Physics Letters A 377 (2013) 2058–2061

Contents lists available at SciVerse ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Chaotic dynamics of flexible beams with piezoelectric and temperature phenomena

V.A. Krysko [d], J. Awrejcewicz [b,c,*], I.E. Kutepov [d], N.A. Zagniboroda [d], I.V. Papkova [d], A.V. Serebryakov [d], A.V. Krysko [a]

[a] Department of Applied Mathematics and Systems Analysis, Saratov State Technical University, Politehnicheskaya 77, 410054 Saratov, Russian Federation
[b] Lodz University of Technology, Department of Automation and Biomechanics, 1/15 Stefanowski St., 90-924 Lodz, Poland
[c] Warsaw University of Technology, Department of Vehicles, 84 Narbutta St., 02-524 Warsaw, Poland
[d] Department of Mathematics and Modeling, Saratov State Technical University, Politehnicheskaya 77, 410054 Saratov, Russian Federation

ARTICLE INFO

Article history:
Received 13 May 2013
Accepted 25 June 2013
Available online 1 July 2013
Communicated by A.R. Bishop

Keywords:
Chaos
Weak turbulence
Beam
Wavelets

ABSTRACT

The Euler–Bernoulli kinematic model as well as the von Kármán geometric non-linearity are used to derive the PDEs governing flexible beam vibrations. The beam is embedded into a 2D temperature field, and its surface is subjected to action of the electric potential. We report how an increase of the exciting load amplitude yields the beam turbulent behavior, and how the temperature changes a scenario from a regular/laminar to spatio-temporal/turbulent dynamics. Both classical Fourier analysis and Morlet wavelets are used to monitor a strong influence of temperature on regular and chaotic beam dynamics.

1. Introduction

Recently an attempt to study wave turbulence exhibited by thin elastic plates within “turbulence theories” has been observed. Since wave (weak) turbulence is less mathematically sophisticated than the classical (hydrodynamic) turbulence (see [1,2]) it is tempting to validate feasibility of vibrations of the two-dimensional solids with respect to the existing theoretical models for turbulence.

A series of recently published reports are devoted to an experimental study of the turbulent behavior of a plate within the Föppl–von Kármán model [3–5]. Despite a qualitative good agreement with the kinetic weak turbulence theory, the obtained energy spectrum has not been confirmed by a theoretical prediction. Mordant [6] applied an experimental method for monitoring of both temporal and spatial evolution of wave turbulence exhibited by a thin elastic plate. Various Fourier spectra of the wave deformations were analyzed. Elastic wave turbulence was also reported experimentally while analyzing thin elastic plates [7]. It was shown that when the total energies in wave fields were small, the obtained energy spectra fitted well with a statically steady solution of the weak kinetic turbulence theory.

Other researchers focused only on the theoretical and numerical simulation studying 2D structural members (mainly flexible plates). Touzé et al. used the von Kármán PDEs to study both experimentally and numerically a scenario of transition to wave turbulence exhibited by thin vibrating plates. Large amplitude plate vibrations were monitored while analyzing two bifurcations which separated three distinct regimes: periodic, quasi-periodic and spatio-temporal chaos. It is claimed that the third regime (turbulent) is characterized by a broad band Fourier spectrum and an energy cascade from large to small wavelength [8].

It should be emphasized that transition from regular to chaotic dynamics in circular cylindrical shells and doubly-curved panels was numerically reported by Amabili et al. [9–11]. Chaotic vibrations of shallow shell/panel with and without concentrated mass were studied both analytically and experimentally by Nagai et al. [12,13] and Maruyama et al. [14]. Recently, Touzé et al. [15] described a transition to chaotic vibrations for harmonically forced perfect and imperfect circular plates.

This report extends a series of our papers [16–23] devoted to the study of transition from regular (periodic and quasi-periodic) vibrations to chaotic ones in continuous mechanical systems (plates, cylindrical shells, panels and sector-type spherical shells).

In this Letter, however, we address the problem related to large deflations of beams, plates and shells, when the structural members are subjected to the action of temperature field and piezoelectric phenomena [24].

* Corresponding author at: Lodz University of Technology, Department of Automation and Biomechanics, 1/15 Stefanowski St., 90-924 Lodz, Poland.
E-mail address: awrejcew@p.lodz.pl (J. Awrejcewicz).

0375-9601/$ – see front matter
http://dx.doi.org/10.1016/j.physleta.2013.06.040

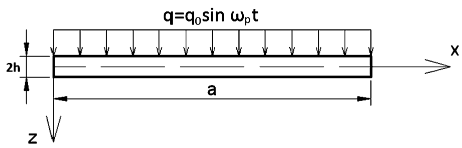


Fig. 1. Scheme of the analyzed beam.

2. Mathematical model

We apply the following assumption to derive the governing equations: (1) the Euler–Bernoulli kinematic model is used; (2) 2D temperature field model is applied; (3) beam surface is subjected to the action of the electric potential difference $V(t)$ – boundary of the studied space is not covered by the electrodes; (4) geometric non-linearity is taken in the von Kármán form – see Fig. 1.

Stress-strain equations including linear formulas for direct and inverse piezoelectric and pyroelectric effects are given in the following form:

$$\sigma_{xx} = c_{11}^E(\varepsilon_{xx} - \alpha_T T) - e_{31}E_z, \qquad D_x = \varepsilon_{11}^S E_x,$$
$$D_z = \varepsilon_{33}^S E_z + e_{31}\varepsilon_{xx} + g_{pyr}T. \tag{1}$$

We use the following physical constants for the material: c_{11}^E – elasticity modulus (for constant electric field); e_{31} – piezoelectric coefficient; $\varepsilon_{11}^S, \varepsilon_{33}^S$ – dielectric permittivity (for constant deformation); α_T – linear heat extension coefficient; $T = \theta(x,z,t) - T_0$ – temperature increase with respect to the initial temperature T_0; g_{pyr} – pyroelectric coefficient; $g_{pyr} = (2\ldots3)\cdot 10^{-3}$ C/(m K) in the direction of initial polarization, and $g_{pyr} = 0$ for remaining directions. State equations (1) refer to the situation when the beam material has already been polarized with respect to the beam thickness. Vector characteristics of the electric field are as follows: $\mathbf{D} = \mathbf{D}(x,z,t)$ – induction, $\mathbf{E} = \mathbf{E}(x,z,t)$ – intensity.

Applying the variation relations the following equations are derived:

$$k_1^2 \frac{\partial^2 w}{\partial x^2} + \frac{1}{\lambda^2}\frac{\varepsilon_{11}^S}{\varepsilon_{33}^S}\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2} - k_{pyr}^2 \lambda^2 \frac{\partial T}{\partial z} = 0, \tag{2}$$

$$\frac{\partial^2 u}{\partial x^2} + L_3(w,w) - \lambda^2(\alpha_T T_0)\int_{-1/2}^{1/2}\frac{\partial T}{\partial x}\,dz = \ddot{u} + \varepsilon_1 \dot{u}, \tag{3}$$

$$\frac{1}{\lambda^2}\left(L_1(u,w) + L_2(w,w) - \frac{1}{12}\frac{\partial^4 w}{\partial x^4} + q + k_2^2 \cdot V(t)\cdot\frac{\partial^2 w}{\partial x^2}\right) - (\alpha_T T_0) \times \left(\int_{-1/2}^{1/2}\frac{\partial^2 T}{\partial x^2} z\,dz + \frac{\partial^2 w}{\partial x^2}\int_{-1/2}^{1/2} T\,dz + \frac{\partial w}{\partial x}\int_{-1/2}^{1/2}\frac{\partial T}{\partial x}\,dz\right) = \ddot{w} + \varepsilon_2 \dot{w}, \tag{4}$$

$$\frac{\partial^2 T}{\partial x^2} + \lambda^2\frac{\partial^2 T}{\partial z^2} = \left[\frac{\rho c_\varepsilon a}{\lambda_q}\sqrt{\frac{c_{11}^E}{\rho}}\right]\frac{\partial T}{\partial t} + \left[\alpha_T a \frac{c_{11}^E + c_{12}^E + c_{13}^E}{\lambda_q}\sqrt{\frac{c_{11}^E}{\rho}}\right] \times \left(\frac{\partial \dot{u}}{\partial x} + \frac{1}{\lambda^2}\frac{\partial w}{\partial x}\frac{\partial \dot{w}}{\partial x} - \frac{1}{\lambda^2} z \frac{\partial^2 \dot{w}}{\partial x^2}\right) - \left[\frac{a}{\lambda_q}\frac{g_{pyr}}{d_{31}}\sqrt{\frac{c_{11}^E}{\rho}}\right] \times \frac{1}{\lambda^2}\frac{\partial \dot{\psi}}{\partial z}, \tag{5}$$

where: $L_1(u,w) = \frac{\partial u}{\partial x}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 u}{\partial x^2}\frac{\partial w}{\partial x}$, $L_2(w,w) = \frac{3}{2}\frac{\partial^2 w}{\partial x^2}(\frac{\partial w}{\partial x})^2$, $L_3(w,w) = \frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2} = \frac{1}{2}L_1(w,w)$, and the electric potential $\psi = \psi(x,z,t)$ satisfies the following electrostatic equations $\frac{\partial D_x}{\partial x} + \frac{\partial D_z}{\partial z} = 0$, $E_x = -\frac{\partial \psi}{\partial x}$, $E_z = -\frac{\partial \psi}{\partial z}$. Eqs. (2)–(5) are associated with the following boundary conditions for the electric potential:

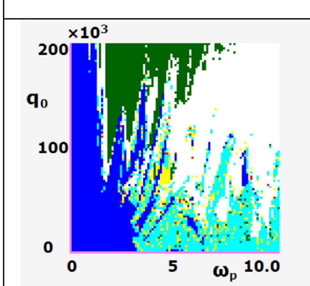
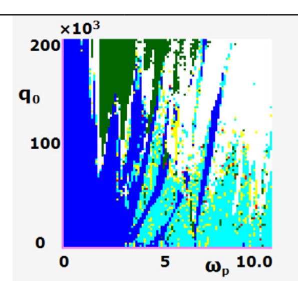
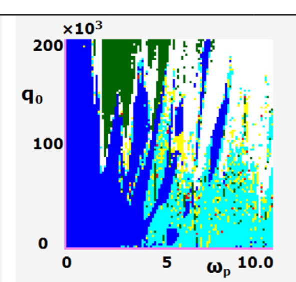
$$\psi(x,-1/2,t) = -V(t)/2,$$
$$\psi(x,1/2,t) = V(t)/2 \quad (0 \leqslant x \leqslant 1, t > 0), \tag{6}$$
$$\frac{\partial \psi}{\partial x} = 0. \tag{7}$$

For the functions characterizing the stress beam state regarding $u(x,t)$, $w(x,t)$ we have,

$$w(0,t) = w(1,t) = u(0,t) = u(1,t) = w'_x(1,t) = 0,$$
$$M_x(0,t) = 0, \tag{8}$$

and in the case of temperature field the first order boundary conditions are applied.

Table 1
Vibration charts.

$\tilde{T} = 20^\circ$C	$\tilde{T} = 200^\circ$C	$\tilde{T} = 300^\circ C$
		
2D quasi-periodicity; superposition of two independent frequencies; regime not defined; damped vibrations; chaotic vibrations; periodic vibrations; chaotic vibrations		