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Jamming transition of a two-dimensional traffic dynamics with consideration of optimal current difference

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ABSTRACT

A modified two-dimensional lattice hydrodynamic traffic flow model is proposed by incorporating the optimal current difference effect of leading vehicles. Phase transitions and critical phenomenon are investigated near the critical point both analytically and numerically. Based on the configuration of vehicles, it is shown that two distinct jamming transitions occur: conventional jamming transition to the kink jam and jamming transition to the chaotic jam. It is shown that consideration of optimal current difference effect stabilizes the traffic flow and suppresses the traffic jam efficiently for all possible configurations of vehicles on a square lattice.

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1. Introduction

Due to rapid increase of automobile on roads, the problem of traffic congestion has been attracted considerable attention of scientists and researchers, nowadays. Many traffic flow models [1–13] have been developed to investigate the properties of traffic congestion and also to reduce congestion. Recently, lattice hydrodynamic model, firstly, proposed by Nagatani [14] has been given much attention. This model incorporates the idea that drivers adjust their velocity according to the observed headway. Subsequently, many extensions [15–29] to the Nagatani's work have been carried out by considering different factors into account like backward effect [15], lateral effect of the lane width [16], density difference effect [17] and anticipation effect of potential lane changing [18], etc. Recently, Peng [19] proposed a two-lane traffic flow model with the considering the effect of optimal flow difference in one-dimension and found that this factor stabilizes the traffic flow efficiently. Most of the above cited models describe some traffic phenomena only on a single lane or two-lane highway.

Traffic networks for example either traffic system of a whole a city or an expressway network, consist of many complicated ingredients. It is very complex to model the whole features of traffic network. Two-dimensional lattice hydrodynamic models, therefore, are very abstract models of traffic network. In this direction, Nagatani also extended the lattice hydrodynamic model to two-dimensional [30] as well as to high-dimensional [31] traffic flow model and investigated jamming transitions. On the other hand, two-dimensional traffic flow has also been investigated by cellular automata models [32]. However, best to our knowledge, the effect of optimal current difference on two-dimensional lattice has not been investigated yet.

In this Letter, we present a modified two-dimensional lattice hydrodynamic model by considering the effect of optimal current difference. Linear and nonlinear stability analysis will be carried out to investigate the impact of optimal current difference on traffic flow. Here, we would like to address two important points: (1) To check whether or not the modified model exhibits the similar jamming transition as reported by Nagatani [30]. (2) The effect of optimal current difference on the stability of two-dimensional traffic flow has to be analyzed. Finally, numerical simulation will be carried out to validate the theoretical findings.

2. A modified two-dimensional lattice hydrodynamic model

The first lattice hydrodynamic model proposed by Nagatani [14] incorporates both the ideas of car following models and macroscopic models to analyze the density wave. Later, he extended the model for a two-dimensional traffic flow [30] by assuming two types of vehicles on a square lattice, where one type of vehicle moves in the positive x-direction and other type of vehicle moves in the positive y-direction. Both types of vehicles interfere only at the crossing and are not allowed to turn. One type of vehicle never changes to the other type, i.e. the number of vehicles under each type remains conserved. The density and speed of vehicles moving in the positive x-direction [y-direction] are denoted by $\rho_x(x, y, t)$ [$\rho_y(x, y, t)$] and $u(x, y, t)$ [$v(x, y, t)$], respectively. Hence, the continuity equations on two-dimensional square lattice are given by

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$$\partial_t \rho_x(x,y,t) + \partial_x\big(\rho_x(x,y,t)u(x,y,t)\big) = 0 \tag{1}$$

$$\partial_t \rho_y(x,y,t) + \partial_y\big(\rho_y(x,y,t)v(x,y,t)\big) = 0 \tag{2}$$

where $\partial_t = \partial/\partial t$, $\partial_x = \partial/\partial x$, and $\partial_y = \partial/\partial y$.

The basic idea behind Nagatani's model [30] is that the traffic current is adjusted by the optimal current with a delay time. However, the above lattice model did not consider the optimal current difference effect which has an important influence on multi-lane traffic flow system [19]. In this work, we propose a new evolution equation for two-dimensional lattice hydrodynamic model with the consideration of optimal current difference as follows:

$$\begin{aligned}
&\rho_x(x,y,t+\tau)u(x,y,t+\tau)\\
&\quad = c\rho_0\big[V\big(\rho(x+x_0,y,t)\big)\big]\\
&\qquad + \lambda c\rho_0\big[V\big(\rho(x+2x_0,y,t)\big) - V\big(\rho(x+x_0,y,t)\big)\big]
\end{aligned} \tag{3}$$

$$\begin{aligned}
&\rho_y(x,y,t+\tau)v(x,y,t+\tau)\\
&\quad = (1-c)\rho_0\big[V\big(\rho(x,y+y_0,t)\big)\big]\\
&\qquad + \lambda(1-c)\rho_0\big[V\big(\rho(x,y+2y_0,t)\big) - V\big(\rho(x,y+y_0,t)\big)\big]
\end{aligned} \tag{4}$$

where ρ_0 is the total average density, $\rho(x,y,t) = \rho_x(x,y,t) + \rho_y(x,y,t)$ is the local density, x_0 and y_0 are the average headways in x- and y-directions, respectively, c is the fraction of vehicles moving in x-direction, λ is the reaction coefficient of optimal current difference, $V(.)$ is the optimal velocity function and τ is the delay requires to reach the traffic current at optimal level. In the proposed model, the traffic current in the x-direction at position (x,y) at time t is adjusted not only by the optimal current at position $(x+x_0,y)$ at time $t-\tau$, but also effected by the optimal current difference at $(x+2x_0,y)$ and $(x+x_0,y)$ at time $t-\tau$. By choosing $x_0 = 1/c\rho_0$ and $y_0 = 1/(1-c)\rho_0$ and combining the difference form of Eqs. (1) and (2) with Eqs. (3) and (4), the extended lattice hydrodynamic model is obtained as follows:

$$\begin{aligned}
&\rho_{j,m}(t+2\tau)\\
&\quad = \rho_{j,m}(t+\tau) - \tau\rho_0^2c^2\big[V\big(\rho_{j+1,m}(t)\big) - V\big(\rho_{j,m}(t)\big)\big]\\
&\qquad - \tau\lambda\rho_0^2c^2\big[V\big(\rho_{j+2,m}(t)\big) - 2V\big(\rho_{j+1,m}(t)\big) + V\big(\rho_{j,m}(t)\big)\big]\\
&\qquad - \tau\rho_0^2(1-c)^2\big[V\big(\rho_{j,m+1}(t)\big) - V\big(\rho_{j,m}(t)\big)\big]\\
&\qquad - \tau\lambda\rho_0^2(1-c)^2\big[V\big(\rho_{j,m+2}(t)\big) - 2V\big(\rho_{j,m+1}(t)\big)\\
&\qquad + V\big(\rho_{j,m}(t)\big)\big]
\end{aligned} \tag{5}$$

where $\rho_{j,m,t} = \rho_{x,j,m}(t) + \rho_{y,j,m}(t)$, $\rho_{x,j,m}(t)$ and $\rho_{y,j,m}(t)$ are the densities respectively, in x- and y-directions at the site (j,m) on the square lattice. Without loss of generality, the horizontal and vertical spacing on the square lattice are considered as x_0 and y_0, respectively. For $c = 0$ or $c = 1$, the two-dimensional modified lattice hydrodynamic model reduces to one-dimensional lattice model, recently proposed by Peng [19] in which reaction coefficient of optimal current difference found effective in suppressing traffic jam. The optimal velocity function is adopted as following [30]:

$$V(\rho_{j,m}) = \frac{v_{\max}}{2}\left[\tanh\left(\frac{2}{\rho_0} - \frac{\rho_{j,m}}{\rho_0^2} - \frac{1}{\rho_c}\right) + \tanh\left(\frac{1}{\rho_c}\right)\right] \tag{6}$$

where $v_{\max}$ and ρ_c denote the maximal velocity and the safety critical density. The optimal velocity function is monotonically decreasing, has an upper bound ($v_{\max}$) and a turning point at $\rho_{j,m} = \rho_c = \rho_0$.

3. Linear stability analysis

We apply the linear stability analysis to model Eq. (5). For this, the state of uniform traffic flow is taken as ρ_0 and optimal velocity $V(\rho_0)$, where ρ_0 is a constant. Hence, the steady-state solution of the homogeneous traffic flow is given by

$$\rho_{j,m}(t) = \rho_0, \qquad u_{j,m}(t) = V(\rho_0) \quad \text{and} \quad v_{j,m}(t) = V(\rho_0).$$

Let $y_{j,m}(t)$ be a small deviation from the steady-state flow: $\rho_{j,m}(t) = \rho_0 + y_{j,m}(t)$. Substituting perturbed density profile into Eq. (5) and linearizing it, we obtain

$$\begin{aligned}
&y_{j,m}(t+2\tau)\\
&\quad = y_{j,m}(t+\tau) - \tau\rho_0^2c^2V'(\rho_0)\big[y_{j+1,m}(t) - y_{j,m}(t)\big]\\
&\qquad - \tau\rho_0^2c^2V'(\rho_0)\big[y_{j+2,m}(t) - 2y_{j+1,m}(t) + y_{j,m}(t)\big]\\
&\qquad - \tau\rho_0^2(1-c)^2V'(\rho_0)\big[y_{j,m+1}(t) - y_{j,m}(t)\big]\\
&\qquad - \tau\rho_0^2(1-c)^2V'(\rho_0)\big[y_{j,m+2}(t) - 2y_{j,m+1}(t) + y_{j,m}(t)\big]
\end{aligned} \tag{7}$$

where $V'(\rho) = [\frac{dV(\rho)}{d\rho}]|_{\rho=\rho_0}$. Putting $y_{j,m}(t) = e^{(ik(j+m)+zt)}$ into Eq. (7), we get

$$\begin{aligned}
e^{2\tau z} &= e^{\tau z} - \tau\rho_0^2V'(\rho_0)\gamma\big[e^{ik} - 1\big]\\
&\quad - \lambda\tau\rho_0^2V'(\rho_0)\gamma\big[e^{2ik} - 2e^{ik} + 1\big]
\end{aligned} \tag{8}$$

where $\gamma = \{c^2 + (1-c)^2\}$. Inserting $z = z_1(ik) + z_2(ik)^2 \ldots$ into Eq. (8) and comparing the coefficient of ik and $(ik)^2$, we obtained

$$z_1 = -\gamma\rho_0^2V'(\rho_0) \tag{9}$$

$$z_2 = -\left[\frac{1+2\lambda}{2} + \frac{3\rho_0^2V'(\rho_0)\tau}{2}\gamma\right]\rho_0^2V'(\rho_0)\gamma \tag{10}$$

If $z_2 < 0$, then the uniform steady state flow becomes unstable for long-wavelength modes while the uniform flow remains stable for $z_2 > 0$. Thus, one obtains the neutral stability condition as follows:

$$\tau = (-1+2\lambda)/3\rho_0^2V'(\rho_0)\gamma \tag{11}$$

For homogeneous traffic flow, the flow is unstable if

$$\tau > (-1+2\lambda)/3\rho_0^2V'(\rho_0)\gamma \tag{12}$$

When $\lambda = 0$, the result of stability condition is same as the two-dimensional lattice hydrodynamic in Ref. [30]. Further, for $c = 0$ or $c = 1$, the stability criteria match with the stability condition of one-dimensional lattice hydrodynamic model proposed by Nagatani [14]. In Fig. 1, the neutral stability curve from linear and coexisting curve through nonlinear stability analysis are indicated by the solid and dotted line, respectively for different values of λ. The apex of each curve represents the critical point (ρ_c, τ_c). It is clear from Figs. 1(a) and 1(b) that with increasing effect of reaction coefficient, the sensitivity of critical point decreases and the region of stability increases for both the values of c. Therefore, the proposed model pulls the neutral stability curve and coexisting curve down, which shows that the optimal current difference effect enlarges the stability region for any configuration of traffic on a square lattice.

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