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Superradiant emission from a partially inverted slab of two-level atoms

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ABSTRACT

Using the eigenmodes of the 1-D Lienard–Wiechert Green's function, I compute the superradiant emission from an initially partially inverted slab of two-level atoms. I find that in regime I, $(T=0) > n_{\max}$, there is no superradiant emission (where $n(T=0) = n^{ground}(T=0) - n^{excit}(T=0)$); in regime II, $n_{\max} > n(T=0) > n_{crit}$, there is weak intensity superradiance; while in regime III, $n_{crit} > n(T=0) \geq -1$, superradiant emission share the same features as that of an initially completely inverted system. I show, further, that in regime II, the superradiant emission intensity increases exponentially as a function of $-n(T=0)$, while in regime III, it increases nearly linearly as a function of $-n(T=0)$.

1. Introduction

Research on superradiance has been extensive since the seminal work of Dicke [1]. The review papers/books [2–4] summarize the advances made in this field over the last few decades. More recently [5], I summarized the analytical results for the expressions of the cooperative decay rate (CDR) and the associated cooperative Lamb shift (CLS) for the cases of both weak excitation and of complete inversion for different simple geometries at initial time. However, in order to follow the time-development of superradiance beyond the linear regime, it is necessary to solve the Maxwell–Bloch set of equations.

In [6], it proved useful for a deeper physical understanding of the superradiance process to formulate the dynamics of the emission from the two-level system if the polarization, the population difference and the electric field were expanded in a basis formed by the eigenfunctions of the 1-D Lienard–Wiechert (L–W) Green's function. Using the eigenfunction expansion, shown in earlier publications to be a powerful tool for analyzing superradiance and other phenomena in the linear regime [7–10], I showed it to be also a powerful tool for analyzing this and associated problems in the nonlinear regime. The mathematical expansion of the dynamical variables in eigenmodes permits the replacement of the Maxwell–Bloch equations describing the atoms–field interaction written as a system of coupled nonlinear partial differential equations to a system of coupled nonlinear ordinary differential equations for the expansion coefficients. Following these expansion coefficients in time allowed for a more transparent physical analysis of superradiance than existing numerical methods directly integrating the coupled Maxwell–Bloch equations [11].

In the present paper, I analyze, using the same eigenmode expansions, the problem of the emission from the slab if the atomic system is initially only partially inverted. This is a problem of physical interest for many experimental instances and in applications.

The main results obtained here are that for values of the initial population difference between the ground and excited state atoms $n(T=0) > n_{\max}$, there is no superradiant emission; for $n_{\max} > n(T=0) > n_{crit}$ there is weak intensity superradiance, as only the coefficient of the dominant eigenmode in the polarization expansion is significant; while for $n_{crit} > n(T=0) \geq -1$ (in our notation, $n(T=0) = -1$, represents a system that is completely inverted) multiple eigenmode expansion coefficients contribute to the electric field at either slab exit planes and the intensity of the superradiant emission is everywhere in this range within two orders of magnitude of the superradiant intensity for the case of an initially complete inversion. I show, further, that the superradiant emission intensity increases exponentially as a function of $-n(T=0)$, in the intermediate range; while it increases approximately linearly as a function of $-n(T=0)$ in the range of parameter including that of the complete inversion case.

The paper is organized as follows: in Section 2, I summarize the key properties of the 1D L–W eigenmodes. In Section 3, I give the results of integrating the coupled ordinary differential equations for the eigenmode expansion coefficients of the physical variables

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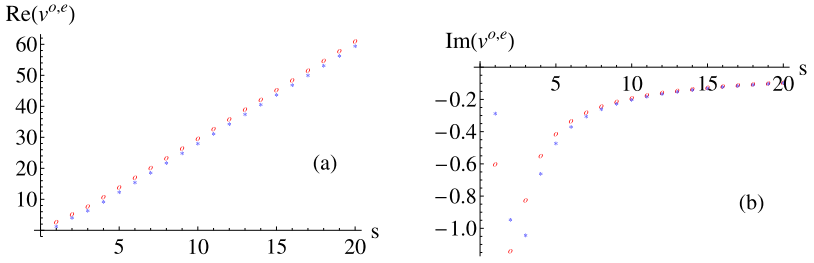

Fig. 1. (a) The real part and (b) the imaginary part of the odd ('o') and even ('∗') wave-vectors are plotted as functions of the eigenmode index. $u_0 = k_0 z_0 = 7\pi/4$.

for a number of values of the initial population difference. I conclude in Section 4. In Appendix A, I give the coupled ordinary differential equations which govern the dynamics of the eigenmode expansion coefficients derived in [6].

2. Eigenmode expansion

As was previously shown in [6], if one decomposes the system's dynamical variables, i.e. the atomic polarization, the difference in population between the two atomic states and the Rabi frequency associated with the electric field, in the basis formed by the eigenfunctions of the integral equation:

$$\Lambda_s \varphi_s(Z) = \frac{u_0}{2} \int_{-1}^{1} dZ' \exp\left(iu_0 |Z - Z'|\right) \varphi_s(Z'), \tag{1}$$

as follows

$$\psi(Z,T) = \sum_s e_s^o(T)\tilde{\varphi}_s^o(T) + \sum_s e_s^e(T)\tilde{\varphi}_s^e(T), \tag{2}$$

$$n(Z,T) = \sum_s \eta_s^o(T)\tilde{\varphi}_s^o(T) + \sum_s \eta_s^e(T)\tilde{\varphi}_s^e(T), \tag{3}$$

$$\chi(Z,T) = \sum_s p_s^o(T)\tilde{\varphi}_s^o(T) + \sum_s p_s^e(T)\tilde{\varphi}_s^e(T), \tag{4}$$

the set of Maxwell–Bloch equations reduce to an infinite set of coupled ordinary first order differential equations in the expansion coefficients summarized in Appendix A. (The tilde over the eigenfunction indicates that the normalized eigenfunctions are used in the expansions expressions.)

In what follows, I shall use the system of units where all quantities are normalized to the parameter of interatomic cooperativity $C = \frac{4\pi N \wp^2}{\hbar V}$, where N is the number of particle, V is the slab volume, and $\wp$ is the reduced dipole moment of the atomic transition (its normalization is uniquely determined when given as a function of the isolated atom decay rate, see below). In these units, the transverse decay rate Γ_2, due to the instantaneous dipole–dipole interaction between atoms, is equal to 2.33/4, and the normalized Lorentz shift is equal to 1/3. The isolated atom decay rate $\gamma_1 = \frac{4}{3}\wp^2 k_0^3/\hbar$ specifies the longitudinal decay rate of the system. The normalized coordinates are respectively given by

$$Z = z/z_0 \qquad T = Ct \qquad \Gamma_1 = \gamma_1/C \qquad \Gamma_2 = \gamma_2/C$$
$$u_0 = k_0 z_0 \qquad \Omega_L = \omega_L/C.$$

The slab thickness is $2z_0$, and $\Gamma_T = \frac{\Gamma_1}{2} + \Gamma_2$.

It is to be noted that the above eigenfunctions belong to one of two families, each with a definite spatial parity (odd, even), given respectively by

$$\varphi_s^o(Z) = \sin\left(\nu_s^o Z\right), \tag{5}$$

$$\varphi_s^e(Z) = \cos\left(\nu_s^e Z\right), \tag{6}$$

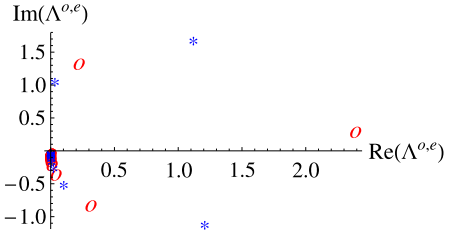

Fig. 2. The complex plane loci of the odd ('o') and even ('∗') eigenvalues. $u_0 = k_0 z_0 = 7\pi/4$.

where the complex wavevectors (ν_s^o, ν_s^e) are solutions of the transcendental equations

$$\cot\left(\nu_s^0\right) = i\frac{u_0}{\nu_s^o}, \tag{7}$$

$$\tan\left(\nu_s^e\right) = -i\frac{u_0}{\nu_s^e}, \tag{8}$$

where s, a positive integer, is the index of the solution.

The eigenvalues associated with these eigenfunctions are given by:

$$\Lambda_s^{o,e} = i\frac{u_0^2}{u_0^2 - (\nu_s^{o,e})^2}. \tag{9}$$

I plot in Fig. 1, the real part and the imaginary part of the characteristic wave-vectors as functions of the index, and in Fig. 2, the locus in the complex plane of the eigenvalues for $u_0 = 7\pi/4$. The important thing to note in these figures is that: (i) the real part of the characteristic vector for the odd and even solutions differ from each other by approximately $\pi/2$, (ii) the eigenvalue with the largest real part (called dominant mode) is that which has the real part of its wavevector approximately equal to u_0, and that (iii) except for the dominant and other finite number of modes (called leading modes) with indices close to that of the dominant mode, all the other modes eigenvalues cluster around the origin of the complex plane.

The eigenfunctions of Eqs. (5)–(6) obey the following pseudo-orthogonal relations

$$\int_{-1}^{1} \sin\left(\nu_s^o Z\right) \sin\left(\nu_{s'}^o Z\right) dZ = N_s^o \delta_{s,s'}, \tag{10}$$

where

$$N_s^o = 1 - \frac{\cos^2(\nu_s^o)}{iu_0}, \tag{11}$$

and

$$\int_{-1}^{1} \cos\left(\nu_s^e Z\right) \cos\left(\nu_s^e Z\right) dZ = N_s^e \delta_{s,s'}, \tag{12}$$