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Ultrawide low frequency band gap of phononic crystal in nacreous composite material

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ABSTRACT

The band structure of a nacreous composite material is studied by two proposed models, where an ultrawide low frequency band gap is observed. The first model (tension-shear chain model) with two phases including brick and mortar is investigated to describe the wave propagation in the nacreous composite material, and the dispersion relation is calculated by transfer matrix method and Bloch theorem. The results show that the frequency ranges of the pass bands are quite narrow, because a special tension-shear chain motion in the nacreous composite material is formed by some very slow modes. Furthermore, the second model (two-dimensional finite element model) is presented to investigate its band gap by a multi-level substructure scheme. Our findings will be of great value to the design and synthesis of vibration isolation materials in a wide and low frequency range. Finally, the transmission characteristics are calculated to verify the results.

1. Introduction

Most natural materials (or biomaterials) are complicated composites with hard and soft materials as their essential components. These composite biomaterials, though consisting of some relatively weak constituents, are featuring excellent mechanical properties after lots of years of evolutions. They have motivated the studies of an increasing number of academics on material and mechanics. Many natural biomaterials are organized in hierarchical structures, featured by multifunctionality, self-healing capability, and outstanding mechanical properties attributing to their material structures [1]. The smallest building blocks in most of these biomaterials are on the nanometer length scale. The nanometer size of the components, in particular, contributes to the optimum strength and maximum tolerance of flaws [2].

With nanocomposites as the major components, nacre, a sort of biomaterials, presents the impressive toughness and stiffness characteristics, as well as the insensitivity to flaws. Nacre has accordingly attracted considerable interest from researchers. Seashells are observed to be well organized in the microstructure in the form of staggered mineral platelets (100–500 nm in length) glued together by the collagen constituent phases [3,4]. The layer of the collagen constituent phases plays an important role for the stress distribution in nacre biocomposites. Nacre materials feature high stiffness when the collagen constituent phases are incompressible, while the mineral platelets between the adjacent collagen layers additionally enhancing the stiffness of the nacre under shear and transverse tension [5]. Through mechanical tensile and shear tests, three mechanisms of failure at the interfaces between the mineral platelets and collagen layers are observed: fracture of the mineral platelets, toughening due to the friction derived from nanoasperities, and toughening due to the collagen glue [6]. Moreover, the crack-induced stress intensification and the dependence on crack size can also be diminished in macro-scale nacreous materials [7]. This feature of nacre materials can be ascribed to the "Brick-and-Mortar" structure as well as the coordination of mechanical properties between "Brick" (hard materials like minerals) and "Mortar" (soft materials like collagens) phases. A recent interesting and meaningful research paper [8] shows that hierarchical staggered structures such as bone and nacre are of great value to the design of new composites for mitigating vibration and absorbing shock with excellent damping properties and high toughness.

Most researchers on nacreous composite materials seem to concentrate on the static characteristics, while sparing much less time for the wave propagation characteristics. It should be noticed that nacreous composite materials exhibit periodicity in the soft and hard hierarchical structures, which is the basic characteristic of phononic crystals [9,10]. They are capable of tailoring the wave propagation through some frequency ranges ("band gaps") in which the propagation of sound and elastic waves is

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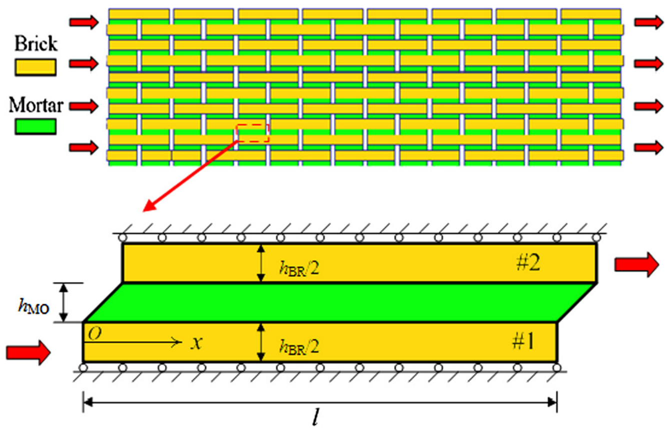

Fig. 1. Sketch of a TSC model with wave propagation.

forbidden [11,12]. In addition, they also exhibit other interesting properties [13,14]. What kind of dispersion relation or band characteristics do nacreous composite materials have? What effect will soft and hard hierarchical structures make on the band characteristics of nacreous composite materials? To answer these questions, further explorations into the wave propagation of nacreous composite materials are in need. This paper will study a tension-shear chain model and a two-dimensional finite element model of a nacreous composite material for its band structure and transmission characteristics. The outline of the paper is as follows: a tension-shear chain model about the wave propagation in the nacreous composite material is described in Section 2; the band structure of the two-dimensional model with finite element method is calculated in Section 3; the transmission characteristics about the two-dimensional model are presented in Section 4; and then the summary is followed in Section 5.

2. One-dimensional tension-shear chain model

To shed light on the stress field in materials with the Brick and Mortar (B-and-M) composite structure, many theoretical attempts have been made based on a variety of assumptions for the structure and the constitutive law of composite materials. Most researchers spare much less time for the dynamic characteristics of the B-and-M composite structure. As our attention in this paper is mainly paid to the band structures and wave propagation characteristics of this B-and-M composite structure, we propose a simplified model for the wave propagation problem by referring to the "tension-shear chain" (TSC) model [2] where the stress in the mortar layer of a B-and-M composite material is dominantly shear [Fig. 1]. Specifically, the model is set to be an infinite and periodic B-and-M composite phononic crystal structure with harmonic plane waves propagating in it. The Bloch theorem and symmetry of the B-and-M structure allow us to tackle the problem simply by considering a unit cell of one-dimensional phononic crystal consisting of a mortar layer sandwiched by two quarter bricks (marked by #1 and #2). The mortar is assumed to be elastic with shear modulus G_{MO} and thickness h_{MO}, and the bricks are modeled as the elastic material with Young's modulus E_{BR}, thickness h_{BR} and density ρ, while the inertial effect of the mortar is neglected. The length of the mortar and brick in the unit cell are both l.

Assuming the normal stresses are distributed uniformly in the two bricks. As the shear stress $\tau(x)$ developed in the mortar layer is applied on the surfaces of brick #1 and brick #2, the dynamic equilibrium of the two bricks suggests that the normal stresses $\sigma_1(x,t)$, $\sigma_2(x,t)$ and the shear stress $\tau(x,t)$ in the bricks are correlated through

$$\frac{h_{BR}}{2}\rho\ddot{u}_1(x,t)+\frac{h_{BR}}{2}\frac{d\sigma_1(x,t)}{dx}+\tau(x,t)=0,$$
$$\frac{h_{BR}}{2}\rho\ddot{u}_2(x,t)+\frac{h_{BR}}{2}\frac{d\sigma_2(x,t)}{dx}-\tau(x,t)=0. \tag{1}$$

The wave propagating equations about brick #1 and brick #2 are established in the frequency domain as

$$\tau(x)=-\frac{h_{BR}}{2}\frac{d\sigma_1(x)}{dx}-\frac{h_{BR}}{2}\rho\omega^2u_1(x),$$
$$\tau(x)=\frac{h_{BR}}{2}\frac{d\sigma_2(x)}{dx}+\frac{h_{BR}}{2}\rho\omega^2u_2(x). \tag{2}$$

Considering the elastic problem, the elastic relations of the two bricks are

$$\frac{d\sigma_1(x)}{dx}=E_{BR}u_1''(x),$$
$$\frac{d\sigma_2(x)}{dx}=E_{BR}u_2''(x). \tag{3}$$

The mortar layer undergoes purely elastic deformation and the shear stress therein is correlated with the relative longitudinal displacements between bricks #1 and #2 through

$$\tau(x)=\left[u_2(x)-u_1(x)\right]\frac{G_{MO}}{h_{MO}}. \tag{4}$$

The governing equations of this model are obtained from Eqs. (2)–(4) as

$$\begin{Bmatrix} u_1''(x) \\ u_2''(x) \end{Bmatrix}=\begin{bmatrix} A_1+A_2 & -A_1 \\ -A_1 & A_1+A_2 \end{bmatrix}\begin{Bmatrix} u_1(x) \\ u_2(x) \end{Bmatrix}, \tag{5}$$

where, $A_1=\frac{2G_{MO}}{h_{MO}h_{BR}E_{BR}}$, $A_2=-\frac{\rho\omega^2}{E_{BR}}$.

The general solutions can be obtained by solving Eq. (5) as

$$u_1(x)=C_1\frac{\exp(\sqrt{A_2}x)}{\sqrt{A_2}}-C_2\frac{\exp(\sqrt{2A_1+A_2}x)}{\sqrt{2A_1+A_2}}$$
$$-C_3\frac{\exp(-\sqrt{A_2}x)}{\sqrt{A_2}}+C_4\frac{\exp(-\sqrt{2A_1+A_2}x)}{\sqrt{2A_1+A_2}},$$
$$u_2(x)=C_1\frac{\exp(\sqrt{A_2}x)}{\sqrt{A_2}}+C_2\frac{\exp(\sqrt{2A_1+A_2}x)}{\sqrt{2A_1+A_2}}$$
$$-C_3\frac{\exp(-\sqrt{A_2}x)}{\sqrt{A_2}}-C_4\frac{\exp(-\sqrt{2A_1+A_2}x)}{\sqrt{2A_1+A_2}}, \tag{6}$$

where, C_1, C_2, C_3 and C_4 are unknown variables.

The boundary conditions are applied at the end of brick #1 ($x=l$) and the front of brick #2 ($x=0$), where the normal stress (or strain) vanishes, implying that,

$$u_1'(l)=0,$$
$$u_2'(0)=0. \tag{7}$$

Substituting Eq. (7) into Eq. (6), we can obtain

$$\mathbf{K}_1\begin{Bmatrix} C_1 \\ C_3 \end{Bmatrix}=\mathbf{H}_1\begin{Bmatrix} C_2 \\ C_4 \end{Bmatrix}, \tag{8}$$

where,

$$\mathbf{K}_1=\begin{bmatrix} \exp(\sqrt{A_2}l) & \exp(-\sqrt{A_2}l) \\ 1 & 1 \end{bmatrix},$$
$$\mathbf{H}_1=\begin{bmatrix} \exp(\sqrt{2A_1+A_2}l) & \exp(-\sqrt{2A_1+A_2}l) \\ -1 & -1 \end{bmatrix}. \tag{9}$$