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Controllable entanglement transfer via two parallel spin chains

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ABSTRACT

Transferring quantum states between nearby quantum processors is important for building up a powerful quantum computer. In this paper, we propose a controllable scheme to transfer bipartite entangled states using two open-ended spin-$\frac{1}{2}$ chains in parallel as a dual-rail quantum channel. We perform two sets of operations, one on one end of the chains at the beginning of the system evolution and the other on the other end of the chains at the time the transferred entanglement needs to be picked up. Among the operations employed in the scheme there are weak measurements with controllable strengths. By suitably choosing the strengths of these weak measurements, the entanglement transferability is pronouncedly improved, compared to that due to the spin chains' natural dynamics. In principle, the entanglement amount at the receiving site can be made arbitrarily close to that at the sending site, i.e., perfect entanglement transfer could be achieved asymptotically.

1. Introduction

The transfer of quantum states is undoubtedly very important in future's quantum information processing technology [1]. For long-distance quantum communication, photons are the most suited candidates to play the role of quantum information carriers flying from one to another in a faraway spatial location. For example, in quantum key distribution, the photons encoding a secret key via their polarization freedoms can easily travel along long optical fibers or through free space and can then be readily measured at an arriving location. However, in distributed quantum computation [2–4], not only transferring quantum states between quantum computers is important but also interfacing a quantum computer (say, arrays of spins or trapped ions) with optics is necessary. An idea to avoid such interfacing problems is to use the same physical systems for both the quantum computers and the quantum channels. For short distances, it is more suitable to adopt the collective phenomena, such as the natural dynamical evolution, of a permanently coupled chain of quantum systems to connect different nearby quantum processors or registers to build up a powerful quantum computer. In fact, by using a 1D spin chain as the data bus, Bose proposed a quantum state transfer protocol in which an unknown state can be efficiently transferred from one to another spin with certain fidelity via the spin chain's natural evolution [5]. Nevertheless, for a spin chain governed by a uniformly coupled Heisenberg Hamiltonian [5], perfect quantum state transfer is only possible for systems with two or three spins [6]. Subsequently, a number of approaches, such as engineered couplings [7–13], Gaussian wave-packet encoding [14–16], employment of specific pulses [17], weak coupling of the sending and receiving qubits to a quantum many-body system [12,18,19] and so on, have been proposed to achieve perfect or near perfect quantum state transfer. In addition to these strategies, Burgarth and Bose also suggested a dual-rail channel by adding an auxiliary spin chain to improve transfer capability of single-spin states [20–22]. With enough measurements carried out, their protocol will achieve conclusively perfect quantum state transfer with a success probability close to 1. The adding of an additional spin chain is actually not problematic and is even much easier in many experiments [23–25] that realize a whole bunch of parallel uncoupled chains rather than just a single one.

As is well known, entanglement is a key resource to realize various intriguing tasks in quantum information processing and quantum computing [1]. The capability of on-demand transfer of entanglement through spin chains is, of course, significant [26,27]. In particular, it is practically interesting to obtain entanglement between two independent spins at a receiving site through the process of transferring the entanglement as a whole prepared between two spins at a sending site. To achieve this task, the dual-rail channel based on using two parallel spin chains (cf. Fig. 1) is

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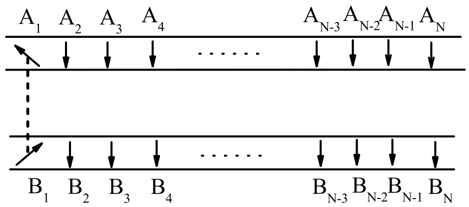

Fig. 1. The schematic setup for entanglement transfer through two parallel spin-$\frac{1}{2}$ chains each contains N spins. An entangled state to be transferred is encoded in spins A_1 and B_1 on each of which the sender performs a weak measurement before the system starts to evolve. Later, at a desired moment of time, the receiver performs a suitable set of operations on spins A_N and B_N to get them in the intended entangled state.

a naturally occurring setting. However, with respect to the issue of entanglement transfer, the answer to the question "Can the entanglement of an arbitrarily prepared bipartite entangled state be perfectly transferred through a pair of parallel spin chains?" is still not known completely. Our study in this work shows that in fact not all the bipartite entangled states can be transferred via two parallel spin chains. More concretely, we find out that there are states of two spins that are initially entangled at a sending site but later become always unentangled at a receiving site (i.e., the two spins at the receiving site remain separable during the entire time evolution). Also, there are states whose entanglement can be transferred to a destination, but during the system's natural evolution their entanglement appears with some delay [28], then suddenly vanishes, and after some time reappears again, etc. Here we propose a controllable scheme that allows us to improve the entanglement transfer in terms of the dual-rail protocol, especially to renew the transferability of those states whose entanglement cannot be transferred by natural evolution. Namely, we find that for bipartite entangled states of the form $\alpha|00\rangle + \beta|11\rangle$ ($|0\rangle \equiv |\downarrow\rangle$: spin-down state, $|1\rangle \equiv |\uparrow\rangle$: spin-up state and $|\alpha|^2 + |\beta|^2 = 1$), a large weight of the $|11\rangle$ component hinders its entanglement transfer. Therefore, in our scheme, we first lower the weight of the $|11\rangle$ component by means of weak measurements [29–39] with strength p on each of the two spins at the sending site. The weak measurement differs from the projective measurement in that the former does not completely collapse the system's measured state. Actually, such kind of measurements has been experimentally realized in several physical contexts [40–45]. Next, we let the system evolve as it should. And, finally, at a desired receiving site, we perform on each spin another weak measurement with strength q. By suitably choosing q we shall be able to transfer entanglement of any bipartite entangled states. In principle, the entanglement degree at the receiving site can be made in our scheme exactly equal to that at the sending site, i.e., perfect entanglement transfer could be achieved.

We structure our paper as follows. After this Introduction, in Section 2 we deal with a solvable model consisting of two parallel open-ended spin-$\frac{1}{2}$ chains each of which is characterized by nearest neighbor interactions and under a common uniform magnetic field. By means of dual-rail encoding, the process of entanglement transfer along the chains is investigated. It is shown that, due to natural evolution, not any entangled states can transfer their entanglement and the entanglement transfer, if it happens, cannot be perfect. Then, in Section 3, we propose a controllable scheme to circumvent such limitations imposed by the system's natural dynamics. By performing appropriate prior and posterior unsharp measurements, the entanglement transferability is considerably enhanced and, in principle, can be made asymptotically perfect. Finally, we conclude in Section 4.

2. Dual-rail transfer of entanglement

Consider, for generality, two 1D spin-$\frac{1}{2}$ graphs A and B, each of which contains N spins. The spins in graph A (B) are labeled $A_1, A_2, \ldots$ and A_N ($B_1, B_2, \ldots$ and B_N). There are no interactions between the graphs so the total Hamiltonian of the system can be written as [20]

$$H = H^{(A)} \otimes I^{(B)} + I^{(A)} \otimes H^{(B)}, \tag{1}$$

where $H^{(S)}$ ($S = A, B$) is the Hamiltonian of spin graph S and $I^{(S)}$ the identity operator. The authors of Refs. [46–48] studied spin rings, so for precise analytical formulation they had to introduce the cyclic boundary conditions which are a good approximation only for rings with a large radius. Here we are interested in linear open-ended spin chains (see Fig. 1), which represent the most natural geometry for an information transfer channel. Assuming the nearest neighbor Heisenberg interactions of equal strength and the common uniform magnetic field h, the Hamiltonians $H^{(A)}$ and $H^{(B)}$ in Eq. (1) are identical in form, i.e., for both $S = A$ and B,

$$H^{(S)} = -\frac{J}{2}\sum_{j=1}^{N-1}\left(\sigma_x^j\sigma_x^{j+1} + \sigma_y^j\sigma_y^{j+1} + \sigma_z^j\sigma_z^{j+1}\right) - h\sum_{j=1}^{N}\sigma_z^j, \tag{2}$$

where $\sigma_{x(y,z)}^j$ are the $x(y,z)$ Pauli matrices for the jth spin and $J > 0$ the coupling strength between nearest neighbors.

Let the two-spin entangled state to be transferred has the form

$$\left|\psi(0)\right\rangle_{A_1B_1} = \cos\theta|0\rangle_{A_1}|0\rangle_{B_1} + e^{i\phi}\sin\theta|1\rangle_{A_1}|1\rangle_{B_1}, \tag{3}$$

with $0 < \theta < \pi/2$ and $0 < \phi < \pi$. Unlike the transfer of single-spin states, the transfer of two-spin entangled states would be more subtle since the entanglement dynamics due to decoherence is very rich and sensitive to the form of the entangled state to be transferred (see, e.g., [28]). Hence, we should consider the whole range of possible values of ϕ and θ to explore the dependence of entanglement transferability on those parameters. The form (3) of the input state means that a dual-rail encoding is adopted: information is encoded in states of the first spin pair A_1B_1 of the two chains. As for the other spins, they are all prepared in the unexcited (i.e., spin-down) state $|0\rangle \equiv |\downarrow\rangle$. As $H^{(S)}$ commutes with $\sum_{j=1}^N \sigma_z^j$, there exists at most one excitation (i.e., one spin-up state) in each chain. For convenience, we denote by $|\mathbf{0}\rangle^{(S)} = |0\ldots0\ldots0\rangle_{S_1\ldots S_j\ldots S_N}$ the state with all the spins being unexcited and by $|\mathbf{j}\rangle^{(S)} = |0\ldots1\ldots0\rangle_{S_1\ldots S_j\ldots S_N}$ ($\mathbf{j} = \mathbf{1}, \mathbf{2}, \ldots, \mathbf{s}, \ldots, \mathbf{r}, \ldots, \mathbf{N}$) the state with only spin j being excited. The eigenstates $|\widetilde{m}\rangle^{(S)}$ and eigenenergies $E_m = E_m^{(A)} = E_m^{(B)}$ of the Hamiltonian (2) relevant to our problem can be derived as [5] $|\widetilde{m}\rangle^{(S)} = \{[\sqrt{2} + \delta_{m,1}(1-\sqrt{2})]/\sqrt{N}\}\sum_{j=1}^N \cos[\pi(m-1)(2j-1)/2N]|\mathbf{j}\rangle^{(S)}$ and $E_m = 2h + 2J\{1 - \cos[\pi(m-1)/N]\}$, with $m = 1, 2, \ldots, N$. Since the two parallel spin chains do not have any direct interactions, the excitation transfer in each chain can be dealt with independently. In this case, the transition amplitude of an excitation from a sth to an rth site in each chain takes the same form as

$$\begin{aligned} c_{sr}^{(S)}(t) &= {}^{(S)}\langle\mathbf{r}|e^{-iH^{(S)}t}|\mathbf{s}\rangle^{(S)} \\ &= \sum_{m=1}^{N} {}^{(S)}\langle\mathbf{r}|\widetilde{m}\rangle^{(S)}\,{}^{(S)}\langle\widetilde{m}|\mathbf{s}\rangle^{(S)} e^{-iE_m t}. \end{aligned} \tag{4}$$

As the values of $c_{sr}^{(A)}(t)$ and $c_{sr}^{(B)}(t)$ of the two identical chains are the same for all possible s and r, we ignore their superscripts (A) and (B) throughout the paper. For concreteness, we set $s = 1$ and $r = N$ in the following (i.e., the sender and the receiver are located near the opposite ends of the chains).