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Quantum instability of two non-parallel flows: Parallel wave propagation

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ABSTRACT

The streaming instability for two non-paralleled plasma shells propagating through a background quantum plasma has been studied. The dispersion relation is calculated, assumed that the perturbation wave vector is in the direction of one of the plasma shell, by linearizing and solving fluid-Maxwell equations. The resulting dispersion equation is analyzed numerically in a wide range of system parameters. Investigation of cut-off wave number and growth rate are shown that, the instability for a wide range of wave number when the velocity of shells reaches to a specific value, $\beta = \beta_c$.

1. Introduction

Many theoretical concepts of plasma systems rely on a thorough understanding of plasma beam instabilities. The free energy in the relative streaming of the two species can give rise to great variety of instabilities and consequent plasma heating. These effects can be important in a variety of laboratory, magnetospheric and astrophysical plasmas, such as cosmical magnetic fields [1] and gamma ray bursts (GRBs) [2–5]. In this Letter, we focus our attention on the growth rate of the electromagnetic beam plasma instabilities in collision of the two plasma shells. We note that the term “collision” refers to the interaction between rarefied plasmas of different bulk motion (= shells), within with the charge particles scattering at the collective fields of plasma instabilities. When two plasma shells propagate in a plasma, two main kinds of streaming instability, multi-stream instability (MSI) or two-stream instability (TSI) [6] and filamentation instability (FI) [7], which may have a considerable effect upon the plasma electrical conductivity can be appeared. So far, these instabilities have been studied by assuming the two counter-streaming plasma shells in the context of classical regime [8–13]. While the two plasma shells can be considered non-parallel [12]. On the other hand quantum plasma, attracting more attention in recent years [14–26], can be used to observe novel features of the system. As is well known, quantum effects appear in ultra small electronic devices, laser plasmas, and dense astrophysical plasmas [27–29]. While the instability of quantum counterstreaming beam systems have been assessed for mode with wave vector aligned or perpendicular with the streams, and recently for mode with arbitrarily oriented wave vector by A. Bret and F. Haas [22], quantum non-parallel streams have not been considered yet. The intent of this Letter is precisely to fill this gap. Therefore in this Letter, the dynamics of two non-parallel plasma shells with zero net charge in quantum regime have been investigated. It should be noted that, presenting of background plasma is very important to design this problem because in the absence of it one can be sitting in the reference frame of one of the shells to cancel any orientation parameter.

In order to simplify the analytical treatment, we focus on the role of the quantum multi-stream and filamentation instability by considering two identical electron–proton plasma shells crossing each other on the background plasma, Fig. 1. In addition, it is assumed that the finite thermal spread of both plasma shells and background plasma are negligible. Consequently, in this Letter the cold beam approximation is valid for the entire linear regime.

This Letter is organized as follows. The physical and theoretical model employed in our analysis is described in Section 2. In Section 3 the MSI and FI are analyzed in detail numerically. Finally, the conclusions are summarized in Section 4.

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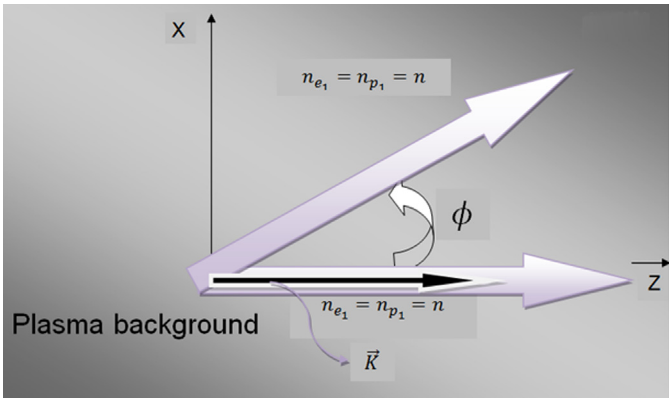

Fig. 1. Schematic of two plasma shells propagating in plasma.

2. Dispersion equation

We consider two infinite, homogeneous, cold, non-parallel, and non-relativistic electron–proton plasma shells, neutralized charge and current, propagating through neutralize and un-magnetized background plasma, composed of electron and proton with equal density N. The equilibrium configuration is illustrated in Fig. 1. As can be seen in this figure, perturbation wave vector's is assumed to be parallel to the one of the plasma shells, in Z direction. It is also assumed that the ions are not affected by the perturbation. Let us study the non-relativistic dynamic of the three electron populations, including two electron populations of plasma shells propagating with same velocity, V, along their respective direction, shown in Fig. 1, and electron population of background plasma at the rest in equilibrium state. The following set of quantum fluid equations are used for the three electron populations,

$$\frac{\partial n_j}{\partial t} + \nabla.(n_j \mathbf{v}_j) = 0, \tag{1}$$

$$\frac{\partial \mathbf{v}_j}{\partial t} = -\frac{q}{m}\left(\mathbf{E} + \frac{\mathbf{v}_j \times \mathbf{B}}{c}\right) + \frac{\hbar^2}{2m^2}\nabla\left(\frac{\nabla^2\sqrt{n_j}}{\sqrt{n_j}}\right), \tag{2}$$

where $q > 0$ and m are the charge and mass of the electron, n_j and v_j are, respectively, the density and velocity of electron population j, the index $j = 1, 2$ stands for the electron population of plasma shells, and index $j = 3$ stands for electron population of background plasma. Quantum corrections is clearly contained within the so-called Bohm potential by means of the $\hbar^2$ term in the above equation [25]. It is assumed that all dynamic variables may be written as a zero order part which is constant in time and space and a first order part which varies as $\exp(i\vec{k}.\vec{r} - i\omega t)$, where $\vec{k} = (0, 0, k_z)$. In terms of the first order variables, the continuity equation becomes

$$n_{j1} = n_{j0}\frac{\vec{k}.\vec{v}_{j1}}{\omega - \vec{k}.\vec{v}_{j0}} \tag{3}$$

while the linearized quantum Euler equation, Eq. (2), gives

$$i(\vec{k}.\vec{v}_{j0} - \omega)\vec{v}_{j1} = -\frac{q}{m}\left(\vec{E}_1 + \frac{\vec{v}_{j0} \times \vec{B}_1}{c}\right) - i\frac{\hbar^2 k^2}{4m^2}\frac{n_{j1}}{n_{j0}}\vec{k}. \tag{4}$$

Note that $\vec{v}_{10} = (0, 0, V)$ and $\vec{v}_{20} = (V \sin\phi, 0, V \cos\phi)$ are equilibrium velocity of electron plasma shells, ϕ is the angle between the two shells, and $\vec{v}_{30} = 0$ is equilibrium velocity of electron background plasma. Substituting the expression for $\vec{B}_1$ from the linearized Maxwell equation, $\vec{k} \times \vec{B}_1 = i\frac{\omega}{c}\vec{E}_1$, in Euler equation one would obtain the perturbed velocities, v_{1j}, as

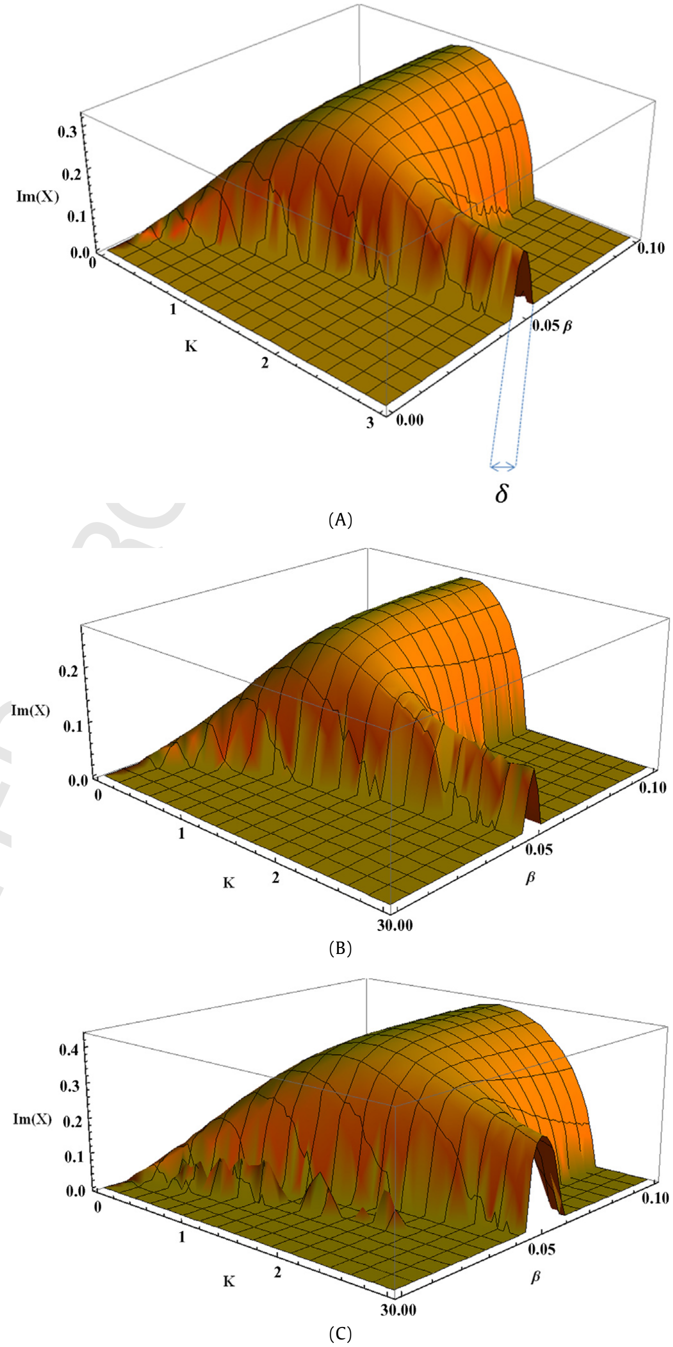

Fig. 2. Normalized growth rate map in terms of the normalized wave number, $K = (k_z V)/\omega_p$, and normalized velocity of the plasma shell, $\beta = V/c$, for the case in which (A) $\phi = 0$ and $\alpha = 0.1$ (two-stream instability), (B) $\phi = \pi$ and $\alpha = 0.1$ (multi-stream instability), and (C) $\phi = \pi$ and $\alpha = 0.5$ (multi-stream instability).

$$\vec{v}_{11} = \frac{-iE_{1x}q}{m\omega_p x}\hat{i} - \frac{iE_{1y}q}{m\omega_p x}\hat{j} - \frac{iE_{1z}q(K_z - x)}{m\omega_p(-(K_z - x)^2 + K_z^4\Theta)}\hat{k}, \tag{5}$$

$$\vec{v}_{21} = \frac{-iE_{1x}q}{m\omega_p x}\hat{i} - \frac{iE_{1y}q}{m\omega_p x}\hat{j} - \frac{iq(x - K_z \mathrm{Cos}[\phi])(E_{1z}x + E_{1x}K_z \mathrm{Sin}[\phi])}{m\omega_p x(x^2 - K_z^4\Theta + K_z \mathrm{Cos}[\phi](-2x + K_z \mathrm{Cos}[\phi]))}\hat{k}, \tag{6}$$

$$\vec{v}_{31} = \frac{-iE_{1x}q}{m\omega_p x}\hat{i} - \frac{iE_{1y}q}{m\omega_p x}\hat{j} - \frac{iE_{1z}qx}{m\omega_p x^2 - K_z^4 m\omega_p\Theta}\hat{k} \tag{7}$$

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