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Exploration of amphoteric and negative refraction imaging of acoustic sources via active metamaterials

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A B S T R A C T

The present work describes the design of three flat superlens structures for acoustic source imaging and explores an active acoustic metamaterial (AAM) to realise such a design. The first two lenses are constructed via the coordinate transform method (CTM), and their constituent materials are anisotropic. The third lens consists of a material that has both a negative density and a negative bulk modulus. In these lenses, the quality of the images is "clear" and sharp; thus, the diffraction limit of classical lenses is overcome. Finally, a multi-control strategy is developed to achieve the desired parameters and to eliminate coupling effects in the AAM.

1. Introduction

Acoustic imaging presents peculiarities and attributes that can provide observation and inspection capabilities in situations where optical systems cannot. As a result, acoustic imaging enjoys widespread applications in engineering, such as medical imaging in the human body, non-destructive inspection of solid objects and sound navigation and ranging in underwater environments [1–3].

The physics of negative refraction (NR) proposed by Veselago in 1967 [4] triggered a great deal of interest in electromagnetic (EM) and optical "flat lens" (also referred to as a "superlens" or "perfect lens") imaging. However, NR imaging remained an academic curiosity for over three decades until Pendry and his colleagues paved the way towards the realisation of such negative refractive-index materials (metamaterials) in the EM field. These materials simultaneously possess negative permittivity ε and negative permeability μ [5,6]. Promptly, a considerable number of theoretical and experimental works appeared in the open literature that attempted to realise EM and optical superlenses using appropriate metamaterials [7–12].

Because acoustic imaging is expected to revolutionise numerous applications in science and medicine, many researchers have committed to constructing acoustic superlenses or acoustic metamaterials (AMs) that behave much like left-handed media. The introduction of locally resonant (LR) sonic crystals (SCs) by Liu in 2000 paved the way towards acoustic analogues of EM metamaterials (EMMs) [13]. By creating artificially designed structures on a deep subwavelength scale, artificial acoustic "atoms" can be purposely engineered into LR SCs to dramatically change the excitation and propagation of acoustic waves; this gives rise to subdiffraction-limited resolution and a myriad of related novel effects such as NR, negative elastic modulus, and negative mass density [14–17]. By analogy with EM and optical superlenses, the same interesting NR phenomena may occur in the acoustic field whereby acoustic waves travel the "wrong" way leading to an inverted Snell–Descartes law and the possibility of a convergent flat lens. In a recent publication, Liang Feng et al. [18] validated acoustic NR with backwards-wave effects in the second band of a two-dimensional (2D) triangular SC both theoretically and experimentally. Guenneau et al. provided a theoretical design for a 2D periodic LR cylindrical structure that displays the NR effect in the low frequency regime; a slab of such a periodic LR structure could work as a sonic flat superlens [19]. Liu and his colleagues established the NR imaging of solid acoustic waves in a three-component SC composed of coated solid inclusions placed in a solid matrix. Additionally, much parallel progress has been reported for a similar problem in SCs in recent works [20–23].

In spite of recent advances, the image quality of the aforementioned NR imaging in SCs is not always guaranteed; in fact, images are usually obscured because the undesired wave reflection and diffraction that existed in such an SC imaging system decreases the image quality. Fortunately, the works of Pendry [6] and Chen

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et al. [7] suggest that the shortcomings in these SC imaging systems could be overcome by a perfect lens; light travelling in a perfect lens will not experience any reflection, and the ray paths will be deformed by the amphoteric/negative refractive index gradient. In view of the exact equivalence between 2D acoustics and electromagnetics in both isotropic and anisotropic media, a perfect acoustic lens could be constructed by simply extending the method to produce a perfect EM lens to the case of an acoustic field. The EMMs in an EM lens always have characteristics of anisotropic and/or negative permittivity ε and negative permeability μ, and these characteristics must be mimicked in the production of AMs for the acoustic lenses (by, e.g., negative densities or/and moduli); however, finding a material in nature with such characteristics is nearly impossible.

No scheme can be of much interest if the means to realise it are not available. Fortunately, the effective medium theory defeats the practical limitations in realising an AM with correctly designed properties [24–34]. This theory, which describes the construction of metamaterials having characteristics such as anisotropic or negative densities using multilayered structures with homogeneous, isotropic materials, has been successfully used in the construction of metamaterials for acoustic cloaks/anti-cloaks [29,30] and noise shielding [31]. Still, problems are often encountered when constructing the materials for each layer of the multilayered structure because the parameters are usually difficult to realise. Luckily, the appearance of active acoustic metamaterials (AAMs) may help to solve the current knotty problem. The effective parameters of an AAM can be optionally changed using external control voltages. Moreover, an active technique to generate AMs with negative densities and/or bulk moduli can realise the desired parameters in a very large bandwidth; conversely, the current passive technique of AM design is inherently limited in bandwidth because it must ultimately use some form of resonant behaviour. Hence, the recipe to design an acoustic superlens that can bend acoustic waves in almost any direction is now clear: AAMs must be constructed. Several theoretical and experimental studies that are relevant to this Letter were conducted by Lee et al. [32] and Baz [35], and they provide ways to achieve the negative effective density and modulus characteristics for AMs.

The Letter is organised into four sections. Section 1 presents a brief introduction of acoustic imaging and superlenses. In Section 2, the detailed design procedures for amphoteric/negative imaging and AAMs are explored. Section 3 presents several numerical examples to validate the designs of the acoustic superlenses and to illuminate how the multi-control strategy for the AAM helps to achieve the required parameters. Finally, a brief summary of the conclusions and suggestions for future work are given in Section 4.

2. Construction of NR metamaterials for acoustic imaging

The fundamental idea behind EMMs, which realise any desired EM function, is built on the invariance of Maxwell's equations under a space-deforming transformation if the material properties are altered accordingly. Maxwell's equations possess a special symmetry to describe waves in a way that most other systems do not [24]. Here, we use an EM field transformation as the starting point to derive the parameters of NR acoustic metamaterials. Considering a time-harmonic electric field and a magnetic field in a uniaxially anisotropic material, the 2D Maxwell's equations for transverse electric (TE) modes can be given as follows:

$$-\frac{\partial E_z}{\partial y} = \mathrm{i}\omega\mu_x H_x, \qquad -\frac{\partial E_z}{\partial x} = \mathrm{i}\omega\mu_y(-H_y), \quad \text{and}$$
$$\frac{\partial H_x}{\partial y} + \frac{\partial(-H_y)}{\partial x} = \mathrm{i}\omega\varepsilon_z(-E_z), \tag{1}$$

where μ and ε are the permittivity and permeability tensors, respectively, which can be given as $\mu = \mathrm{diag}(\mu_x, \mu_y, \mu_z)$ and $\varepsilon = \mathrm{diag}(\varepsilon_x, \varepsilon_y, \varepsilon_z)$; the subscripts x, y and z denote the corresponding components of μ and ε in the three axial directions in Cartesian coordinates. With respect to the acoustic equation in a metamaterial domain where the bulk modulus κ is scalar and the fluid density ρ is anisotropic, the linear relationships between pressure and density are

$$\nabla p = \mathrm{i}\omega \overleftrightarrow{\rho}\, \vec{v} \quad \text{and} \quad \kappa\nabla\cdot\vec{v} = \mathrm{i}\omega p, \tag{2}$$

where $\overleftrightarrow{\rho}$ and $\vec{v}$ are the density tensor and the velocity vector of the acoustic medium, respectively. Expanding these acoustic equations in Cartesian coordinates with z-invariance and suppressing the $\exp(-\mathrm{i}\omega t)$ term throughout, the equations governing the acoustic wave motion are simplified to

$$\frac{\partial p}{\partial x} = \mathrm{i}\omega\rho_x v_x, \qquad \frac{\partial p}{\partial y} = \mathrm{i}\omega\rho_y v_y, \quad \text{and}$$
$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = \mathrm{i}\omega p\kappa^{-1}. \tag{3}$$

By comparing Eqs. (3) and (1), we find that 2D acoustics and TE electromagnetics in anisotropic media are exactly equivalent under the following exchange of variables:

$$\begin{bmatrix} -E_z & H_x & -H_y \end{bmatrix} \leftrightarrow \begin{bmatrix} p & v_y & v_x \end{bmatrix} \quad \text{and}$$
$$\begin{bmatrix} \mu_x & \mu_y & \varepsilon_z \end{bmatrix} \leftrightarrow \begin{bmatrix} \rho_y & \rho_x & \kappa^{-1} \end{bmatrix}. \tag{4}$$

The boundary conditions are also preserved under this exchange [24]. This manipulation method has been successfully used to generate a perfectly invisible cloak and to define the characteristics of the corresponding AMs [24,29]. Inspired by this method and the work of Pendry [6], which proposed an unconventional EM lens consisting of a negative refractive index material (i.e., a material with negative permittivity ε and negative permeability μ), an acoustic superlens can be obtained easily via the exchange of variables. In the acoustic lens slab, the NR phenomenon can be induced (this phenomenon will be validated in the next section), and such an acoustic lens slab requires its constituent materials to simultaneously possess negative density and bulk modulus. In what follows, another type of metamaterial for acoustic superlenses will be discussed that is designed by the so-called coordinate transformation method (CTM) [26,28,29]; the characteristics of the coordinate-mapping-induced materials are anisotropic but not negative, and they exhibit positive or negative refraction depending on the angle of incidence.

Now, consider a coordinate transformation, as sketched in Fig. 1, to manipulate the propagation path of travelling waves. The mapping formulae for this coordinate transformation are

$$x < x_0\colon \quad x_t = x, \quad y_t = y, \quad z_t = z;$$
$$x_0 < x < x_0 + n\lambda_0\colon \quad x_t = x,$$
$$y_t = y + (x - n\lambda_0)\tan(\alpha), \quad z_t = z;$$
$$x > x_0 + n\lambda_0\colon \quad x_t = x, \quad y_t = y + n\lambda_0\tan(\alpha), \quad z_t = z, \tag{5}$$

where λ_0 is the wavelength of the incident waves and $n\lambda_0$ is the thickness of the lens. Hence, the coordinate-mapping-induced medium is described by

$$\overleftrightarrow{\mu}_t = \Lambda \overleftrightarrow{\mu} \Lambda^T / \det(\Lambda) \quad \text{and} \quad \overleftrightarrow{\varepsilon}_t = \Lambda \overleftrightarrow{\varepsilon} \Lambda^T / \det(\Lambda), \tag{6}$$

and the set of EM parameters for the TE wave is

$$\overleftrightarrow{m}_{TE} = \begin{bmatrix} \mu_x & \mu_x\tan\alpha & 0 \\ \mu_x\tan\alpha & \mu_x\tan^2\alpha + \mu_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}, \tag{7}$$