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Synchronization of map-based neurons with memory and synaptic delay

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ABSTRACT

Synchronization of two synaptically coupled neurons with memory and synaptic delay is studied using the Rulkov map, one of the simplest neuron models which displays specific features inherent to bursting dynamics. We demonstrate a transition from lag to anticipated synchronization as the relationship between the memory duration and the synaptic delay time changes. The neuron maps synchronize either with anticipation, if the memory is longer than the synaptic delay time, or with lag otherwise. The mean anticipation time is equal to the difference between the memory and synaptic delay independently of the coupling strength. Frequency entrainment and phase-locking phenomena as well as a transition from regular spikes to chaos are demonstrated with respect to the coupling strength.

1. Introduction

Biological neural networks consist of a large number of individual neurons interconnected in a complex manner usually via synapses through dendrodendritic microcircuits. The information processing tasks of neural networks are performed on the individual neuron level by generation of temporal sequences of action potentials, and then elaborated at mesoscales and macroscales by means of a network of neuron–neuron interaction. On the level of a single neuron, mechanisms and the nature of the neuron activity have been extensively investigated over the past decade; and the available literature is already redundant of rigorous and important results concerning the neuron ability to process and compute [1].

Synchronization of coupled neurons is relevant for coding and signal transmission allowing better understanding of the brain functionality and revealing distinctive features of some brain diseases. The interest in mathematical modeling of neuron synchronous behavior has significantly increased after real neurobiological experiments with two electrically coupled neurons [2,3], where various synchronization types have been identified. To simulate cooperative neuron dynamics, different models of coupled neurons based on either iterative maps [4–16] or differential equations [17–19,2,20–23] in various coupling configurations have been developed. Depending on both the coupling strength and the delay time, coupled neurons can be matched either in timings of their bursts (burst synchronization) [24], in phase [21], with lag, or anticipated [9,15,25,26]. Neuron dynamics were studied regarding intrinsic and external parameters including time constants, e.g., the influence of the rate of synaptic activation and deactivation on synchronization of bursting biological neurons. Furthermore, under specific conditions, intermittency between synchronized states was found when the time constant increased [3].

One of the important neuron functions is information transmission through a neuron network. This process is characterized by a certain delay time due to a finite velocity of the action potential propagation along the neuron axon and time lapses in dendritic and synaptic processes [27]. The delay in synaptic connections [28] is required for a neurotransmitter to be released from a presynaptic membrane, diffuse across the synaptic cleft, and bind to a receptor site on the postsynaptic membrane. On the other hand, feedback loops involving one or more neurons are ubiquitous in a nervous system [29]. The brain-stem feedback loops are thought to be responsible for short-term memory [30] that was predicted by Hermann Ebbinghaus in 1885 [31].

Since there are two different delay times, the interesting question arises: How do these time delays affect synchronization of synaptically coupled neurons? To answer this question, we explore one of the simplest neuron models, the Rulkov map [32,33]. Although this map is not explicitly referred to physiological processes in the membrane, it is capable of extraordinary complexity and quite specific neuron dynamics (silence, periodic spiking, and chaotic bursting), thus replicating a great deal of experimentally observed regimes [2,3,9], e.g., spike adaptation [34], routes from silence to bursting mediated by subthreshold oscillations [35],

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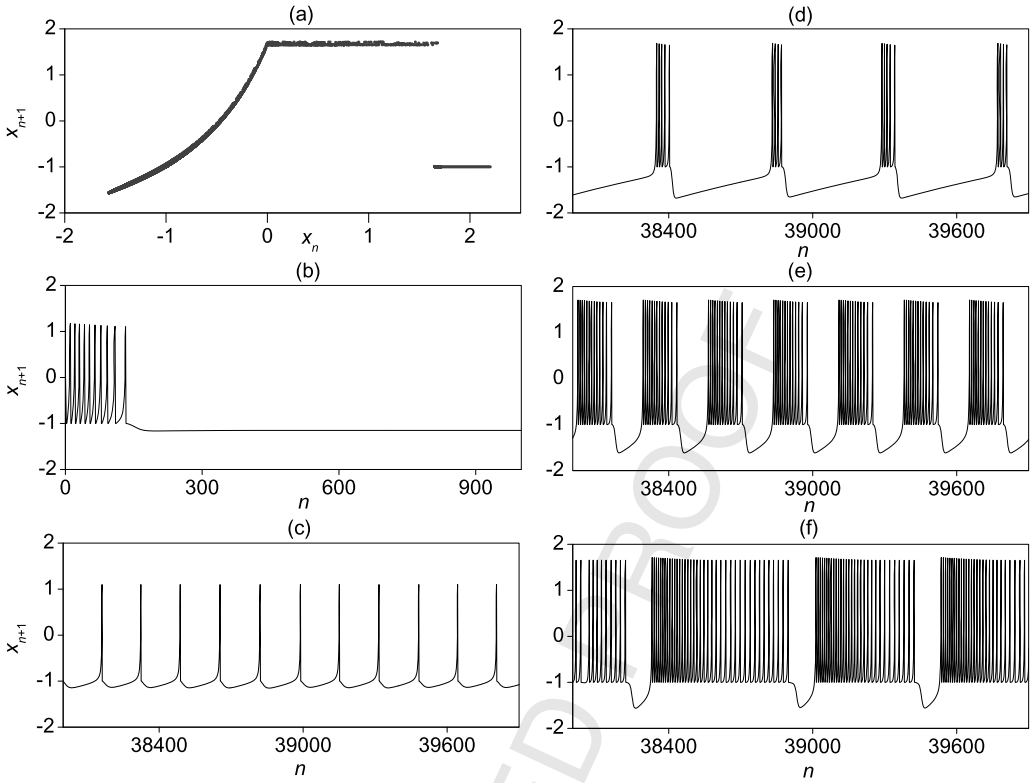

Fig. 1. (a) Function $f(x_n, y_n)$ for $\sigma = 0.3$ and (b–f) time series showing different dynamical regimes: (b) Silence after transients for $\sigma = -0.15$, (c) tonic spikes for $\sigma = -0.025$, and (d–f) bursts for (d) $\sigma = -0.30$, (e) 0.15, and (f) 0.30. $\alpha = 5.3$, $\mu = 0.001$.

emergent bursting [32], phase and antiphase synchronization with chaos regularization [9,33], as well as complete and burst synchronization [36–38].

Recently, Matias et al. [22,23] demonstrated a smooth transition from lag to anticipated synchronization of coupled Hodgkin–Huxley neurons [39] when the inhibitory synaptic conductance was increased. In the map-based models the time delay is independent of the coupling strength and only determined by the difference between the delay in coupling and neuron memory. In this work we will study the transition from lag to anticipated synchronization as a direct function of this difference. Being computationally more efficient than complex phenomenological models [39,40], the map-based models can improve qualitative understanding of the synchronous neuron behavior.

The paper is organized as follows. In Section 2 we review the theoretical framework of the Rulkov neuron and describe parameters explored in the model. Section 3 is devoted to synchronization of two coupled neurons; we show how two delay times affect synchronization. Finally, in Section 4 we conclude our results and outline a possible extension of this work.

2. Model equations

2.1. Dynamics of a single Rulkov neuron

The Rulkov map is defined by the following equations [9]

$$x_{n+1} = f(x_n, y_n), \tag{1}$$

$$y_{n+1} = y_n - \mu(x_n + 1) + \mu\sigma, \tag{2}$$

$$f(x_n, y_n) = \begin{cases} \alpha/(1 - x_n) + y_n & \text{for } x_n \le 0, \\ \alpha + y_n & \text{for } 0 < x_n < \alpha + y_n \text{ and } x_{n-1} \le 0, \\ -1 & \text{for } x_n \ge \alpha + y_n \text{ or } x_{n-1} > 0, \end{cases} \tag{3}$$

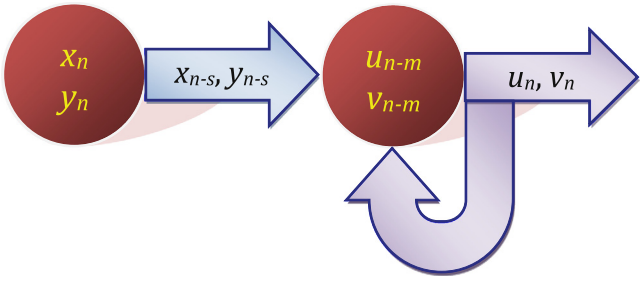

Fig. 2. Coupling scheme of two Rulkov neurons with synaptic delay s and memory m.

where x_n and y_n are the fast and slow variables and α, μ, and σ are intrinsic parameters. The map dynamics depends mostly on α and σ as shown in Fig. 1, where we plot the map function Eq. (3) [Fig. 1(a)] and typical times series illustrating different dynamical regimes [Figs. 1(b–f)]. The parameter σ regulates the neuron response under the action of the external dc bias current and synaptic inputs and therefore it is used as a control parameter to select a desired dynamical regime. For $\sigma < -0.3$ the neuron is in a silent state (subthreshold oscillations). For larger σ, the neuron generates repetitive spike bursts; the number of spikes in a burst train increases with σ, as seen from Figs. 1(c–f). Such a behavior of the Rulkov map mimics real neuron dynamics.

2.2. Two coupled Rulkov neurons

Now, we consider the scheme of two coupled Rulkov neurons shown in Fig. 2, where a memorized state of a postsynaptic neuron is coupled with a delayed state of a presynaptic neuron. While the presynaptic neuron is described by Eqs. (1)–(3), the postsynaptic neuron is modeled by the following equations