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11 Dayleard composite regularization the coupled Uirate equations 77 $\frac{1}{12}$ Dark and composite rogue waves in the coupled Hirota equations $\frac{1}{18}$

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¹⁸ ARTICLE INFO ABSTRACT ⁸⁴ 19

 α 27 Rogue wave the contraction of the contraction of the contraction of α C 2014 Elsevier B.V. All rights reserved. *Article history:* Received 29 May 2014 Received in revised form 17 July 2014 Accepted 2 August 2014 Available online xxxx Communicated by C.R. Doering *Keywords:* Rogue wave

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1. Introduction

³⁶ gigantic ocean waves of extreme amplitude that appear seemingly typical, most-studied example is the Peregrine soliton [12], the ¹⁰² ³⁷ out of nowhere and have been held responsible for a large num-
simplest of a hierarchy of rational solutions of the scalar nonlin-³⁸ ber of maritime disasters [\[1\].](#page--1-0) As these colossal surface waves are ear Schrödinger (NLS) equation. As it involves dynamics featuring ¹⁰⁴ ³⁹ intrinsically difficult to monitor because of their fleeting existences a peak amplitude three times the background height and a dou-⁴⁰ and dangerousness, more and more effort has been devoted to the ble localization on a finite background, such a rational soliton was 106 ⁴¹ study of such rare extreme events in contexts that are similar in frequently used as a prototypical profile for a single rogue wave 107 ⁴² physical models but can be readily conducted in a laboratory en-
event occurred in reality. Recent breakthroughs in observation of ¹⁰⁸ ⁴³ vironment [\[2\].](#page--1-0) The recent experimental progress includes the first peregrine soliton in deep water [4], plasmas [13], and an optical 108 ⁴⁴ ever observation of optical rogue waves in a photonic crystal fiber $\frac{f}{f}$ fiber [14] provide direct evidences for this hypothesis. Experiments $\frac{110}{f}$ 45 [\[3\],](#page--1-0) the resurgence of surface rogue waves in a water wave tank also showed that the actual dynamics of the so-called super rogue ⁴⁶ [\[4,5\],](#page--1-0) the realization of capillary rogue waves via the four-wave $\frac{1}{2}$ waves can be well mimicked by the higher-order rational solutions ⁴⁷ coupling [\[6\],](#page--1-0) the generation of dissipative rogue waves in a mode-
 $\frac{113}{113}$ whose neak amplitude increases progressively to a factor of more 48 locked laser [\[7\],](#page--1-0) the appearance of spatiotemporal rogue waves via $_{144}$ then five [5] ⁴⁹ optical filamentation [\[8\],](#page--1-0) and the refocusing of the time-reversed **115** many scenarios several scalar waves or several components 50 rogue waves in deep water [\[9\],](#page--1-0) to name a few. These experimen-
 $\frac{1}{2}$ rocker wave pool to be considered in erge to accurat for the security of a vector wave pool to be considered in erge to accurat for 51 tal studies show that rogue waves possess some hallmark phe-
the significant interaction processes Undertanding the underly 52 nomenological features, e.g., they are extremely large and steep $\frac{1}{16}$ in physics particular counter consideration of the counted system. 53 compared with typical events, occur in a nonlinear medium, and $\frac{1}{2}$ for existence mathematical popular medial December the function of $\frac{1}{2}$ 54 120 follow an unusual *L*-shaped statistics [\[3,10,11\].](#page--1-0) Despite all this, the 55 fundamental origin of these protean rogue wave phenomena is still $\frac{121}{121}$ is a subject of the station of the Measlem Cylin 121 Figure 1.1 September 10 and the coupled NLS equations (or the Manakov system) 122
56 far from being completely understood Originally, the rogue waves terminology refers to the transient out of nowhere and have been held responsible for a large numfar from being completely understood.

 58 physical contexts, much activity has been undertaken on the non-
 58 physical contexts, much activity has been undertaken on the non- 59 linear dynamics which may contribute to shedding light on some the two- of three-wave resonance equations $\left(21-25\right)$, were ob-60 of basic features of rogue waves, within the framework of inte-
60 of basic features of rogue waves, within the framework of inte-

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²⁰ Article history: **Article history:** The intriguing dark and composite rogue wave dynamics in coupled Hirota systems are unveiled, based ⁸⁶ 21 87 on the exact explicit rational solutions obtained under the assumption of equal background height. It is 22 Received in revised form 17 July 2014 **Found that a dark rogue wave state would occur** as a result of the strong coupling between two field 88 23 Acception 2014 Components with large wavenumber difference, and there would appear plenty of composite structures 89 24 Communicated by CP Docting that are attributed to the specific wavenumber difference and the free choice of three independent struc- 90 25 91 tural parameters. The coexistence of different fundamental rogue waves in such coupled Hirota systems 26 *Reywords:* 92 92 is also demonstrated.

33 **1. Introduction 1. Introduction 1. Introduction 1. It has now been generally** $\frac{99}{2}$ 34 **100** accepted that the rational solutions, if localized on both space 100 ³⁵ Originally, the rogue waves terminology refers to the transient and time, can be appropriate for the rogue wave description. One ¹⁰¹ typical, most-studied example is the Peregrine soliton [\[12\],](#page--1-0) the ear Schrödinger (NLS) equation. As it involves dynamics featuring a peak amplitude three times the background height and a double localization on a finite background, such a rational soliton was frequently used as a prototypical profile for a single rogue wave event occurred in reality. Recent breakthroughs in observation of Peregrine soliton in deep water $[4]$, plasmas $[13]$, and an optical fiber [\[14\]](#page--1-0) provide direct evidences for this hypothesis. Experiments also showed that the actual dynamics of the so-called super rogue waves can be well mimicked by the higher-order rational solutions whose peak amplitude increases progressively to a factor of more than five $[5]$.

 57 While the experimental investigation is booming in diverse $(15-17)$, the Davey-Stewartson equation [10], the coupled finola 123 $\frac{61}{127}$ able in the scalar NLS model. Of particular interest are the dark $\frac{127}{127}$ 62 128 and composite structures; the former features a single hole on a 63 129 nonzero background [\[19,21\],](#page--1-0) while the latter can be identified as In many scenarios, several scalar waves or several components of a vector wave need to be considered in order to account for the significant interaction processes. Understanding the underlying physics naturally requires consideration of the coupled system of equations rather than of the scalar model. Recently, the fundamental rogue wave solutions for some coupled integrable equa-[\[15–17\],](#page--1-0) the Davey–Stewartson equation $[18]$, the coupled Hirota (CH) equations [\[19\],](#page--1-0) the Hirota–Maxwell–Bloch equations [\[20\],](#page--1-0) and the two- or three-wave resonance equations [\[21–23\],](#page--1-0) were obtained, exhibiting intricate structures that are generally unattain-

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3 An issue closely related to the complex dynamics research is $\begin{array}{ccc} a & \upharpoonright & i \end{array}$ ($\begin{array}{ccc} a & \downarrow \upharpoonright & j \end{array}$ 5 turbations or how the modulational instability (MI) of the back-
 $\frac{2c}{L}$ $\frac{2c}{L}$ $\frac{4c}{L}$ $\frac{4c}{L}$ 6 ground fields develops. In the past few years, there appeared a where a ($>$ 0), k_j , and ω_j ($j=1,2$) are real parameters and are $^{-72}$ ⁷ number of significant works that were dedicated to this interest-
⁷³ 8 ing topic, mainly in the framework of the focusing NLS equation the contract of the state of the focusing NLS equation ⁷⁶ eration of deterministic rogue waves were also discussed in the the second service in the second the second to the ⁷⁶ 11 parity-time (PT)-symmetric coupled NLS equations [\[29\].](#page--1-0) Most re-
(i.e. $\delta > 0$) without loss of generality Moreover, for the sake of 12 cently, Grelu, Soto-Crespo, and the author showed numerically that the set of the state of the same of the same of $\frac{1}{28}$ ¹³ the bright–dark rogue waves formed in a two-wave resonance sys-
¹³ 14 tem could be stable in spite of the onset of MI $\begin{bmatrix} 30 \end{bmatrix}$.

15 In this Letter, we only explore the intriguing dark and com-
 $1/2$ in the interactional contract the interaction of the int 16 posite rogue wave dynamics in the CH equations, by presenting $\frac{d}{dx}$ and $\frac{d}{dx}$ in Ref. [15], can be simplified to their most explicit forms, $\frac{82}{x}$ 17 the rational solutions in an exact explicit form and in a perspic-
17 the rational solutions in an exact explicit form and in a perspic-18 uous manner that the physics community can follow. The specific the state individually parts. The algebraic manipulations involved are 19 parameter conditions under which these dark or composite rogue and straighted ward, and on carrier to Ref. [50] for eccentral 85 20 waves can form will also be provided. In addition, we demonstrate we will be encoded, these rational solutions can be written 21 a further the possibility that different rogue wave structures can co-22 exist for the same initial parameters. $\begin{bmatrix} 2i\epsilon^2(3n + K)t + i(\delta - n)\xi + 32\epsilon^2 \end{bmatrix}$ 88

24 **2. The CH equations and exact rogue wave solutions**

27 doctor as the contract of t form, as

$$
iu_{t} + \frac{1}{2}u_{xx} + (|u|^{2} + |v|^{2})u
$$

\n
$$
+ i\epsilon[u_{xxx} + (6|u|^{2} + 3|v|^{2})u_{x} + 3uv^{*}v_{x}] = 0,
$$
\n
$$
iv_{t} + \frac{1}{2}v_{xx} + (|v|^{2} + |u|^{2})v
$$
\n
$$
+ i\epsilon[v_{xxx} + (6|v|^{2} + 3|u|^{2})v_{x} + 3vu^{*}u_{x}] = 0,
$$
\n(1)\n
$$
m = \pm \frac{\sqrt{2}}{2}[\sqrt{\delta^{2}(8a^{2} + \delta^{2})} - 4a^{2} + \delta^{2}]^{1/2},
$$
\n(2)\n
$$
S = \frac{\sqrt{2}}{2}[\sqrt{\delta^{2}(8a^{2} + \delta^{2})} + 4a^{2} - \delta^{2}]^{1/2}.
$$
\n(3)\n
$$
u_{0} = 0,
$$
\n(4)\n
$$
m = \pm \frac{\sqrt{2}}{2}[\sqrt{\delta^{2}(8a^{2} + \delta^{2})} + 4a^{2} - \delta^{2}]^{1/2}.
$$
\n(5)\n
$$
u_{0} = 0,
$$
\n(6)\n
$$
u_{0} = 0,
$$
\n(7)\n
$$
u_{0} = \frac{\sqrt{2}}{2}[\sqrt{\delta^{2}(8a^{2} + \delta^{2})} + 4a^{2} - \delta^{2}]^{1/2}.
$$
\n(8)\n
$$
u_{0} = 0,
$$
\n(9)\n
$$
u_{0} = 0.
$$

³⁶ where $u(t, x)$ and $v(t, x)$ are the complex envelopes of the two otherwise for $\frac{1}{2}$ a we can express the row wave solutions as 37 103 fields, *t* is the evolution variable, and *x* is a second independent ³⁸ variable. The subscripts stand for the partial derivatives and the $\sqrt{2}iK(4a^2-\delta^2+2n'\zeta)t+i\delta\theta+32\epsilon^2$ 39 asterisk represents the complex conjugation. The real parameter ϵ $u = u_0 \left[1 - \frac{u_0 u_0}{\sqrt{a^2 + u_0^2}} \right]$, (10) 105 40 scales the integrable perturbations of the simple Manakov system. ⁴¹ The terms inside square brackets in Eqs. (1) and (2) account for $v_1 = v_2 \left| 1 \right\rangle$ $21K(4a^2 - \delta^2 + 2\eta' \zeta)t - 1\delta\theta + 32\epsilon^2$ (11) ¹⁰⁷ ⁴² the third-order dispersion, self-steepening, and delayed nonlinear $A(2a^2 + \eta' \zeta)$ $A(2a^2 + \eta' \zeta)$ ⁴³ response effect, respectively. In optics, these higher-order nonlin-44 ear and dispersive effects turned out to be non-negligible when $\mu_{\text{M}} = \theta^2/(64\epsilon^2) + K^2(\eta^2 + \zeta)^2/(16\epsilon^2) + 16\epsilon^2/(\eta^2 + \zeta)^2$ and μ_{M} 45 the pulses become shorter than 100 fs [\[31\].](#page--1-0) Physically, because of $\theta = 16 \epsilon \chi + (\kappa - 6\eta \zeta) t$, with η being given by ⁴⁶ these extra terms, such a coupled system can be more appropri- $\sqrt{2}$ $\sqrt{2}$ ⁴⁷ ate than the Manakov one for describing the interaction of two $\eta' = \pm \frac{v}{c} \left[-\sqrt{\delta^2 (8a^2 + \delta^2) + 4a^2 - \delta^2} \right]^{1/2}$. (12) ¹¹³ ⁴⁸ surface waves (brought on by severe weather) in deep ocean $[1]$ and 2 surface $\frac{1}{2}$ surface waves (brought on by severe weather) in deep ocean $[1]$ ⁴⁹ and as well as for describing the propagation of ultrashort optical We need to point out that the solutions given by Eqs. (10) and (11) 115 ⁵⁰ pulses in a birefringent fiber or simultaneous propagation of two can also be applied to the $\delta = a$ case, because they are identical to ¹¹⁶ fields in a nonlinear channel [\[32\].](#page--1-0)

⁵² Mathematically, thanks to the integrability, Eqs. (1) and (2) can Moteworthy, the above piecewise solution forms have been ex-
⁵² 53 be solved by using an array of analytical tools such as the inverse pressed as a ratio of second-order polynomials, with the real and 119 ⁵⁴ scattering transform [\[33\],](#page--1-0) the Riemann problem method [\[34\],](#page--1-0) the imaginary parts being clearly separated so that the rogue wave ¹²⁰ 55 Darboux transformation [\[35\],](#page--1-0) the Hirota bilinear method [\[36\],](#page--1-0) and dynamics can be readily analyzed. Meanwhile, we have translated 121 ⁵⁶ others. Recently, we presented the fundamental rogue wave solu-
both forms of solutions along the x axis to make their center ex-
 122 57 tions to Eqs. (1) and (2) by use of the Darboux transformation [\[19\],](#page--1-0) actly on the origin. Obviously, due to the two-wave coupling, such 123 ⁵⁸ but with their interesting dark structures and the related param-
⁵⁸ but with their interesting dark structures and the related param-
124 rational solutions can exist in the whole paramer and the regime. ⁵⁹ eter conditions not clearly revealed. Particularly, a great variety of ent from those in the scalar Hirota equation which only exist in ¹²⁵ 60 novel composite rogue wave dynamics as well as the remarkable a limited regime [\[28,39\].](#page--1-0) More significantly, as the real parameter 126 ⁶¹ coexistence feature were not unveiled yet for such a coupled sys- η [in Eq. (8)] or η' [in Eq. (12)] has two possible values for given ¹²⁷ 62 tem. Our objective here is to find solutions to these basic problems. parameters *a* and *δ* (excluding the case *δ* = *a* where $η = η' = 0$), 128 63 As a matter of fact, the rogue wave solutions define the limit Eqs. (1) and (2) usually admit two families of fundamental rogue 129 64 of either Ma solitons or Akhmediev breathers arising from the wave solutions that could coexist for the same initial parameters. 130 ⁶⁵ modulationally unstable plane waves [\[14,37\].](#page--1-0) For this reason, we **Nevertheless, for the very special case** $\delta = a$, Eqs. (1) and (2) ¹³¹ novel composite rogue wave dynamics as well as the remarkable coexistence feature were not unveiled yet for such a coupled system. Our objective here is to find solutions to these basic problems. As a matter of fact, the rogue wave solutions define the limit introduce first the plane-wave solutions of Eqs. (1) and (2) ,

1 67 a composite of two rogue waves which in form is characterized by 2 68 the rational dependence on the fourth-order polynomials [\[22,23\].](#page--1-0) *^u*0*(t, ^x)* ⁼ *^a* 2exp − *i* 2*- ^k*1*^x* [−] *^ω*¹ 4*t ,* (3)

4 70 whether these rogue wave solutions can immune from small per*^v*0*(t, ^x)* ⁼ *^a* 2exp − *i* 2*- ^k*2*^x* [−] *^ω*² 4*t ,* (4)

where *a* (> 0), k_j , and ω_j ($j = 1, 2$) are real parameters and are connected by the dispersion relations

$$
\omega_{\text{Hg topl, Hainly in the harmonic of the focusing NLS equation}} \omega_j = a^2(3\kappa + 4) + k_j(6a^2 - k_j - k_j^2). \tag{5}
$$

Here we define $\kappa = k_1 + k_2$ and $\delta = k_1 - k_2$ and we assume $k_1 > k_2$ (i.e., $\delta > 0$) without loss of generality. Moreover, for the sake of simplicity, we have assumed the two interacting field components to have an equal background amplitude.

Then, the fundamental rogue wave solutions, given by Eqs. (44) and (45) in Ref. [\[19\],](#page--1-0) can be simplified to their most explicit forms, with the aid of separating the complex spectral parameter into the real and imaginary parts. The algebraic manipulations involved are fairly straightforward, and one can refer to Ref. [\[38\]](#page--1-0) for technical details. Specifically, if $\delta \geqslant a$, these rational solutions can be written explicitly as

exist for the same initial parameters.
\n²² **23**
$$
u = u_0 \left[1 - \frac{2i \zeta^2 (3\eta + K)t + i(\delta - \eta)\xi + 32\epsilon^2}{\Delta[(\delta - \eta)^2 + \zeta^2]} \right],
$$

25. The Ch equations and each figure wave solutions
\n
$$
v = v_0 \left[1 - \frac{2i \zeta^2 (3\eta + K)t - i(\delta + \eta)\xi + 32\epsilon^2}{\Delta[(\delta + \eta)^2 + \zeta^2]} \right],
$$
\n(7)

where $\Delta = \xi^2/(128\epsilon^2) + \zeta^2(3\eta + K)^2t^2/(32\epsilon^2) + 8\epsilon^2/\zeta^2$ and $\xi = 94$

$$
\eta = \pm \frac{\sqrt{2}}{2} \left[\sqrt{\delta^2 (8a^2 + \delta^2)} - 4a^2 + \delta^2 \right]^{1/2},\tag{8}
$$

$$
\varsigma = \frac{\sqrt{2}}{2} \left[\sqrt{\delta^2 (8a^2 + \delta^2)} + 4a^2 - \delta^2 \right]^{1/2}.
$$
 (9)

Otherwise, for $\delta < a$, we can express the rogue wave solutions as

$$
u = u_0 \left[1 - \frac{2iK(4a^2 - \delta^2 + 2\eta' \varsigma)t + i\delta\theta + 32\epsilon^2}{\Lambda(2a^2 + \eta' \varsigma)} \right],
$$
 (10)

$$
v = v_0 \left[1 - \frac{2iK(4a^2 - \delta^2 + 2\eta' \zeta)t - i\delta\theta + 32\epsilon^2}{\Lambda(2a^2 + \eta' \zeta)} \right],
$$
 (11)

 $M = θ²/(64ε²) + K²(η' + ζ)²t²/(16ε²) + 16ε²/(η' + ζ)²$ and $\theta = 16\epsilon x + (R - 6\eta' \zeta)t$, with η' being given by

$$
\eta' = \pm \frac{\sqrt{2}}{2} \left[-\sqrt{\delta^2 (8a^2 + \delta^2)} + 4a^2 - \delta^2 \right]^{1/2}.
$$
 (12)

51 fields in a nonlinear channel $[32]$. Eqs. (6) and (7) at this special parametric point. $\frac{117}{2}$ We need to point out that the solutions given by Eqs. (10) and (11) can also be applied to the $\delta = a$ case, because they are identical to

> Noteworthy, the above piecewise solution forms have been expressed as a ratio of second-order polynomials, with the real and imaginary parts being clearly separated so that the rogue wave dynamics can be readily analyzed. Meanwhile, we have translated both forms of solutions along the *x* axis to make their center exactly on the origin. Obviously, due to the two-wave coupling, such rational solutions can exist in the whole parameter regime, different from those in the scalar Hirota equation which only exist in wave solutions that could coexist for the same initial parameters.

 66 introduce first the plane-wave solutions of Eqs. (1) and (2), $\qquad \qquad$ indeed allow a family of more general fundamental rogue wave $\qquad 132$ Nevertheless, for the very special case $\delta = a$, Eqs. (1) and (2)

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