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Darboux transformation and dark rogue wave states arising from two-wave resonance interaction

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ABSTRACT

Exact rogue wave solutions of the long wave–short wave resonance equation are obtained via Darboux transformations. Compared to the real long-wave field which always features a single hump structure, the short-wave field can be manifested as bright rogue wave, intermediate rogue wave, or dark rogue wave, depending on the subtle nonlinearity driven by the long-wave field.

1. Introduction

Rogue waves, which are originally coined for vivid description of the mysterious and monstrous ocean waves [1], have recently attracted much interest in their fundamental origin and complex dynamics [2]. In addition to in the open ocean, these extreme wave events are also observed in a wide class of physical systems including deep water [3,4], capillary waves and surface ripples [5,6], plasmas [7], optical fibers [8,9], and versatile lasers [10–13] (including multistable systems [14,15]). These studies uncover general features of nonlinearity and complexity shared by the rogue waves. Basically, rogue waves are doubly localized wavepackets in the sense that they seem to appear from nowhere and disappear without a trace [16]. They in fact define the limit of either Ma solitons [17] or Akhmediev breathers [18] which take an otherwise exponential form. The Peregrine soliton [19] is the simplest rogue wave pertinent to the nonlinear Schrödinger (NLS) equation, and has recently been observed in deep water [3,4], plasmas [7], and optical fibers [9]. In order to model the physical systems in a relevant way, there is now a trend to study the rogue wave phenomena beyond the NLS description. Recently, analytical rogue wave solutions were also obtained for the more complicated physical models such as the Hirota equation [20], the Sasa–Satsuma equation [21,22], the coupled Manakov system [23], the coupled Hirota equations [24], and the three-wave resonance equation [25], to name a few.

In this Letter, we focus on the simple long wave–short wave (LWSW) resonance equation [26,27] and manage to gain an insight into the formation of the so-called dark rogue waves. This coupled equation can be obtained from the Davey–Stewartson system [28] under the resonance condition, namely, when the group velocity of a short wave (high-frequency wave) is equal to the phase velocity of a long wave (low-frequency wave). Despite the simplicity, this equation can describe a variety of nonlinear wave phenomena such as capillary–gravity waves in fluids [28] and optical–terahertz waves in negative index media [29]. We note that the rogue wave solutions to this equation had recently been derived by using the Hirota bilinear method [30]. Our objective here is to establish an alternative, more powerful Darboux transformation method for such rogue wave solutions, and furthermore, to reveal an interesting crossover dynamics ranging from bright rogue wave to dark counterpart.

For our studies, we write the LWSW equation in dimensionless form [26,27,29]

$$\begin{aligned} &iu_t + \frac{1}{2}u_{xx} + u\phi = 0, \\ &\phi_t - \left(|u|^2\right)_x = 0, \end{aligned} \qquad (1)$$

where $u(t,x)$ represents the complex envelope of the rapidly varying field and $\phi(t,x)$ stands for the real low-frequency field, with t and x the two independent evolution variables. In the following, we exploit a Darboux dressing technique [31] to find the rogue wave solutions of Eq. (1).

We note that Eq. (1) is integrable [27,28] and thus can be cast into a 3×3 linear eigenvalue problem [31]

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$$\mathbf{R}_x = \mathbf{UR}, \qquad \mathbf{R}_t = \mathbf{VR}, \tag{2}$$

where $\mathbf{R} = (r, s, w)^{\mathsf{T}}$ (T means a matrix transpose and r, s, and w are (t, x, λ)-dependent functions), and

$$\mathbf{U} = \lambda \mathbf{U}_0 + \mathbf{U}_1, \tag{3}$$

$$\mathbf{V} = \lambda^2 \mathbf{V}_0 + \lambda \mathbf{V}_1 + \mathbf{V}_2, \tag{4}$$

with $\mathbf{U}_0 = \mathrm{diag}(2i, 0, -2i)$, $\mathbf{V}_0 = \mathrm{diag}(-2i/3, 4i/3, -2i/3)$, and

$$\mathbf{U}_1 = \begin{pmatrix} 0 & u & -i\phi \\ 0 & 0 & -u^* \\ -2i & 0 & 0 \end{pmatrix}, \qquad \mathbf{V}_1 = \begin{pmatrix} 0 & -u & 0 \\ 0 & 0 & -u^* \\ 0 & 0 & 0 \end{pmatrix}, \tag{5}$$

$$\mathbf{V}_2 = \begin{pmatrix} 0 & \frac{i}{2}u_x & -\frac{i}{2}|u|^2 \\ -u^* & 0 & \frac{i}{2}u_x^* \\ 0 & u & 0 \end{pmatrix}. \tag{6}$$

Here λ is the complex spectral parameter and $\mathbf{R}$ is a column-matrix function of t, x, and λ, i.e., $\mathbf{R} \equiv \mathbf{R}(t, x, \lambda)$. One can prove that Eq. (1) can be exactly reproduced from the compatibility condition $\mathbf{U}_t - \mathbf{V}_x + \mathbf{UV} - \mathbf{VU} = 0$.

By use of the dressing operator $\mathbf{D}$ as in Ref. [31], we obtain the correct Darboux transformations

$$u = u_0 + \frac{2i(\lambda - \lambda^*)\lambda(\alpha - \beta^*)}{\Delta} s^* w, \tag{7}$$

$$\phi = \phi_0 + \frac{8(\lambda - \lambda^*)}{\Delta}\big[\alpha|\lambda|^2|w|^2 + i\lambda(\alpha - \beta^*)\,\mathrm{Im}(r^* w)\big], \tag{8}$$

where (u_0, ϕ_0) and (u, ϕ) denote the seeding pair and the new pair of solutions of Eq. (1), respectively, and

$$\alpha = -\frac{\lambda - \lambda^*}{\lambda + \lambda^*}\mathbf{R}(\lambda)^\dagger \sigma_1 \mathbf{R}(\lambda) = -\alpha^*, \tag{9}$$

$$\beta = -\mathbf{R}(\lambda)^\dagger \sigma_1 \mathbf{R}(-\lambda), \tag{10}$$

$$\gamma = -\frac{\lambda - \lambda^*}{\lambda + \lambda^*}\mathbf{R}(-\lambda)^\dagger \sigma_1 \mathbf{R}(-\lambda) = -\gamma^*, \tag{11}$$

$$\Delta = \alpha\gamma - |\beta|^2. \tag{12}$$

Here σ_1 is the 3×3 analogue of the first Pauli spin matrix [31] and the dagger indicates the complex-conjugate transpose. Noting that we have assumed $s(\lambda)$ and $w(\lambda)$ to be even functions of λ and $r(-\lambda) = r(\lambda) + 2\lambda w(\lambda)$ [31]. Besides, one can find that α, β and γ satisfy $\alpha + \beta = \gamma + \beta^*$ and $\lambda(\alpha - \beta^*) + \lambda^*(\alpha + \beta) = 0$.

Since the rogue waves define the limit of either Ma solitons or Akhmediev breathers on an unstable background [3,9,16], we start directly with the plane-wave solutions

$$u_0(t, x) = a\exp(ikx - i\omega t), \tag{13}$$

$$\phi_0(t, x) = b, \tag{14}$$

where $a > 0$, $b \geqslant 0$, $k \in \mathbb{R}$, and $\omega = \frac{1}{2}k^2 - b$. Then, substituting Eqs. (13) and (14) into Eq. (2), we obtain

$$w(\lambda) = e^{i\theta_1} + \Gamma_1 e^{i\theta_2} + \Gamma_2 e^{i\theta_3}, \tag{15}$$

$$s(\lambda) = f_1 u_0^* w(\lambda), \tag{16}$$

$$r(\lambda) = f_2 w(\lambda), \tag{17}$$

where Γ_1 and Γ_2 are arbitrary constants, and

$$\theta_j = \mu_j x + \nu_j t, \tag{18}$$

$$\nu_j = \frac{4}{3}\lambda^2 + b - \frac{1}{2}\mu_j^2, \tag{19}$$

$$f_1 = \frac{r_{11}e^{i\theta_1} + \Gamma_1 r_{12} e^{i\theta_2} + \Gamma_2 r_{13} e^{i\theta_3}}{e^{i\theta_1} + \Gamma_1 e^{i\theta_2} + \Gamma_2 e^{i\theta_3}}, \tag{20}$$

$$f_2 = \frac{r_{21}e^{i\theta_1} + \Gamma_1 r_{22} e^{i\theta_2} + \Gamma_2 r_{23} e^{i\theta_3}}{e^{i\theta_1} + \Gamma_1 e^{i\theta_2} + \Gamma_2 e^{i\theta_3}}, \tag{21}$$

with

$$r_{1j} = \frac{i}{\mu_j - k}, \qquad r_{2j} = -\frac{1}{2}\mu_j - \lambda. \tag{22}$$

The index j in Eqs. (18), (19), and (22) runs over 1, 2, and 3, and μ_j are three roots of the cubic equation

$$(\mu - k)\big(\mu^2 - 2b - 4\lambda^2\big) + 2a^2 = 0. \tag{23}$$

It is easily seen from Eqs. (18)–(23) that $\theta_j(-\lambda) = \theta_j(\lambda)$, $f_1(-\lambda) = f_1(\lambda)$, and $f_2(-\lambda) = f_2(\lambda) + 2\lambda$, which gives rise to the symmetry properties that $r(\lambda)$, $s(\lambda)$, and $w(\lambda)$ should satisfy, see discussions for Eqs. (7) and (8).

At this stage, one can obtain the Ma soliton or Akhmediev breather solutions of Eq. (1) by substituting Eqs. (15)–(17) into Eqs. (7) and (8), with an appropriate choice of the complex parameter λ. Specially, in the cubic equation (23), if a specific value of λ is chosen such that

$$\lambda^2 = \frac{1}{6}k^2 - \frac{1}{2}b - \frac{1}{8}\left(\ell + \frac{\sigma}{\ell}\right) \mp \frac{i\sqrt{3}}{8}\left(\ell - \frac{\sigma}{\ell}\right), \tag{24}$$

we then get two equal roots

$$\mu_1 = \mu_2 = \kappa = m + in, \tag{25}$$

where m and n are the real and imaginary parts of the complex constant κ, respectively, and are given by

$$m = \frac{1}{6}\left[5k - \sqrt{3\big(k^2 + \ell + \sigma/\ell\big)}\right], \tag{26}$$

$$n = \pm\sqrt{(3m - k)(m - k)}. \tag{27}$$

Noting that the plus or minus sign in Eq. (27) results from the pair of conjugate values of λ^2 [see Eq. (24)]. The other parameters σ, ρ, and ℓ are defined by

$$\sigma = \frac{1}{9}k^4 + 6a^2 k, \tag{28}$$

$$\rho = \frac{1}{2}k^6 - \frac{1}{54}\big(27a^2 + 5k^3\big)^2, \tag{29}$$

$$\ell = \begin{cases} -(\rho - \sqrt{\rho^2 - \sigma^3})^{1/3}, & k \leqslant -3k_n, \\ (-\rho + \sqrt{\rho^2 - \sigma^3})^{1/3}, & -3k_n < k \leqslant \frac{3}{2}k_n, \end{cases} \tag{30}$$

with $k_n = (2a^2)^{1/3}$. Note that the allowed regime of k is $k \leqslant \frac{3}{2}k_n$, otherwise ℓ and n will be complex, in conflict with the fact that m and n should be real.

Under the circumstances, taking $\Gamma_1 = -1$ and $\Gamma_2 = 0$ can reduce Eqs. (20) and (21) to the simple rational forms

$$f_1(\lambda) = \frac{i}{\kappa - k} + \frac{1}{(\kappa - k)^2(\kappa t - x)}, \tag{31}$$

$$f_2(\lambda) = -\frac{1}{2}\kappa - \lambda - \frac{i}{2(\kappa t - x)}. \tag{32}$$

As a result, from the Darboux transformations (7) and (8), we obtain the exact fundamental rogue wave solutions

$$u(t, x) = u_0\left[1 - \frac{it + \frac{ix}{2m - k} + \frac{1}{2(2m-k)(m-k)}}{(x - mt)^2 + n^2t^2 + 1/(4n^2)}\right], \tag{33}$$

$$\phi(t, x) = b + 2\frac{n^2t^2 - (x - mt)^2 + 1/(4n^2)}{[(x - mt)^2 + n^2t^2 + 1/(4n^2)]^2}. \tag{34}$$

Here the solutions have been translated along the x axis so that their central values locate on the origin [22,24]. It is clear that the short-wave rogue wave, u, is characterized by the second-order polynomial of t and x, while the real long-wave rogue wave, ϕ,