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Compressibility-near-zero acoustic metamaterial

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ABSTRACT

This letter theoretically analyzes and experimentally demonstrates a novel class of compressibility-near-zero (CNZ) acoustic metamaterials, achieved by using resonant-type metamaterials, namely the Helmholtz resonator. We first present a closed analytical formula for the effective compressibility of the proposed unit cell and then show that two frequencies exist which may support CNZ propagation. We demonstrate how the choice of the actual operating CNZ frequency depends on the properties of the host and finally experimentally verify CNZ propagation of acoustic waves.

1. Introduction

Near-zero (NZ) metamaterials present a specific subclass of metamaterials, initially demonstrated in the EM domain [1–5]. Their operation relies on the fact that a non-zero frequency exists at which the wave number is equal to zero. This results in propagation characterized by a constant phase over physically long distances, and gives rise to interesting phenomena such as energy tunneling, supercoupling and energy squeezing.

By analogy with the EM case, an NZ acoustic metamaterial can be obtained in two ways: by tailoring either its effective mass density or its effective compressibility (the reciprocal of the bulk modulus) to obtain near-zero values at certain frequencies. Recently, different NZ acoustic metamaterials have been proposed based either on tailoring the effective mass density [6–9], or on locally resonant phononic crystals [10].

In this paper, we analyze and experimentally demonstrate a novel class of one-dimensional NZ acoustic metamaterials based on tailoring the effective compressibility. This is achieved using resonant-type metamaterials based on the Helmholtz resonator (HR), which has been previously used as a building block for single-negative acoustic metamaterials, [11–13]. Although at first it may seem that any metamaterial unit cell will support NZ propagation at a certain frequency, we show that the host structure needs to be carefully designed and that not one but two frequencies exist capable of supporting NZ propagation, finally leading to the development of NZ acoustic devices with novel characteristics.

2. Theory

For a section of an acoustic duct of length l and cross-sectional area S loaded with an HR with effective neck length l_h and cross-sectional area S_h, the continuity equation can be expressed as:

$$dV = S\frac{\partial v}{\partial x} l\, dt + S_h\, d\xi, \tag{1}$$

provided that the proposed structure is small enough to be considered as a unit cell of an acoustic metamaterial.

Under the assumption that sound propagation is an adiabatic process, i.e. for $pV^{\kappa} = \text{const.}$, (1) can be rewritten as:

$$-\beta_0\, dp = \frac{\partial v}{\partial x} dt + \frac{S_h}{V} d\xi, \tag{2}$$

where β_0 is the compressibility of the gas. The neck of the HR thus acts as a sink, effectively modifying the continuity equation. The HR can be viewed as the "internal inhomogeneity"; of the cell, and its influence can be expressed through the effective compressibility β_{eff} as follows:

$$-\beta_{eff}\, dp = \frac{\partial v}{\partial x} dt. \tag{3}$$

Eqs. (2) and (3), together with the complex form of the differential equation for the displacement of the gas in the neck of an HR (the lossless case), yield the analytical expression for the effective compressibility:

$$\beta_{eff} = \beta_0\left(1 + \frac{S_h}{V}\cdot\frac{1}{\frac{S_h}{V_h} - l_h\frac{\omega^2}{c^2}}\right), \tag{4}$$

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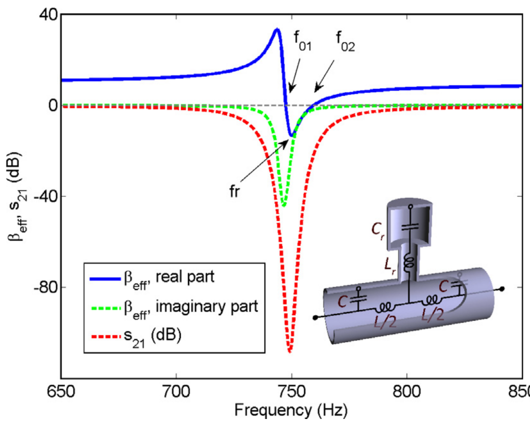

Fig. 1. (Color online.) Typical transmission coefficient and effective compressibility of the unit cell (shown in the inset, with its equivalent electrical circuit).

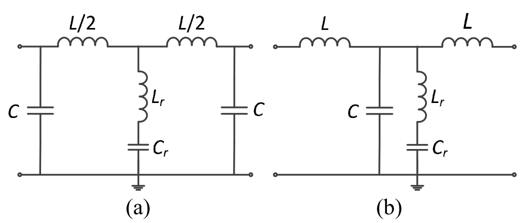

Fig. 2. Equivalent circuit of the HR-loaded (a) CLC–LPF, and (b) LCL–LPF. To preserve the symmetry, the inductor of CLC–LPF is modeled with two inductances equal to $L/2$.

where c is the velocity of sound. This expression suggests that it is possible to achieve zero effective compressibility at a certain frequency.

The simulated response of the proposed structure (with losses due to air viscosity only) and its effective compressibility, extracted using the approach analogous to the EM case [14,15], reveal a notch in the frequency response due to negative values of effective compressibility around the resonance of the HR, Fig. 1. Furthermore, β_{eff} equals zero at not one but two frequencies in the vicinity of f_r, namely at f_{01} and f_{02}, where propagation of CNZ nature might be expected. However, the imaginary part of β_{eff} at f_{01} is very large, and therefore no propagation occurs due to high losses. At f_{02}, the imaginary part of β_{eff} is smaller, but still non-zero, and consequently the insertion loss is lower but still not negligible, even though only losses due to air viscosity have been taken into account. Therefore, by using a simple short section of an acoustic duct as a host, CNZ propagation cannot be practically achieved either at f_{01} or at f_{02}.

We will now analyze in detail the conditions needed to achieve CNZ propagation in practice. Instead of a simple duct, as a host structure we propose a simple lowpass filter (LPF) of the 3rd order, but we note that this analysis can be applied to other hosts as well. The central section of the LPF is loaded with one HR, designed so that its resonant frequency falls within the stopband of the LPF. We analyze two LPFs: CLC–LPF modeled as shown in Fig. 2(a), and its dual, denoted LCL–LPF, Fig. 2(b).

In such an environment, HR is driven by pressure variations in the channel regardless of whether it is located at the inductive or the capacitive section of the acoustic filter. It should be noted that the equivalent circuits from Fig. 2 are in essence the same as those of a short section of an HR-loaded simple duct. The only difference is in the actual values of L and C: whereas in the case of LPF, L and C are significantly larger than L_r and C_r, in the case of a simple duct they are very similar. Therefore, to analyze the influence of the type of the host to CNZ propagation, we analyze the influence of decreasing L and C.

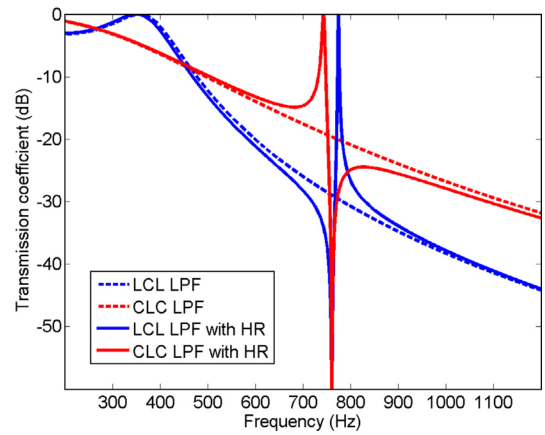

Fig. 3. (Color online.) Comparison of simulated lossless responses of equivalent electrical circuits of acoustic CLC- and LCL- low-pass filters (dashed lines), and the same low-pass filters loaded with one HR (full lines) at their central sections.

In Fig. 3, the responses of both circuits are compared to the responses of original LPFs. It can be seen that the inclusion of the HR in both cases results in a new transmission peak and a new transmission zero in the stopband of LPF. As expected, the transmission zero always occurs at f_r. However, the transmission peak occurs at the frequency higher than f_r for HR coupled to the capacitive section, and lower than f_r for HR coupled to the inductive section of the filter. To analyze the nature of obtained transmission peaks, we analytically derive the frequency at which CNZ propagation occurs, for both HR-loaded CLC–LPF and LCL–LPF.

Provided that the overall length l of the analyzed structures is sufficiently small, they can be regarded as homogeneous, and their effective wave number can be calculated. We note that this condition holds, although it might seem that in the actual implementation both structures are long with respect to the guided wavelength. However, as it will be shown, at the CNZ frequency the structures support CNZ propagation with constant phase over physically long distances, so they are in fact short with respect to the wavelength and the homogenization process can be applied.

The complex wavenumber $k = k' + jk''$ can be calculated from the ABCD matrix as $kl = \arccos(D)$. In the case of the HR-loaded CLC–LPF, D can be obtained as:

$$D = 1 + \frac{\frac{1}{Z_L+2Z_r} + j\omega C}{\frac{Z_r}{Z_L(Z_L+2Z_r)}} \tag{5}$$

where $Z_L = j\omega L/2$ and $Z_r = (1 - \omega^2 L_r C_r)/j\omega C_r$. Similarly, for the HR-loaded LCL–LPF, D is equal to:

$$D = 1 - \frac{\omega^2 LC}{1 - \omega^2 L_r C_r} \cdot \left(1 + \frac{C_r}{C} - \omega^2 L_r C_r\right) \tag{6}$$

The attenuation constant k' and the wave number k'' obtained from (5) and (6) are plotted in Fig. 4. It can be seen that k'' equals zero at two different frequencies, f_{01} and f_{02}, the first one before and the other one after the stopband. In fact, CNZ propagation will be supported only at the one of these frequencies where the condition $D = 1$ is fulfilled, i.e. where $k' = 0$. In the case of the HR-loaded CLC–LPF, this frequency is f_{01}, which is obtained from (5) by equating D to 1:

$$f_{01} = \frac{1}{2\pi\sqrt{\frac{2CC_r}{2C+C_r}(L_r + \frac{L}{4})}} \tag{7}$$

In the case of the HR-loaded LCL–LPF, this frequency is f_{02}, obtained similarly from (6):