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Calculation of the tunneling time using the extended probability of the quantum histories approach

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ABSTRACT

The dwell time of quantum tunneling has been derived by Steinberg (1995) [7] as a function of the relation between transmission and reflection times τ_t and τ_r, weighted by the transmissivity and the reflectivity. In this paper, we reexamine the dwell time using the extended probability approach. The dwell time is calculated as the weighted average of three mutually exclusive events. We consider also the scattering process due to a resonance potential in the long-time limit. The results show that the dwell time can be expressed as the weighted sum of transmission, reflection and internal probabilities.

1. Introduction

Quantum tunneling is one of the most important quantum phenomena. Various tunneling times can be determined based on characterizing the time spent by a particle under the barrier. They are expressed in terms of the derivatives of the transmission coefficient $T = |T|e^{i\theta_t}$ or the reflection coefficient $R = |R|e^{i\theta_r}$. For example, the Larmor time was first introduced by Baz [1,2] in a thought experiment designed to measure the time associated with scattering events. The Larmor times for transmission and reflection can be obtained respectively from

$$\tau_t^{LM} = -\hbar\frac{\partial}{\partial V}\theta_t \quad \text{and} \quad \tau_r^{LM} = -\hbar\frac{\partial}{\partial V}\theta_r, \tag{1}$$

where V is the hight of a square potential barrier.

Another example is the Büttiker–Landauer time [3,4] that invokes an oscillatory barrier to estimate the tunneling time. The original static barrier is augmented by small oscillations in the barrier height. The Büttiker–Landauer expressions for transmission and reflection times are given as

$$\tau_t^{BL} = -\hbar\frac{\partial}{\partial V}\ln|T| \quad \text{and} \quad \tau_r^{BL} = -\hbar\frac{\partial}{\partial V}\ln|R|, \tag{2}$$

respectively.

Sokolovski and Baskin [5] presented an identity connecting the dwell time $\tau_d(k)$ and the complex time $\tau_{t(r)}^{\Omega} = \tau_{t(r)}^{LM} - i\tau_{t(r)}^{BL}$ as

$$\tau_d(k) = |T|^2\tau_t^{\Omega} + |R|^2\tau_r^{\Omega}, \tag{3}$$

where $\hbar^2k^2/2m$ is an energy of the free particle.

Hauge and Støvneng [6] also proved Eq. (3) and suggested that the transmission and the reflection are mutually exclusive events. Steinberg [7] used conditional probability to define the dwell time. He proposed that the dwell time distribution consists of two parts, one from transmission and another from reflection. In spite of these results, we still do not know how the dwell time can be decomposed. In 2004, Yamada [8] derived four tunneling times in a unified manner without relying on any specific models by using the Gell-Mann and Hartle (GMH) decoherence functional [9] $D_{GMH}^{(\gamma)}(\tau,\tau')$ to define the following quantity

$$I[F] = \frac{1}{P(\Theta_\gamma)}\int d\tau \int d\tau'\, F(\tau,\tau')D_{GMH}^{(\gamma)}(\tau,\tau'). \tag{4}$$

To understand the physical meaning of the quantity $I[F]$, it is necessary to explore the relationship between $D_{GMH}^{(\gamma)}(\tau,\tau')$ and the essential ideas of extended probability in quantum history [10–12].

In this paper, we discuss how the dwell time distribution is determined by the extended probabilities of alternative histories, which are decomposed into three elements. This formulation is compared with the GMH decoherence functional. The natural occurrence of the dwell time is given by the weighted average of

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three mutually exclusive events. We introduce a dwell time τ_{in} as the time for the particle to remain in a given region. Then we consider the dwell time for a resonance potential barrier. This gives a time τ_{in} that contributes to the dwell time in the long-time limit. Finally, we present our conclusions in Section 4.

2. The conditional probability for the dwell time

From quantum theory [13], let us consider the statistics of making a measurement as given by the projection operator $|\gamma\rangle\langle\gamma|$:

$$P(\gamma) = tr\big[|\gamma\rangle\langle\gamma|\rho_0\big], \tag{5}$$

where $P(\gamma)$ is the probability of having the outcome γ and ρ_0 is initially an arbitrary density matrix. The conventional statistics of the histories are governed by a chain operator $\mathbf{C}_\alpha$ for discrete measurement

$$\mathbf{C}_\alpha = \mathbf{P}^n_\alpha(t_n)\cdots\mathbf{P}^1_\alpha(t_1) \tag{6}$$

where $\mathbf{P}^k_\alpha(t_k)$ are projection operators corresponding to an event α at time t_k.

In most pictures of the quantum history, the statistics of histories are governed by the chain operator representing a sequence of events at various times. Griffiths [14] claimed that these history approaches do not always define probabilities. The condition determining when probabilities can be defined is called *the consistency condition*,

$$D\big(k,k'\big) = \mathrm{tr}\big[\mathbf{K}\big(k'\big)\rho_0\mathbf{K}^\dagger(k)\big] = P_{GH}(k)\delta_{k,k'}, \tag{7}$$

where ρ_0 is the initial density matrix of the system and $D(k,k')$ is called the decoherence functional. Each history is described by the chain operator $\mathbf{K}(k)$, $k = 1, 2, \ldots,$ as

$$\mathbf{K}(k) = \mathbf{P}^k_f(t)\cdots\mathbf{P}^k_j(t_j)\cdots\mathbf{P}^k_1(t_0) \tag{8}$$

where $\mathbf{P}^k_j(t_j)$ is a projection operator corresponding to the event j in the sequence of the history k at time t_j and $k = 1, 2, 3, \ldots$.

Gell-Mann and Hartle defined the probability of a history k as

$$P_{GH}(k) = \mathrm{tr}\big[\mathbf{K}(k)\rho_0\mathbf{K}^\dagger(k)\big]. \tag{9}$$

Griffiths used the consistency condition to show that sets of alternative histories may be assigned to the probabilities and showed what these probabilities are. One example of this is the probability of tunneling time. Yamada [15] argued that the quantum traversal time defined by the clocked Schrödinger equation, does not satisfy the weak decoherence condition. Therefore the definition of the probability distribution of tunneling times is impossible. As an alternative definition, Goldstein and Page [16] introduced the probability distribution in the form of the expectation value of the chain operator

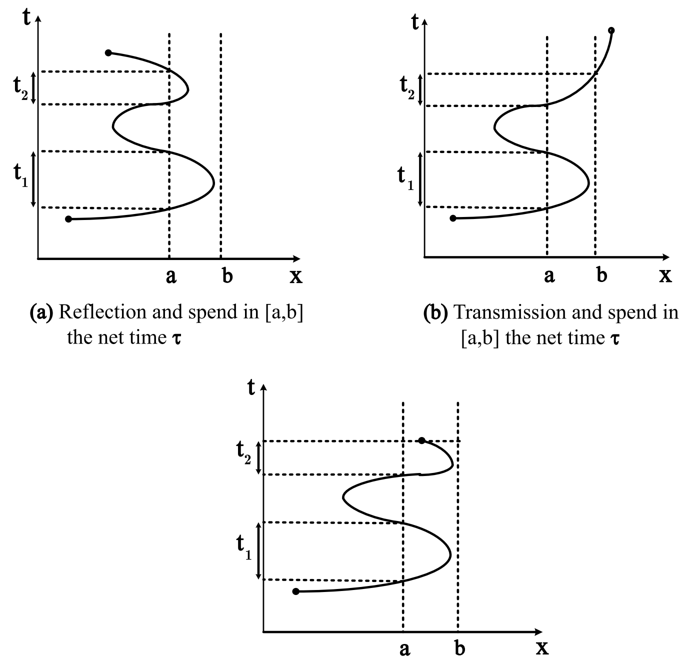
$$\pi(k) = \mathrm{tr}\big[\rho_0\mathbf{K}(k)\big]. \tag{10}$$

Generally, Eq. (10) is negative. Therefore, they imposed the linear positivity condition

$$\pi(k) \geqslant 0. \tag{11}$$

A set of histories obeying the linear positivity condition will obey the standard sum rules and gives a positive value. Now Hartle [10,11] proposed that all histories, either fine-grained or coarse-grained are definable in the extended probability form. The extended probability $P(k)$ is given by

$$P(k) = Re\big(\pi(k)\big). \tag{12}$$



(a) Reflection and spend in [a,b] the net time τ (b) Transmission and spend in [a,b] the net time τ (c) Finally trapped and spend in [a,b] the net time τ

Fig. 1. Illustrating 3 mutually exclusive events of the Feynman path which contribute to the dwell time. (a) Reflection paths, (b) transmission paths and (c) trapped paths.

Eq. (12) was also discussed in Refs. [17,18]. These probabilities are not necessarily all positive nor less than one, but all the other requirements of usual probability theories are maintained. However, the concept of the negative probability was mentioned in Refs. [19,20]. Some authors have suggested that the probabilities can be extended to complex numbers [21]. In this letter, we pursue a complex joint probability.

Using the Feynman path integral approach, Sokolovski and Baskin [5] calculated a traversal time using the functional

$$t^{cl}_{ab}\big[x(t)\big] = \int_0^t dt'\,\Theta_{ab}\big(x\big(t'\big)\big), \tag{13}$$

where $\Theta_{ab}(x) = 1$ for $a \leqslant x \leqslant b$ and 0 otherwise. Here we introduce a matrix element of the chain operator in the functional form to represent the quantum history of a particle that has been, for a duration τ in a given region prior to time t,

$$C_\tau\big[x(t)\big] = \big\langle x\big|\mathbf{C}(\tau)\big|x'\big\rangle = \int D[x]\delta\big(\tau - t^{cl}_{ab}\big[x(t)\big]\big)e^{\frac{i}{\hbar}S[x(t)]}. \tag{14}$$

The Dirac δ function selects a class of paths that obeys condition $t^{cl}_{ab}[x(t)] = \tau$. Integrating over all values τ, we can obtain the time evolution operator as

$$\mathbf{U}(t) = \int d\tau\,\mathbf{C}(\tau). \tag{15}$$

Now we can classify a quantum trajectory of a particle, having the value of $t^{cl}_{ab}[x(t)]$ exactly equal to τ according to three classes as Fig. 1 suggests. The trajectories are decomposed by using the end point of the paths.

To measure whether the particle is in the given region Ω_γ, we define the projection operator $\mathbf{P}_\gamma[x]$ in the form

$$\mathbf{P}_\gamma[x] = \int dx\,\Theta_\gamma(x)|x\rangle\langle x|, \quad \gamma = 1, 2, 3, \tag{16}$$