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Quantum metrology in correlated environments

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ABSTRACT

We analytically obtain the precision bounds of frequency measurements in correlated Markovian and non-Markovian environments using a variational approach. The metrological equivalence of product states and maximally entangled states persisting in maximally correlated Markovian and non-Markovian environments is verified using a standard Ramsey spectroscopy setup. Furthermore, we find that optimal measurements can be used to achieve a much higher resolution than standard Ramsey spectroscopy in correlated environments.

1. Introduction

Quantum metrology is a fundamental and important subject concerning the estimation of parameters under the constraints of quantum dynamics [1–5]. In order to assess the performance of a parameter estimation technique, one often utilizes analytic bounds on the error. For the mean-square error criterion, the Cramér–Rao bounds [6–8] are the most well known.

Environments can significantly impact quantum systems, playing a very important role in quantum metrology. Without suffering from environments, entangled states can achieve a higher resolution than the precision limits achievable with uncorrelated probes [9,10]. In real experiments, environments induce decoherence of systems, which can affect the measurement precision. Huelga et al. [11] first studied precision spectroscopy in the presence of Markovian dephasing, and showed that given a fixed number of particles n and a total available time T, uncorrelated and maximally entangled particles could achieve exactly the same precision when subject to Markovian dephasing. Recently, Matsuzaki et al. [12] and Chin et al. [13] independently explored quantum metrology in uncorrelated non-Markovian environments, demonstrating that metrological equivalence does not hold when the system is subject to non-Markovian dephasing.

Although environments can induce negative effects on the system, correlated environments may be useful. Correlations between environments can increase the quantum efficiency of transport [14]. In photosynthetic light harvesting, environmental correlation effects assist the excitation energy transfer [15]. Correlated environments can generate strong nonlocal memory effects, even though the local dynamics are Markovian [16]. The roles of environmental correlations have also been investigated in the non-Markovian dynamics of a spin chain system [17]. A generalized nonequilibrium fluctuation–dissipation relation has been found for strongly correlated environments [18]. Within a correlated environment, the structure of a quantum computer protected from decoherence has been analyzed, and the scaling equation obtained reflects competition between the dimension of the quantum system and the scaling dimension of the correlations [19].

In this study, we analyze whether metrological equivalence persists when a complete system composed of n two-level particles suffers from correlated Markovian and non-Markovian environments, which has been unexplored thus far. Trivially, correlated environments improve the precision by reducing the global dephasing rate. Remarkably, the correlation between environments could potentially enhance the precision when correlated environments maintain or increase the dephasing rate of the entire system. Therefore, we are interested in the best resolution that can be achieved by performing optimal measurements in correlated environments.

To study the precision bounds, we use the variational approach [20] in combination with some symmetries. Furthermore, we solve the open question left by Ref. [13]: whether the

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improvement of the precision bounds by the optimal measurement is on the order of 1 in uncorrelated Markovian as well as non-Markovian environments. For solving this problem with respect to correlated environments, we define a general function $F(\omega)$ (in Section 3), which can effectively express the impact of correlations among environments on the dephasing rate. Interestingly, we find that standard Ramsey spectroscopy is not optimal in correlated environments, and the optimal measurement can yield a much lower frequency uncertainty (better resolution). If the initial probes are in the product state, the precision bound decreases with the interrogation time t when the number of particles n is even. Thus, if experiments allow, the precision bound can be close to 0 when the interrogation time t is very large. Furthermore, when the number of particles n is odd, the precision bound is independent of the interrogation time t. The same situation exists when the initial probes are prepared in the maximally entangled state. These results indicate that certain symmetries play an important role in the achievable resolution. On the other hand, in some special non-Markovian environments, the opposite case exists. Moreover, compared with the uncorrelated environment case, the correlations allows us to obtain a better resolution by using the optimal measurement.

The remainder of this article is arranged as follows. In Section 2, we study the precision bounds in uncorrelated Markovian and non-Markovian environments with the help of the variational approach. The precision bounds in correlated Markovian and non-Markovian environments are primarily explored in Section 3. In Section 4, we discuss about the definition of Markovianity and non-Markovianity, the influence of odd and even particles on the precision bound, and the maximally correlated environments in the experiment. We draw our conclusions in Section 5.

2. Uncorrelated environments

Let us consider a global system composed of n particles. The Hamiltonian of each particle is described by $w_0 Z$ ($\hbar = 1$ throughout), where the eigenvectors of the Pauli operator Z are denoted by $(|0\rangle, |1\rangle)$. The n particles suffer from the corresponding n uncorrelated environments, which induce pure dephasing. The time evolution of the reduced density matrix of the system (for one particle) is given by

$$\rho_{ii}(t) = \rho_{ii}(0), \tag{1}$$

$$\rho_{01}(t) = \rho_{01}(0) e^{-2\gamma(t)}, \tag{2}$$

for $i = 0, 1$.

When an environment induces pure Markovian dephasing, the function $\gamma(t) = \gamma t$, where γ is the decay rate. Ramsey spectroscopy [21] obtains the same frequency resolution for both the maximally entangled state and the product state (using the same notation of Refs. [11] and [13]):

$$|\delta\omega_0|_e = |\delta\omega_0|_u = \sqrt{\frac{2e\gamma}{nT}}, \tag{3}$$

where T denotes the total duration of the experiment, and ω_0 is the atomic frequency.

The optimal frequency resolution from the best measurement can be obtained using the variational approach in Ref. [20]. The quantum Fisher information (QFI) is given by

$$\mathcal{F}_Q\left[\hat{\rho}_S(\phi)\right] = \min_{\hat{h}_E(\phi)} 4\left\langle\left[\hat{\mathcal{H}}(\phi) - \left\langle\hat{\mathcal{H}}(\phi)\right\rangle\right]^2\right\rangle_{\Phi}, \tag{4}$$

in which, $\hat{\mathcal{H}}(\phi) = \hat{H}_{S,E}(\phi) - \hat{h}_E(\phi)$ (see Appendix A), $|\Phi_{S,E}(\phi)\rangle$ is any purification of $\hat{\rho}_S(\phi)$, and $\hat{h}_E$ is the Hermitian operator in the environment space. Moreover, ϕ is the detuning between the frequency ω of the external oscillator and the atomic frequency ω_0 to which we intend to lock it. The best resolution is described by the expression

$$\delta\omega_0^2 = \frac{1}{N\mathcal{F}_Q\left[\hat{\rho}_S(\phi)\right]}, \tag{5}$$

where the total number of experiment data $N = T/t$.

According to the principle of variational approach in Ref. [20] (see Appendix A), we choose the state

$$\left|\Phi_{S,E}(\phi)\right\rangle = \prod_{i=1}^{n} e^{-i\phi t Z_i/2} e^{-i\arccos\left(\sqrt{P(\gamma t)}\right) Z_i Y_i^E} |\psi\rangle |0\rangle_E^{\otimes n}, \tag{6}$$

where $P(\gamma t) = \frac{1+\exp(-\gamma t)}{2}$; Z_i and Y_i^E are Pauli operators for the ith system and environment, respectively; and $|\psi\rangle$ denotes the initial state of the whole system. Suppose that the n systems are completely identical and that the corresponding n environments are also identical. Based on the symmetry, the Hermitian operator is given by

$$\hat{h}_E(\phi) = \sum_{i=1}^{n} \alpha X_i^E + \beta Y_i^E + \delta Z_i^E, \tag{7}$$

where α, β, and δ are three variational parameters. Then, substituting Eq. (6) and Eq. (7) into Eq. (4), we can obtain the minimum value by taking the derivative of above three variational parameters. As a result, the resolution is given by

$$\delta\omega_0^2(t) = \left(1 - \left\langle \sum_{i=1}^{n} Z_i/n \right\rangle_{\psi}^2\right) \frac{1 + nq[\exp(2\gamma t) - 1]}{qn^2 Tt}, \tag{8}$$

where $q = \frac{\Delta(\sum_{i=1}^n Z_i/n)^2}{1-\langle\sum_{i=1}^n Z_i/n\rangle_{\psi}^2}$ [22]. The best resolution is described by $|\delta\omega_0|_{opt} = \sqrt{\frac{2\gamma}{nT}}$ at the optimal interrogation time $t = \frac{1}{2\gamma\sqrt{n}}$, which occurred when $q = 1$, $\langle\sum_{i=1}^n Z_i/n\rangle_{\psi}^2 = 0$, and $n \gg 1$. Compared to the Ramsey spectroscopy case in Eq. (3), the maximum improvement of $\sqrt{e}$ in the resolution can be achieved by the optimal measurement.

Next, we consider that the environments induce pure non-Markovian dephasing. Similarly to Ref. [13], we utilize the simple power law form of $\gamma(t) = \gamma t^{\nu}$ for non-Markovian dephasing. Note that the $\nu = 1$ case corresponds to the Markovian case. Moreover, the $\nu = 2$ case is not a specific feature of the chosen model; it is a general consequence of the unitary evolution of the total system and environmental state.

Using the Ramsey spectroscopy setup, the resolution for an initial preparation of n particles in the product state $(\frac{|0\rangle+|1\rangle}{\sqrt{2}})^{\otimes n}$ and the maximally entangled state $\frac{|0\rangle^{\otimes n}+|1\rangle^{\otimes n}}{\sqrt{2}}$ are given by

$$|\delta\omega_0|_u^R = \sqrt{\frac{(2e\gamma\nu)^{1/\nu}}{nT}}, \tag{9}$$

with the optimal interrogation time $t_u = (\frac{1}{2\gamma\nu})^{(1/\nu)}$;

$$|\delta\omega_0|_e^R = \sqrt{\frac{(2e\gamma\nu)^{1/\nu}}{n^{(2-1/\nu)}T}}, \tag{10}$$

with the optimal interrogation time $t_e = (\frac{1}{2n\gamma\nu})^{\frac{1}{\nu}}$, respectively.

For the non-Markovian case, the function $P(\gamma t) = \frac{1+\exp(-\gamma t^{\nu})}{2}$ in Eq. (6). The optimal measurement achieves the following resolution, which is similar to that shown in Eq. (8):