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Solitary waves and rogue waves in a plasma with nonthermal electrons featuring Tsallis distribution

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ABSTRACT

In this Letter, we discuss the electron acoustic (EA) waves in plasmas, which consist of nonthermal hot electrons featuring the Tsallis distribution, and obtain the corresponding governing equation, that is, a nonlinear Schrödinger (NLS) equation. By means of Modulation Instability (MI) analysis of the EA waves, it is found that both electron acoustic solitary wave and rogue wave can exist in such plasmas. Basing on the Darboux transformation method, we derive the analytical expressions of nonlinear solutions of NLS equations, such as single/double solitary wave solutions and single/double rogue wave solutions. The existential regions and amplitude of solitary wave solutions and the rogue wave solutions are influenced by the nonextensive parameter q and nonthermal parameter α. Moreover, the interaction of solitary wave and how to postpone the excitation of rogue wave are also studied.

1. Introduction

Electron acoustic (EA) waves not only occur in space plasmas such as in the Earth's bow shock [1] and in the auroral magnetosphere [2], but also can be obtained in laboratory experiments [3–5]. For instance, the excitation of EA waves has been obtained in recent experiments with non-neutral plasmas by the non-neutral plasma physics group at University of California at San Diego [5]. The propagation of EA waves in plasmas with two-temperature (cold and hot) electrons has received a great deal of attention because of its vital role in understanding different types of collective processes in laboratory devices as well as in space environments [6,7]. EA waves in such plasmas are typically high frequency waves because its frequency is much higher than the ion plasma frequency. Therefore, ions remain stationary and form a neutralized background. They provide charge neutrality but do not play an essential role in the dynamics. The cool electrons provide the inertia necessary to maintain the electrostatic oscillations, while the restoring force comes from the hot electrons' pressure.

A lot of studies focus on the nonlinear evolution of EA waves in such plasmas. One of the most famous nonlinear EA waves is the electron acoustic solitary wave, which is observed in experiments with pure electron plasmas [8] and in laser-produced plasmas [9]. Many researchers have done excellent job in studying EA solitary waves theoretically and numerically [10–13]. For instance, in Ref. [13], the excitation of long-lived electrostatic solitons were obtained in unmagnetized plasma of electrons and ions by driving the system with an external electric field. The excitation of such acoustic solitons is triggered by the resonant interaction of electrostatic waves and particles that deform the particle distribution function through the generation of trapped particle regions. All these researches consider the electrons follow the Maxwellian distribution, which is believed to be valid universally for the macroscopic ergodic equilibrium systems. Even so, numerous observations of space plasmas are often characterized by a particle distribution function with high energy tail, but such distribution may deviate from Maxwellian distribution [14,15].

The non-equilibrium stationary states exist in the systems with the long-range interactions such as plasma and gravitational systems. Therefore, Maxwellian distribution might be inadequate for the description of these systems. Tsallis [16] consistently extended Boltzmann–Gibbs (BG) thermodynamics by extending the concept of entropy to the nonextensive fields. In these fields, the entropic

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index q characterizes the degree of nonextensivity of the considered system ($q = 1$ corresponds to the standard extensive, BG statistics). Indeed, many physical systems, which cannot be correctly explained by using the classical statistical description, can be described by the appropriate framework of nonextensive statistics. For instance, some researchers have discussed details of the wave–particle interaction in the case of plasmas of nonextensive electrons [17–19]. In these papers, the effect of the Landau damping saturation is analyzed as a function of different values of the nonextensive parameter q. It is found that effect of nonlinear wave–particle interaction produces distortion of the particle distribution function.

Much more evidence has shown that Tsallis q-entropy and the ensuing nonextensive statistics may be very important for systems endowed with long-range interactions as usually happens in astrophysics and plasma physics. It is proved to provide a convenient frame for the analysis of problems in plasma physics, long-range Hamiltonian systems, gravitational systems, astrophysical environments, and so on [20,21]. For instance, the Tsallis statistics is commonly used in core-halo distributions frequently detected under typical interplanetary conditions [22], and the analysis of turbulence and intermittency phenomena in solar wind plasmas [23]. The presence of the nonextensive electron component has been proved to have a great influence upon the Modulational Instability (MI) of the EA waves and play a vital role in the formation of solitary waves. Meanwhile, a lot of studies have already confirmed the existence of nonthermal electrons through the observation of a variety of astrophysical plasma environments [24,25]. Therefore, we mainly discuss the solitary waves and rogue waves in EA waves containing the population of Tsallis electrons.

In recent years, rogue wave also arouses people's great interest. The rogue wave (or freak wave), which is firstly found in the ocean with amplitude much higher than the average wave crests around it [26], has attracted more and more attention of the researchers. The rogue wave is a short-lived phenomenon which will suddenly appear out of normal waves but with a small probability. It was considered mysterious until direct measurements confirmed its appearance in real life [27]. Indeed, rogue wave can be detected in various nonlinear physical environments, such as optical systems [28,29], Bose–Einstein condensates [30], superfluid helium [31], atmosphere [32], and even the financial market [33]. Recently, more and more researchers pay attention to the rogue wave phenomena in plasmas. For instance, in Ref. [34], authors investigated the nonlinear Langmuir rogue wave in collisionless electron–positron plasmas. Authors [35] presented an investigation for the generation of a dust acoustic rogue wave in a dusty plasma composed of negatively charged dust grains, as well as nonextensive electrons and ions. Ref. [36] showed that solitary and freak waves could propagate in a dusty plasma composed of positive and negative ions, as well as nonextensive electrons. The generation of nonlinear ion acoustic waves in a plasma with nonextensive electrons and positrons was also studied [37].

Linear wave theories have difficulties in explaining such waves [38,39]. In contrast, nonlinear theories are promising [40,41]. Generally speaking, rogue waves represent an extreme sensitivity of the nonlinear system to the initial conditions. These waves may arise from the instability of a certain class of initial conditions that tend to grow exponentially and thus have the possibility of increasing up to very high amplitudes due to MI [42]. Moreover, many researchers confirm that one of the best ways to describe rogue waves mathematically is the rational solutions of the nonlinear Schrödinger (NLS) equation. Besides the theoretical study on rogue wave in plasma, the experimental study has also made great progress. Experimental observations of rogue waves in a multicomponent unmagnetized plasma have been reported [43].

As the nonlinear features of EA wave have not been understood thoroughly, it is meaningful to investigate not only the EA solitary wave but also the EA rogue wave in plasmas with nonthermal hot electrons featuring the Tsallis distribution. To the best of our knowledge, the EA solitary wave and EA rogue wave in such plasmas has not been investigated yet. Moreover, the rogue wave may cause disaster in ocean, but in optical media, the high amplitude feature of rogue wave can be applied to excite high intensity optical pulses. Thus, it is important and meaningful to study how to control rogue wave in plasmas. In this Letter, we will not only focus on the study of the Modulation Instability (MI) which causes the emergence of the solitary wave and rogue wave, but also investigate the effect of nonextensive parameter q and nonthermal electrons parameter α on the existential region and the amplitude of these nonlinear waves. Then, we give the analytical solitary wave solutions and rogue wave solutions for NLS equation by using the Darboux transformation method [44–46]. Finally, the collision of two solitary waves and rogue waves as well as the control of rogue waves are investigated.

2. Basic equations and derivation of the NLS equation

2.1. Basic equations

We consider a collisionless unmagnetized plasma consisting of cold fluid electrons, stationary ions and hot nonthermal electrons featuring Tsallis distribution. The normalized equations are as follows [47]:

$$\begin{cases} \dfrac{\partial n_c}{\partial t} + \dfrac{\partial (n_c u_c)}{\partial x} = 0, \\ \dfrac{\partial u_c}{\partial t} + u_c \dfrac{\partial u_c}{\partial x} = \delta \dfrac{\partial \phi}{\partial x}, \\ \dfrac{\partial^2 \phi}{\partial x^2} = \dfrac{1}{\delta} n_c + n_h - \left(1 + \dfrac{1}{\delta}\right), \end{cases} \tag{1}$$

where n_c (n_h) is the cold (hot) electron number density normalized by its equilibrium value n_{c0} (n_{h0}), u_c represents the cold electron fluid velocity normalized by $\sqrt{k_B T_h/\delta m_e}$, where $\delta = n_{h0}/n_{c0}$. ϕ is the electrostatic potential normalized by $k_B T_h/e$. The time and distance are in units of the cold electron plasma period $\omega_{pc}^{-1} = \sqrt{m_e/4\pi n_{c0} e^2}$ and the hot electron Debye length $\lambda_{Dh} = \sqrt{k_B T_h/(4\pi n_{h0} e^2)}$, respectively.

The hot electrons are assumed to follow the nonextensive nonthermal velocity distribution function and the normalized hot electron density is given as

$$n_h = \left[1 + (q-1)\phi\right]^{\frac{q+1}{2(q-1)}} \left[1 + A\phi + B\phi^2\right], \tag{2}$$