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Effects of interactions on the generalized Hong–Ou–Mandel effect

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ABSTRACT

We numerically investigate the influence of interactions on the generalized Hong–Ou–Mandel (HOM) effect for bosonic particles in a (quasi-)one-dimensional set-up with weak harmonic confinement and show results for the cases of $N = 2$, $N = 3$ and $N = 4$ bosons interacting with a beam splitter, whose role is played by a δ-barrier. In particular, we focus on the effect of attractive interactions and compare the results with the repulsive case, as well as with the analytically available results for the non-interacting case (that we use as a benchmark). We observe a fermionization effect both for growing repulsive and attractive interactions, i.e., the dip in the HOM coincidence count is progressively smeared out, for increasing interaction strengths. The role of input asymmetries is also explored.

1. Introduction

When two indistinguishable photons simultaneously hit a 50%–50% beam splitter, one in each input port, they interfere destructively and no coincidence counts of particles detected in both output ports can be observed. This inherently quantum mechanical effect is a striking manifestation of the bosonic properties of the photons. Since its first observation in 1987, the celebrated Hong–Ou–Mandel (HOM) [1] effect has triggered a multitude of investigations and has been the focus of ongoing research: it has found application in the creation of post-selected entanglement between photon pairs [2], and can be exploited for logic gates in linear optical quantum computation [3], or to demonstrate the purity of a solid-state single-photon source [4]. The HOM effect has also been experimentally demonstrated with single photons emitted by two independently trapped single atoms [5]. The influence of varying properties of the beam-splitter on the joint probability distribution (of finding both photons on the same side) has been investigated in [6]. Experimental realizations have been extended to larger particle numbers, namely three photons impinging on a multiport mixer [7], and to one- and two-photon pairs [8]. HOM interference has also been observed with macroscopic states containing on the order of 10^6 photons per mode [9]. A review on multi-photon experiments and the generalized Hong–Ou–Mandel effect is given in [10].

Both theoretically and experimentally, the generalization of the HOM effect to massive particles is currently a very active field of research. Theoretical investigations include N bosons or fermions passing simultaneously through a symmetric Bell multiport beam splitter [11], and a large number of particles impinging on a single beam splitter [12,13]. In the latter case it has been shown for the balanced beam-splitter that if an even number of particles impinges on the beam-splitter from either side, an even number must also emerge from each side. A recent proposal suggests to observe the HOM effect with colliding Bose–Einstein condensates [14] for a set-up that has already been used to demonstrate the violation of the Cauchy–Schwarz inequality [15]. Bosonic atoms have also been proposed as a suitable candidate for Knill–Laflamme–Milburn quantum computation, with the advantage of very controllable input state preparation [16].

It is relevant to highlight here that two-particle quantum interference akin to the Hong–Ou–Mandel effect has recently been observed in the context of independently prepared bosonic atoms in tunnel-coupled optical tweezers [17] in a regime where the role of interactions was negligible. The latter experiment follows up on another experimental realization of such quantum interference effects in the context of electrons [18]. A further atomic HOM experiment has been recently realized with twin-atom pairs [19]. Moreover, the remarkable advances in the setting of ultracold gases enable the very accurate counting [20] and controllability [21,22] of bosonic atoms towards generalized HOM experiments, potentially involving different atom numbers. Motivated by the fact that the parity of the number of atoms in a potential well is experi-

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mentally accessible more straightforwardly than the exact particle number it has been pointed out that parity measurements can yield useful signatures of the generalized HOM effect [13]. Recent experimental results even demonstrate the simultaneous determination of the number of atoms in each well of a double-well trap with single-atom resolution for up to $N = 500$ atoms per well [23].

An interesting and important question concerning atomic realizations of the HOM effect is the influence of interparticle interactions on the dynamics. Theoretically, effects of the interparticle interaction have been investigated in [24] within a Bose–Hubbard set-up where a transition from bunching to antibunching can be observed for growing repulsive interparticle interactions, corresponding to a fermionization of the particles [25,26]. For two atoms in a double-well potential analytical considerations show that the reduction of the Hong–Ou–Mandel dip remarkably is independent of the sign of the interaction [27].

Our investigation considers the (generalized) HOM effect for a 1D Bose gas on the N-particle level, focusing on the cases of $N = 2$, $N = 3$ and $N = 4$. In addition to examining the somewhat less studied attractive case, we compare the results with those of the repulsive case and importantly with the non-interacting case that can be addressed, in principle, in its full generality for arbitrary N, as will be discussed below. Here, we will chiefly focus on the effect of *attractive* interparticle interactions on the HOM dip and demonstrate that, as for the double-well case [27], also in the present setting a transition from bunching to antibunching with growing attractive interaction strength can be observed. That somewhat counter-intuitive (on the basis of the nature of the interaction) feature can again be interpreted as a fermionization effect, in line with [28], where the Bose–Fermi mapping has been demonstrated for attractive 1D bosons. The latter generalization enables the formation of gas-like states that fermionize as the attraction becomes stronger.

In the original HOM experiment the photons emerge as a result of quantum interference in a superposition of states $|2,0\rangle$ and $|0,2\rangle$ and thus in a measurement would always be found on the same side. As an aside, we note in passing, effectively regarding the large N limit of mean-field matter waves that a recent proposal suggests an analogue of the HOM effect with bright solitons [29]. In this classical case it is found that the indistinguishability of the particles yields a 0.5 split mass on either side for solitary waves (by parity symmetry at this mean-field level). But for very slight deviations it can be observed that all the particles always end up on the same side of the barrier potential. Recently, also the collisional dynamics of matter-wave solitons (in the absence of a barrier potential) have been investigated experimentally [30]. Additionally, the setting of interactions of individual single [31] and multi-component [32] solitary waves with barriers (that play the role of the beam splitter here) is certainly within experimental reach. Collisions have also been suggested as a way to create entanglement between indistinguishable solitons [33] and initially independent and indistinguishable boson pairs [34]. Collisions of distinguishable solitons have been proposed as a possibility to create mesoscopic Bell states [35].

Our presentation is structured as follows: Section 2 gives an introduction to the model system, the Lieb–Liniger(–McGuire) model with a quantum beam-splitter. In Section 3 we present numerical results for the collision of two monomers, two dimers and the asymmetric case of a monomer and a dimer on an additional barrier potential. Section 4 summarizes our findings and presents a number of future challenges.

2. Model system

The N-particle dynamics of interacting bosons in (quasi-)one-dimensional geometries (corresponding to a 3D geometry with tightly confined radial degrees of freedom) can be described within the exactly solvable Lieb–Liniger(–McGuire) model [36,37]

$$\hat{H}_{\rm LL} = -\sum_{j=1}^{N} \frac{\hbar^2}{2m}\partial_{x_j}^2 + \sum_{j=1}^{N-1}\sum_{n=j+1}^{N} g\delta(x_j - x_n). \quad (1)$$

Here, contact interaction between the N bosons is assumed and quantified with the interaction parameter g. In the following we will investigate the scattering dynamics within an additional harmonic confinement (emulating the typical parabolic trap relevant to experimental settings; cf. [31,32]) at a repulsive delta-like barrier potential in the middle of the harmonic confinement, yielding the full Hamiltonian

$$\hat{H} = \hat{H}_{\rm LL} + \sum_{j=1}^{N} V_{\rm ext}(x_j) \quad (2)$$

with the external potential $V_{\rm ext}(x_j) = \frac{1}{2}m\omega^2 x_j^2 + v_0\delta(x_j)$.

For the numerical implementation the system can be discretized via the Bose–Hubbard Hamiltonian

$$\hat{H}_{\rm discretized} = -J\sum_j(\hat{c}_j^\dagger\hat{c}_{j+1} + \hat{c}_{j+1}^\dagger\hat{c}_j) + \frac{U}{2}\sum_j \hat{n}_j(\hat{n}_j - 1) + A\sum_j \hat{n}_j j^2 + v_0\delta_{j,0}, \quad (3)$$

where $\hat{c}_j^{(\dagger)}$ denotes the annihilation (creation) operator for lattice site j, $\hat{n}_j = \hat{c}_j^\dagger\hat{c}_j$ is the particle number operator, and U the on-site interaction strength. The tunneling strength is given by $J \sim \hbar^2/2mb^2$ with lattice spacing b for $b \to 0$, $A \equiv \frac{1}{2}m\omega^2 b^2$ defines the strength of the harmonic confinement and v_0 the strength of the delta-like barrier potential. For sufficiently small lattice spacing $b \to 0$ the Lieb–Liniger model (2) with additional harmonic confinement is recovered.

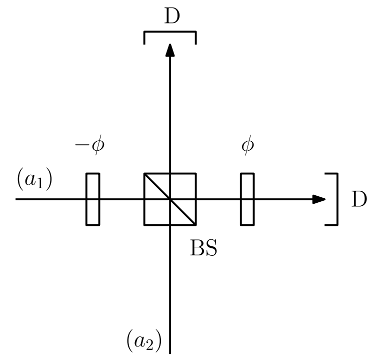

Fig. 1. Schematic depiction of a quantum beam-splitter (BS) with input modes (a_1) and (a_2), detectors (D) at both outputs and optional additional phase shifts.

2.1. Quantum beam-splitter

The repulsive delta-like barrier potential acts as a beam-splitter. In this section we present a theoretical description for the noninteracting case, following [38] (see also [12,13]). Assume a generalized beam-splitter with two incoming modes (a_1) and (a_2) being coupled at the beam-splitting device and possible additional phase shifts, as depicted in Fig. 1. This system is described by the time-evolution operator

$$U_c(\theta,\phi) = \begin{pmatrix} \cos(\theta/2) & ie^{i\phi}\sin(\theta/2) \\ ie^{-i\phi}\sin(\theta/2) & \cos(\theta/2) \end{pmatrix}, \quad (4)$$