillates harmonically in time and is spatially constant for $0 \le x < L$ and vanishes outside.

$$\hat{H} = \begin{cases} \frac{\hat{p}^2}{2m} + V\cos(\omega t + \varphi), & \text{if } 0 \le x < L \text{ (region II)} \\ \frac{\hat{p}^2}{2m}, & \text{else (region I + III)} \end{cases} \tag{1}$$

It therefore corresponds to a harmonically oscillating rectangular potential barrier (see Fig. 1).

The Schrödinger equation is solved in each of the three regions separately and then the wave functions are matched by continuity conditions. A general approach based on Floquet theory [24] can be found in [25]. Since the calculation of transmission and reflection coefficients requires the solution of an infinite dimensional equation system no closed analytic expression for them is feasible (see Chapter II of [25]). However, we can deduce from Chapter III

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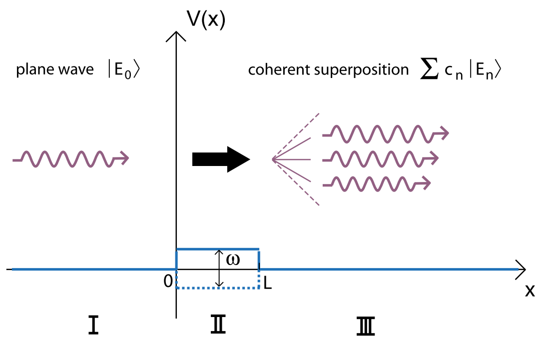

Fig. 1. (Color online.) Spatial characteristics of the considered potential V. It is harmonically oscillating in time with frequency ω in region II and vanishes elsewhere. Behind the barrier the incoming plane wave is split up into a coherent superposition of plane waves with energy $E_n = E_0 + n\hbar\omega$.

of [25] that the transmission probability approaches one if the incoming energy E_0 becomes large with respect to the potential amplitude V and the associated "photon" energy $\hbar\omega$ ($E_0 \gg V, \hbar\omega$). For the further, we restrict ourselves to incoming waves whose energy E_0 is sufficiently high so that reflection at the barrier can be neglected. In that case, standard methods for differential equations suffice to find the solution [26,27]. If we assume the wave function $|\psi_I\rangle$ in region I to be a plane wave with wave vector k_0 we get for the wave function $|\psi_{III}\rangle$ behind the potential barrier

$$|\psi_I\rangle = |k_0\rangle \implies |\psi_{III}\rangle = \sum_{n=-\infty}^{+\infty} J_n(\beta)\, e^{-in\eta}\, |k_n\rangle \tag{2}$$

where

$$\beta = 2\frac{V}{\hbar\omega}\sin\frac{\omega\tau}{2}, \quad \eta = \varphi + \frac{\omega\tau}{2} + \frac{\pi}{2} \tag{3}$$

$$\tau = \frac{mL}{\hbar k_0} = \frac{L}{v_0},\ k_n^2 = k_0^2 + \frac{2m}{\hbar}n\omega \tag{4}$$

For a more detailed derivation including the solution for region II as well we refer to [26,28].

In summary, a plane wave $|k_0\rangle$ gets split up into a coherent superposition of plane waves $|k_n\rangle$ whose energy is given by the incident energy E_0 plus integer multiples of $\hbar\omega$. The transition probability for an energy exchange of $n\hbar\omega$ is just the square of the Bessel function J_n^2 of the n-th order. The argument of the Bessel function shows that an increasing amplitude V of the potential also increases the probability for exchanging larger amounts of energy.

Apart from this expected result, it also exhibits a geometric "resonance"-condition. If the "time-of-flight" τ through the field region and the oscillation frequency are tuned such that $\omega\tau = 2l\pi$, $l \in \mathbb{N}$, all Bessel functions J_n with $n \neq 0$ vanish and no energy is transferred at all. The plane wave even passes the potential completely unaltered since $J_0(0) = 1$. That's a remarkable difference between an oscillating and a static potential where at least phase factors are always attached to the wave function. An experimental implementation of the classical potential can be found in [28,29].

3. Quantized treatment of the field

Since the energy exchange between the harmonically oscillating potential and the particle is quantized by integer multiples of $\hbar\omega$ most authors already speak of photon exchange processes although the potential stems from a purely classical field. This notion is problematic since a formally correct introduction of the photon concept requires a quantization of the field generating the potential. For this purpose, the corresponding field equation has to be solved and a canonical quantization condition for Fourier amplitudes of the field is introduced which are then no longer complex-valued coefficients but interpreted as creation and annihilation operators.

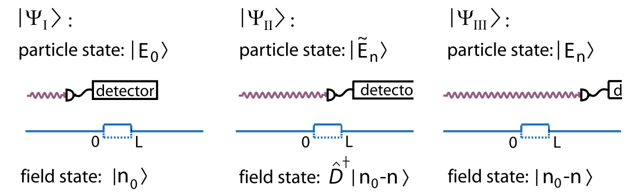

Fig. 2. (Color online.) In the quantized field treatment, the particle's position determines which of the overall wavefunctions $|\Psi_I\rangle$, $|\Psi_{II}\rangle$ or $|\Psi_{III}\rangle$ describes the state of the composite quantum system. The spatial characteristics of the field do not change, it is always present between 0 and L, but the field state changes in accordance with the particle due to their interaction.

For the further, we assume that such a quantum field whose spatial mode is well approximated by the rectangular form generates the potential. The quantum system we observe now consists of particle and field together. The total state $|\Psi\rangle$ of the composite quantum system is an element of the product Hilbert space $\mathcal{H}_{\text{total}} = \mathcal{H}_{\text{particle}} \otimes \mathcal{H}_{\text{field}}$. If the particle is outside the field region the evolution of the state is given by $\hat{H}_0$ composed of the free single-system Hamiltonians $\hat{h}_0^{\text{p}}$ and $\hat{h}_0^{\text{f}}$ of particle and field

$$\hat{H}_0 = \hat{h}_0^{\text{p}} \otimes \mathbb{1} + \mathbb{1} \otimes \hat{h}_0^{\text{f}} \tag{5}$$

$$\hat{h}_0^{\text{p}} = \frac{\hat{p}^2}{2m}, \quad \hat{h}_0^{\text{f}} = \hbar\omega\left(\hat{a}^\dagger\hat{a} + \tfrac{1}{2}\right) \tag{6}$$

Interaction between field and particle takes place if the particle is inside the field region which can be formally expressed by using the Heavyside θ-function in the quantized version of the sinusoidal driving term

$$\hat{H}_{\text{int}} = \lambda\left(\theta(\hat{x}) - \theta(\hat{x} - L)\right) \otimes \left(\hat{a}^\dagger + \hat{a}\right) \tag{7}$$

where all constants have already been absorbed in the coupling parameter λ. Since the sheer presence of an interaction is connected to the particle's position we again distinguish between three different states $|\Psi_I\rangle$, $|\Psi_{II}\rangle$, and $|\Psi_{III}\rangle$ for the composite quantum system (see Fig. 2).

3.1. Fock states

As in the classical field case, we assume that the kinetic energy of the incoming particle is sufficiently high so that reflection at field entry can be neglected. Then, we can choose as ansatz for $|\Psi_I\rangle$ the particle's state to be a single plane wave with wave vector k_0 and the field to be present in a distinct Fock state n_0

$$|\Psi_I\rangle = |k_0\rangle \otimes |n_0\rangle \tag{8}$$

In order to get $|\Psi_{II}\rangle$, we switch to the position space representation of the particle's part of the wave function and match $|\Psi_I\rangle$ at $x_{\text{particle}} \equiv x = 0$ for all times t with the general solution in region II. It is given by an arbitrary linear superposition of plane waves for the particle and displaced Fock states for the field [23]. The continuity conditions uniquely determine the expansion coefficients and yet $|\Psi_{II}\rangle$. At $x = L$, $|\Psi_{II}\rangle$ has to be matched with the general solution of the free Hamiltonian which is given by an arbitrary superposition of plane waves and Fock states. The state $|\Psi_{III}\rangle$ behind the field region then reads

$$|\Psi_{III}\rangle = \sum_{n=0}^{\infty} t_{n_0 n}|k_{n_0-n}\rangle \otimes |n\rangle, \qquad k_l^2 = k_0^2 + \frac{2m}{\hbar}l\omega \tag{9}$$