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Modified diffusion with memory for cyclone track fluctuations

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ABSTRACT

Fluctuations in a time series for tropical cyclone tracks are investigated based on an exponentially modified Brownian motion. The mean square displacement (MSD) is evaluated and compared to a recent work on cyclone tracks based on fractional Brownian motion (fBm). Unlike the work based on fBm, the present approach is found to capture the behavior of MSD versus time graphs for cyclones even for large values of time.

1. Introduction

Predicting trajectories of tropical cyclones, also referred to as typhoons or hurricanes, remains to be a challenging task in spite of full scale numerical simulations [1] and application of various statistical analyzes [2–4]. Fluctuations of cyclone tracks in a given geographical area, however, seem to follow patterns that appear to obey a universal law [5,6]. In a recent paper, for example [6], the longitude and latitude coordinates of cyclones were plotted against time to analyze fluctuations from the mean track of a cyclone. Designating the position of a cyclone as x, Ref. [6] essentially investigated fluctuations using a power law corresponding to a mean square displacement (MSD), $\langle [x(t+t') - x(t)]^2 \rangle \sim t^{\alpha}$, where t is time. This type of MSD is obtained when fluctuations are those of fractional Brownian motion and the exponent α takes values smaller (subdiffusive) or larger (superdiffusive) than 1 [7]. A log–log plot of MSD versus time would yield an α as a slope of a straight line graph. The results obtained for nine cyclones in Ref. [6], however, showed that there is a spread in the values of α, and a plot of the log of MSD versus log time (Figs. 3a and 3b of [6]) exhibits a downward curve at longer times rather than a continuous straight line. This downward curve for almost all cyclones investigated was attributed to a probable lack of statistics. In this paper, we present a non-Markovian stochastic process which could account for downward curves appearing at longer times in an MSD versus time graph of cyclone track fluctuations.

In the next section, we begin by parametrizing random fluctuations where a factor $f(T-t)\,h(t)$ modulates ordinary Brownian motion for time $0 \le t \le T$, with $f(T-t)$ a memory function [8]. The probability density function is evaluated as well as the corresponding mean square displacement. The form, $\langle [x(t+t') - x(t)]^2 \rangle \sim t^{\alpha}$, is shown as a special case. Motivated by the challenge to match empirical data with the model, we select from an array of possible memory functions [8]. An exponentially modified Brownian motion is then taken and shown to yield the observed downward curve at longer times found in Ref. [6] for log–log plots of MSD versus time.

2. Parametrizing the effects of memory

We model a fluctuating variable x with memory by parametrizing its evolution in time as,

$$x(T) = x_0 + \int_0^T f(T-t)\,h(t)\,dB(t), \qquad (1)$$

where x_0 is an initial value, $f(T-t)$ is a memory function, $h(t)$ a function of time, and $B(t)$ is the usual Wiener process. As time t ranges from 0 to T, the $f(\tau - t)h(t)$ modulates the evolution of ordinary Brownian motion $B(t)$ thereby affecting the value or history of $x(T)$. In general, the explicit form of $f(T-t)$ and $h(t)$ can be chosen depending on the process being modeled.

The probability density function for this type of fluctuations with strong correlations or memory can be evaluated. Given Eq. (1), one could have an ensemble of all possible paths which

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start at x_0 at time $t = 0$ and ask what the probability would be that these paths end at a specific endpoint $x(T) = x_T$ at a later time, $t = T$. Following Feynman's sum-over-all possible histories [8–10], we consider all possible paths $x(T)$ which satisfy the δ-function constraint,

$$\delta\big(x(T) - x_T\big) = \delta\left(x_0 + \int_0^T f(T-t)\,h(t)\,\omega(t)dt - x_T\right). \quad (2)$$

Here, we used Eq. (1) and expressed $dB(t) = \omega(t)\,dt$, where $\omega(t)$ is a random white noise variable [11]. For paths satisfying the δ-function constraint, the conditional probability density function $P(x_T, T; x_0, 0)$ can be obtained by evaluating the expectation value $E(\delta(x(T) - x_T))$, i.e.,

$$P(x_T, T; x_0, 0) = E\big(\delta\big(x(T) - x_T\big)\big) = \int \delta\left(x_0 + \int_0^T f(T-t)\,h(t)\,\omega(t)dt - x_T\right) d\mu, \quad (3)$$

where $d\mu$ is the Gaussian white noise measure [11]. Writing the delta function in terms of its Fourier representation we have,

$$P(x_T, T; x_0, 0) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} dk \exp\big\{ik\big[(x_0 - x_T)\big]\big\} \times \int \exp\left\{ik\int_0^T f(T-t)\,h(t)\,\omega(t)dt\right\} d\mu. \quad (4)$$

For the integration over $d\mu$, we can use the characteristic functional [11],

$$\int \exp\left\{i\int_0^T \omega(t)\xi(t)\,dt\right\} d\mu = \exp\left\{-\frac{1}{2}\int_0^T \xi^2(t)\,dt\right\}, \quad (5)$$

which is the Fourier transform of the Gaussian white noise measure $d\mu$. Using Eq. (5) with, $\xi(t) = k\,f(T-t)\,h(t)$, we obtain from Eq. (4) the equation,

$$P(x_T, T; x_0, 0) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \exp\left\{ik\big[(x_0 - x_T)\big] - \frac{k^2}{2}\int_0^T \big[f(T-t)\,h(t)\big]^2 dt\right\} dk. \quad (6)$$

The remaining integral is a Gaussian integral which can be evaluated to yield the form [8],

$$P(x_T, T; x_0, 0) = \left(2\pi\int_0^T \big[f(T-t)\,h(t)\big]^2 dt\right)^{-\frac{1}{2}} \times \exp\left(-\left[\int_0^T \big[f(T-t)\,h(t)\big]^2 dt\right]^{-1} \times \frac{(x_T - x_0)^2}{2}\right). \quad (7)$$

Further simplification of the probability density function, Eq. (7), would depend on the explicit choice of the memory function $f(T-t)$ and $h(t)$.

3. Mean square displacement with memory

Given Eq. (7), displacements of a fluctuating variable x, on the average, can be obtained by looking at the MSD which measures the degree of deviation from a mean value $\langle x\rangle$ given by [12],

$$\mathrm{MSD} = \big\langle \big(x - \langle x\rangle\big)^2\big\rangle = \big\langle x^2\big\rangle - \langle x\rangle^2. \quad (8)$$

A calculation of the second moment,

$$\big\langle x^2\big\rangle = \int_{-\infty}^{+\infty} x^2\,P(x, T; x_0, 0)\,dx = \left(2\pi\int_0^T \big[f(T-t)h(t)\big]^2 dt\right)^{-\frac{1}{2}} \times \int_{-\infty}^{+\infty} x^2 \exp\left(-\left[\int_0^T \big[f(T-t)h(t)\big]^2 dt\right]^{-1} \frac{(x-x_0)^2}{2}\right) dx, \quad (9)$$

yields,

$$\big\langle x^2\big\rangle = x_0^2 + \int_0^T \big[f(T-t)h(t)\big]^2 dt. \quad (10)$$

With this, Eq. (8) becomes (let, $\langle x\rangle = x_0$),

$$\mathrm{MSD} = \int_0^T \big[f(T-t)h(t)\big]^2 dt. \quad (11)$$

Clearly, when the memory function $f(T-t)$ is simply a constant $\sqrt{2D}$ and $h(t) = 1$, Eq. (11) yields the mean square displacement MSD_B of ordinary Brownian motion, i.e.,

$$\mathrm{MSD}_B = 2DT, \quad (12)$$

where D is the diffusion coefficient. On the other hand choosing a memory function of the form,

$$f(T-t) = \frac{(T-t)^{H-1/2}}{\Gamma(H+1/2)}, \quad (13)$$

with $h(t) = 1$, allows us to write Eq. (1) as,

$$x(T) = x_0 + B^H(T), \quad (14)$$

where $B^H(T)$ is a fractional Brownian motion defined in the Riemann–Liouville representation by [7],

$$B^H(T) = \frac{1}{\Gamma(H+\frac{1}{2})}\int_0^T (T-t)^{H-1/2} dB(t). \quad (15)$$

In Eq. (15), H is the Hurst exponent with values $0 < H < 1$. With Eq. (13), the mean square displacement, Eq. (11), becomes,

$$\mathrm{MSD}_{\mathrm{fBm}} = AT^\alpha, \quad (16)$$

where T is time, $\alpha = 2H$, and $A = 1/2H\ \Gamma(H+1/2)^2$. Eq. (16) is essentially the form of MSD considered in Ref. [6] which investigated fluctuations in cyclone tracks. A considerable spread of values for α was obtained and a histogram showed a peak-value of $\alpha = 1.65$. In a log–log plot of MSD versus time, however, Eq. (16) is not able to account for a downward curve appearing at longer