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# Fuzzy modeling and synchronization of different memristor-based chaotic circuits ☆

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ABSTRACT

This Letter is concerned with the problem of fuzzy modeling and synchronization of memristor-based Lorenz circuits with memristor-based Chua's circuits. In this Letter, a memristor-based Lorenz circuit is set up, and illustrated by phase portraits and Lyapunov exponents. Furthermore, a new fuzzy model of memristor-based Lorenz circuit is presented to simulate and synchronize with the memristor-based Chua's circuit. Through this new fuzzy model, two main advantages can be obtained as: (1) only two linear subsystems are needed; (2) fuzzy synchronization of these two different chaotic circuits with different numbers of nonlinear terms can be achieved with only two sets of gain $K$. Finally, numerical simulations are used to illustrate the effectiveness of these obtained results.



## 1. Introduction

In 1971, Leon Chua postulated from symmetry arguments that a fourth passive circuit element which links the fundamental quantities of charge and magnetic flux mush exist [1]. This passive circuit element is named the memristor, and behaves as a resistor with a memory effect that is functionally dependent upon charge. In 2008, the HP lab showed the basic $i$–$v$ characteristics of the memristor in a nanoscale device [2]. These two terminal physical models have been investigated in novel applications [3–13]. For example, this new circuit element can be useful for low-power computation and storage to store information/data without the need of power using [3]. And memristor can be used to implement programmable analog circuits, leveraging memristor's fine-resolution programmable resistance without causing perturbations due to parasitic components [13] and so on.

The HP memristor is a passive two terminal electronic device described by a nonlinear constitutive relation [5]

$$v = M(q)i, \quad \text{or} \quad i = W(\varphi)v,$$

between the device terminal voltage $v$ and terminal current $i$, where $\varphi = \int v\,dt$, the memristance

$$M(q) = \frac{d\varphi(q)}{dq} = \begin{cases} a, & |q| \leqslant 1, \\ b, & |q| > 1. \end{cases}$$

On the other hand, as increasingly more applications of chaos synchronization were proposed [14–19], there are many control techniques to synchronize chaotic systems, such as fuzzy control [20–23], fuzzy logic control [24,25], linear and nonlinear error feedback control [26,27], and so on. Among various kinds of fuzzy methods, the Takagi–Sugeno fuzzy systems are widely accepted as a useful tool for design and analysis of fuzzy control system. Currently, some chaos control and synchronization for memristor-based chaotic systems have been proposed [28–37], in which, the number of the linear subsystem is decided by how many minimum nonlinear terms should be linearized in original system. As a result, there are $m \times 2^n$ linear subsystems (according to $2^n$ fuzzy rules) and $2^n$ equations in the T–S fuzzy system, where $n$ is the number of minimum nonlinear terms, $m$ is the order of the system. If $n$ is large, the number of linear subsystems in the T–S fuzzy system is huge. At the same time, when synchronization is investigated for different nonlinear systems, which may include different numbers of linear subsystems, the traditional method by the idea

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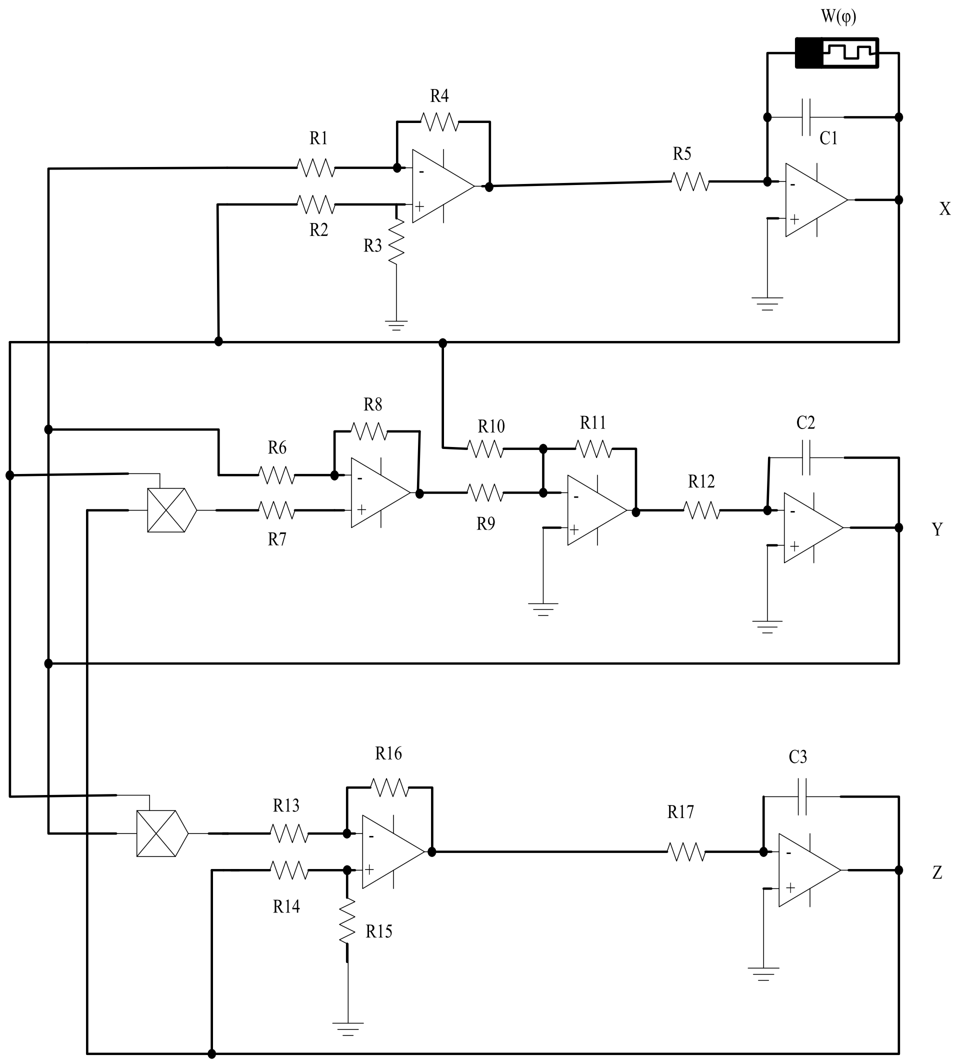


**Fig. 1.** Memristor-based Lorenz circuit.

of parallel distributed compensation (PDC) to design the fuzzy control law for stabilization of the error dynamics cannot be taken as the mismatch of the numbers of subsystems.

Therefore, the fuzzy model proposed in this Letter gives a new way to linearize memristor-based Lorenz circuits in which only two linear subsystems are included. Furthermore, through this new fuzzy model the idea of PDC can be employed to achieve synchronization. In the simulation section, the synchronization of the fuzzy memristor-based Lorenz circuits with fuzzy memristor-based Chua's circuits is presented to show the effectiveness of the proposed method.

## 2. Modeling and fuzzy synchronization of memristor-based Lorenz circuits with memristor-based Chua's circuits

In this section, three issues are concerned such as design of memristor-based Lorenz circuits, fuzzy modeling of memristor-based Lorenz circuits, synchronization of memristor-based Lorenz circuits with memristor-based Chua's circuits.

### 2.1. Design of memristor-based Lorenz circuits

Several new circuits about the memristor have been proposed and investigate in [3,9]. Based on these results, a new kind of memristor-based Lorenz circuits is presented in Fig. 1. We can obtain the following equations

$$\begin{cases} -R_5C_1\dot{X}(t) = -\frac{R_4}{R_1}Y(t) + \left(1+\frac{R_4}{R_1}\right)\frac{R_3}{R_2+R_3}X(t) \\ \quad + R_5W\big(\varphi(t)\big)X(t), \\ -R_{12}C_2\dot{Y}(t) = -\frac{R_{11}}{R_{10}}X(t) + \frac{R_{11}}{R_9}\left(\frac{R_8}{R_6}Y(t) + \frac{R_8}{R_7}X(t)Z(t)\right), \\ -R_{17}C_3\dot{Z}(t) = -\frac{R_{16}}{R_{13}}X(t)Y(t) + \left(\frac{1}{R_{16}} + \frac{1}{R_{13}}\right)\frac{R_{15}R_{16}}{R_{14}+R_{15}}Z(t), \\ \dot{\varphi}(t) = -X(t). \end{cases} \tag{1}$$

For the cause of convenience, denote $x(t) = [x_1(t), x_2(t), x_3(t), x_4(t)]^T = [X(t), Y(t), Z(t), \varphi(t)]^T$, $(1+\frac{R_4}{R_1})\frac{R_3}{R_2+R_3}\frac{1}{R_5C_1} = \sigma_1$, $\frac{R_4}{R_1}\frac{1}{R_5C_1} = \sigma_2$, $\frac{R_{11}}{R_{10}}\frac{1}{R_{12}C_2} = \sigma_3$, $(\frac{1}{R_{16}}+\frac{1}{R_{13}})\frac{R_{15}R_{16}}{R_{14}+R_{15}}\frac{1}{R_{17}C_3} = \sigma_4$, $C_1 = 1$, $\frac{R_{11}}{R_9}\frac{R_8}{R_6}\frac{1}{R_{12}C_2} = 1$, $\frac{R_{11}}{R_9}\frac{R_8}{R_7}\frac{1}{R_{12}C_2} = 1$, $\frac{R_{16}}{R_{13}}\frac{1}{R_{17}C_3} = 1$. Therefore, system (1) can be rewritten as

$$\begin{cases} \dot{x}_1(t) = -\sigma_1 x_1(t) - W\big(x_4(t)\big)x_1(t) + \sigma_2 x_2(t), \\ \dot{x}_2(t) = \sigma_3 x_1(t) - x_2(t) - x_1(t)x_3(t), \\ \dot{x}_3(t) = x_1(t)x_2(t) - \sigma_4 x_3(t), \\ \dot{x}_4(t) = -x_1(t), \end{cases} \tag{2}$$

where the piece-wise linear function $W(x_4(t))$ is given by

$$W\big(x_4(t)\big) = \begin{cases} a, & |x_4(t)| \leqslant 1; \\ b, & |x_4(t)| > 1. \end{cases}$$