Physics Letters A ••• (••••) •••–•••

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Dark and composite rogue waves in the coupled Hirota equations

Shihua Chen

Department of Physics, Southeast University, Nanjing 211189, China

ARTICLE INFO

Article history:
Received 29 May 2014
Received in revised form 17 July 2014
Accepted 2 August 2014
Available online xxxx
Communicated by C.R. Doering

Keywords:
Rogue wave
Soliton
Coupled Hirota equations
Nonlinear dynamics

ABSTRACT

The intriguing dark and composite rogue wave dynamics in coupled Hirota systems are unveiled, based on the exact explicit rational solutions obtained under the assumption of equal background height. It is found that a dark rogue wave state would occur as a result of the strong coupling between two field components with large wavenumber difference, and there would appear plenty of composite structures that are attributed to the specific wavenumber difference and the free choice of three independent structural parameters. The coexistence of different fundamental rogue waves in such coupled Hirota systems is also demonstrated.

1. Introduction

Originally, the rogue waves terminology refers to the transient gigantic ocean waves of extreme amplitude that appear seemingly out of nowhere and have been held responsible for a large number of maritime disasters [1]. As these colossal surface waves are intrinsically difficult to monitor because of their fleeting existences and dangerousness, more and more effort has been devoted to the study of such rare extreme events in contexts that are similar in physical models but can be readily conducted in a laboratory environment [2]. The recent experimental progress includes the first ever observation of optical rogue waves in a photonic crystal fiber [3], the resurgence of surface rogue waves in a water wave tank [4,5], the realization of capillary rogue waves via the four-wave coupling [6], the generation of dissipative rogue waves in a mode-locked laser [7], the appearance of spatiotemporal rogue waves via optical filamentation [8], and the refocusing of the time-reversed rogue waves in deep water [9], to name a few. These experimental studies show that rogue waves possess some hallmark phenomenological features, e.g., they are extremely large and steep compared with typical events, occur in a nonlinear medium, and follow an unusual *L*-shaped statistics [3,10,11]. Despite all this, the fundamental origin of these protean rogue wave phenomena is still far from being completely understood.

While the experimental investigation is booming in diverse physical contexts, much activity has been undertaken on the nonlinear dynamics which may contribute to shedding light on some of basic features of rogue waves, within the framework of integrable nonlinear wave equations [10]. It has now been generally accepted that the rational solutions, if localized on both space and time, can be appropriate for the rogue wave description. One typical, most-studied example is the Peregrine soliton [12], the simplest of a hierarchy of rational solutions of the scalar nonlinear Schrödinger (NLS) equation. As it involves dynamics featuring a peak amplitude three times the background height and a double localization on a finite background, such a rational soliton was frequently used as a prototypical profile for a single rogue wave event occurred in reality. Recent breakthroughs in observation of Peregrine soliton in deep water [4], plasmas [13], and an optical fiber [14] provide direct evidences for this hypothesis. Experiments also showed that the actual dynamics of the so-called super rogue waves can be well mimicked by the higher-order rational solutions whose peak amplitude increases progressively to a factor of more than five [5].

In many scenarios, several scalar waves or several components of a vector wave need to be considered in order to account for the significant interaction processes. Understanding the underlying physics naturally requires consideration of the coupled system of equations rather than of the scalar model. Recently, the fundamental rogue wave solutions for some coupled integrable equations, such as the coupled NLS equations (or the Manakov system) [15–17], the Davey–Stewartson equation [18], the coupled Hirota (CH) equations [19], the Hirota–Maxwell–Bloch equations [20], and the two- or three-wave resonance equations [21–23], were obtained, exhibiting intricate structures that are generally unattainable in the scalar NLS model. Of particular interest are the dark and composite structures; the former features a single hole on a nonzero background [19,21], while the latter can be identified as

E-mail address: cshua@seu.edu.cn.

http://dx.doi.org/10.1016/j.physleta.2014.08.004

a composite of two rogue waves which in form is characterized by the rational dependence on the fourth-order polynomials [22,23].

An issue closely related to the complex dynamics research is whether these rogue wave solutions can immune from small perturbations or how the modulational instability (MI) of the background fields develops. In the past few years, there appeared a number of significant works that were dedicated to this interesting topic, mainly in the framework of the focusing NLS equation [24–26] or its extended form [27,28]. The MI and the related generation of deterministic rogue waves were also discussed in the parity-time ($\mathcal{PT}$)-symmetric coupled NLS equations [29]. Most recently, Grelu, Soto-Crespo, and the author showed numerically that the bright–dark rogue waves formed in a two-wave resonance system could be stable in spite of the onset of MI [30].

In this Letter, we only explore the intriguing dark and composite rogue wave dynamics in the CH equations, by presenting the rational solutions in an exact explicit form and in a perspicuous manner that the physics community can follow. The specific parameter conditions under which these dark or composite rogue waves can form will also be provided. In addition, we demonstrate further the possibility that different rogue wave structures can coexist for the same initial parameters.

2. The CH equations and exact rogue wave solutions

For our studies, we write the CH equations, in dimensionless form, as

$$iu_t + \frac{1}{2}u_{xx} + \left(|u|^2 + |v|^2\right)u + i\epsilon\left[u_{xxx} + \left(6|u|^2 + 3|v|^2\right)u_x + 3uv^*v_x\right] = 0, \tag{1}$$

$$iv_t + \frac{1}{2}v_{xx} + \left(|v|^2 + |u|^2\right)v + i\epsilon\left[v_{xxx} + \left(6|v|^2 + 3|u|^2\right)v_x + 3vu^*u_x\right] = 0, \tag{2}$$

where $u(t,x)$ and $v(t,x)$ are the complex envelopes of the two fields, t is the evolution variable, and x is a second independent variable. The subscripts stand for the partial derivatives and the asterisk represents the complex conjugation. The real parameter ϵ scales the integrable perturbations of the simple Manakov system. The terms inside square brackets in Eqs. (1) and (2) account for the third-order dispersion, self-steepening, and delayed nonlinear response effect, respectively. In optics, these higher-order nonlinear and dispersive effects turned out to be non-negligible when the pulses become shorter than 100 fs [31]. Physically, because of these extra terms, such a coupled system can be more appropriate than the Manakov one for describing the interaction of two surface waves (brought on by severe weather) in deep ocean [1] and as well as for describing the propagation of ultrashort optical pulses in a birefringent fiber or simultaneous propagation of two fields in a nonlinear channel [32].

Mathematically, thanks to the integrability, Eqs. (1) and (2) can be solved by using an array of analytical tools such as the inverse scattering transform [33], the Riemann problem method [34], the Darboux transformation [35], the Hirota bilinear method [36], and others. Recently, we presented the fundamental rogue wave solutions to Eqs. (1) and (2) by use of the Darboux transformation [19], but with their interesting dark structures and the related parameter conditions not clearly revealed. Particularly, a great variety of novel composite rogue wave dynamics as well as the remarkable coexistence feature were not unveiled yet for such a coupled system. Our objective here is to find solutions to these basic problems.

As a matter of fact, the rogue wave solutions define the limit of either Ma solitons or Akhmediev breathers arising from the modulationally unstable plane waves [14,37]. For this reason, we introduce first the plane-wave solutions of Eqs. (1) and (2),

$$u_0(t,x) = \frac{a}{2\epsilon}\exp\left[-\frac{i}{2\epsilon}\left(k_1x - \frac{\omega_1}{4\epsilon}t\right)\right], \tag{3}$$

$$v_0(t,x) = \frac{a}{2\epsilon}\exp\left[-\frac{i}{2\epsilon}\left(k_2x - \frac{\omega_2}{4\epsilon}t\right)\right], \tag{4}$$

where a (>0), k_j, and ω_j $(j=1,2)$ are real parameters and are connected by the dispersion relations

$$\omega_j = a^2(3\kappa + 4) + k_j\left(6a^2 - k_j - k_j^2\right). \tag{5}$$

Here we define $\kappa = k_1 + k_2$ and $\delta = k_1 - k_2$ and we assume $k_1 > k_2$ (i.e., $\delta > 0$) without loss of generality. Moreover, for the sake of simplicity, we have assumed the two interacting field components to have an equal background amplitude.

Then, the fundamental rogue wave solutions, given by Eqs. (44) and (45) in Ref. [19], can be simplified to their most explicit forms, with the aid of separating the complex spectral parameter into the real and imaginary parts. The algebraic manipulations involved are fairly straightforward, and one can refer to Ref. [38] for technical details. Specifically, if $\delta \geqslant a$, these rational solutions can be written explicitly as

$$u = u_0\left[1 - \frac{2i\varsigma^2(3\eta + K)t + i(\delta - \eta)\xi + 32\epsilon^2}{\Delta[(\delta - \eta)^2 + \varsigma^2]}\right], \tag{6}$$

$$v = v_0\left[1 - \frac{2i\varsigma^2(3\eta + K)t - i(\delta + \eta)\xi + 32\epsilon^2}{\Delta[(\delta + \eta)^2 + \varsigma^2]}\right], \tag{7}$$

where $\Delta = \xi^2/(128\epsilon^2) + \varsigma^2(3\eta + K)^2t^2/(32\epsilon^2) + 8\epsilon^2/\varsigma^2$ and $\xi = 16\epsilon x + (R + 2\eta K)t$, with $K = 3\kappa + 2$, $R = 3\delta^2 + 3\kappa^2 + 4\kappa - 36a^2$, and two real parameters η and ς being defined by

$$\eta = \pm\frac{\sqrt{2}}{2}\left[\sqrt{\delta^2\left(8a^2 + \delta^2\right)} - 4a^2 + \delta^2\right]^{1/2}, \tag{8}$$

$$\varsigma = \frac{\sqrt{2}}{2}\left[\sqrt{\delta^2\left(8a^2 + \delta^2\right)} + 4a^2 - \delta^2\right]^{1/2}. \tag{9}$$

Otherwise, for $\delta < a$, we can express the rogue wave solutions as

$$u = u_0\left[1 - \frac{2iK(4a^2 - \delta^2 + 2\eta'\varsigma)t + i\delta\theta + 32\epsilon^2}{\Lambda(2a^2 + \eta'\varsigma)}\right], \tag{10}$$

$$v = v_0\left[1 - \frac{2iK(4a^2 - \delta^2 + 2\eta'\varsigma)t - i\delta\theta + 32\epsilon^2}{\Lambda(2a^2 + \eta'\varsigma)}\right], \tag{11}$$

where $\Lambda = \theta^2/(64\epsilon^2) + K^2(\eta' + \varsigma)^2t^2/(16\epsilon^2) + 16\epsilon^2/(\eta' + \varsigma)^2$ and $\theta = 16\epsilon x + (R - 6\eta'\varsigma)t$, with η' being given by

$$\eta' = \pm\frac{\sqrt{2}}{2}\left[-\sqrt{\delta^2\left(8a^2 + \delta^2\right)} + 4a^2 - \delta^2\right]^{1/2}. \tag{12}$$

We need to point out that the solutions given by Eqs. (10) and (11) can also be applied to the $\delta = a$ case, because they are identical to Eqs. (6) and (7) at this special parametric point.

Noteworthy, the above piecewise solution forms have been expressed as a ratio of second-order polynomials, with the real and imaginary parts being clearly separated so that the rogue wave dynamics can be readily analyzed. Meanwhile, we have translated both forms of solutions along the x axis to make their center exactly on the origin. Obviously, due to the two-wave coupling, such rational solutions can exist in the whole parameter regime, different from those in the scalar Hirota equation which only exist in a limited regime [28,39]. More significantly, as the real parameter η [in Eq. (8)] or η' [in Eq. (12)] has two possible values for given parameters a and δ (excluding the case $\delta = a$ where $\eta = \eta' = 0$), Eqs. (1) and (2) usually admit two families of fundamental rogue wave solutions that could coexist for the same initial parameters.

Nevertheless, for the very special case $\delta = a$, Eqs. (1) and (2) indeed allow a family of more general fundamental rogue wave

Please cite this article in press as: S. Chen, Dark and composite rogue waves in the coupled Hirota equations, Phys. Lett. A (2014), http://dx.doi.org/10.1016/j.physleta.2014.08.004