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Broadband transmission enhancement of acoustic waves through tapered metallic gratings

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ABSTRACT

The phenomenon of the broadband transmission enhancement of acoustic waves is important for its prospective applications in various devices. To achieve broadband transmission enhancement of acoustic waves, we investigate the acoustic transmission in a metallic grating device with linearly tapered slits through eigen-mode matching theory (EMMT) and finite element (FE) simulation. The tapering provides a gradual filling fraction variation from the entrance to the exit of the slits, destroying the Fabry–Perot resonance and thus leading to broadband and wide-angle enhanced transmission in the audible regime. In addition, acoustic waves are strongly localized and enhanced at the slit exits, in contrast with straight slits.

1. Introduction

The extraordinary acoustic transmission (EAT) through one-dimensional acoustic gratings has attracted much attention due to its theoretical and practical interests [1–6]. In practice, the extraordinary transmission can lead to a wide range of future applications such as imaging and screening, antireflection cloaking, grating interferometry and crack detection. In principle, the EAT results from either the Fabry–Perot resonance in the holes/slits, or the coherent diffraction induced by the periodicity [7]. Lu et al. have taken further step to point out that the physical origin of EAT is attributed to the continuum crossover behavior of these two mechanisms [8]. Wang uses acoustical impedances to describe the effect of evanescent waves on the aperture resonance, which reveals that the anomaly of the induced reactances changes the conditions and behaviors of the aperture resonance leading to the EAT phenomenon [9]. Qi et al. demonstrate that 1D gratings can become transparent and largely antireflective under oblique incidence of acoustic waves within multiple frequency bands separated by Wood's anomalies [10]. It should be noted that the EAT mentioned above are directly associated with resonances and hence are inherently narrowband. Most recently Qiu et al. report a broadband transmission enhancement of acoustic waves through a hybrid grating, which states that the EAT results from the impedance matching between the background and the hybrid grating since the latter effectively behaves as a homogeneous metamaterial slab [11]. This, however, leads to a complicate structure, which may be limited for its applications. Thus, it is very meaningful to achieve broadband transmission enhancement of acoustic waves in a simple structure. On the other hand, due to the anisotropic of material and/or fabrication imperfections in reality, the straight sidewall is more likely to have a tapered shape with a given slope angle. In the optical counterpart, the transmission enhancement can be improved by using periodic arrays of tapered slits [12]. Shen et al. find that the tapering leads to broadband and wide-angle enhanced transmission in the infrared [13]. Here, the geometry of tapered structures inspires us to investigate acoustic transmission through tapered metallic gratings, which has not been studied yet.

In this Letter, we report the transmission of an incident acoustic wave through a one-dimensional acoustic grating with tapered slits. By gradually varying the filling fraction from input to output plane, we effectively destroy the Fabry–Perot type resonant conditions of guided modes in the slit, yielding a broadband and wide-angle large transmission in the audible regime. In addition, the localization of the field can be confined to one plane of the structure, instead of over the whole lossy waveguide as in the traditional structures with straight sidewalls.

The Letter is organized as follows. The setup of the numerical model and the method of calculation based on the EMMT and the FE simulation are briefly explained in Section 2. In Section 3, numerical results and discussions are given. We finally summarize the Letter in Section 4.

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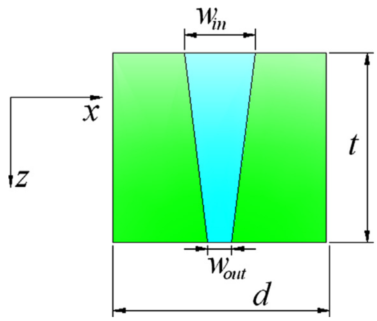

Fig. 1. Schematic of 1D metallic grating with linearly tapered slits. The gratings are periodic in the x-direction.

2. Model and basic theory

Schematic of 1D metallic grating with linearly tapered slits is illustrated in Fig. 1. The grating with tapered slits is characterized by the thickness t, period d, and widths w_{in} and w_{out} at the entrance and exit of the slit, respectively. Surrounding and substrate are semi-infinite. In addition, we assume surrounding, substrate, and slit material to be air or water and the metal grating is assumed to be steel. The material parameters are $\rho_a = 1.21\ \text{kg/m}^3$, $\rho_w = 1000\ \text{kg/m}^3$, $\rho_s = 7800\ \text{kg/m}^3$, $c_a = 344.5\ \text{m/s}$, $c_w = 1498\ \text{m/s}$ and $c_s = 6100\ \text{m/s}$. Here the parameters of ρ_a, ρ_w and ρ_s denote the mass densities of air, water and steel, respectively; c_a, c_w and c_s refer to the acoustic velocity in the air, water and the longitudinal velocity in the steel, respectively.

In order to investigate the acoustic transmission spectra, we introduce now the employed EMMT, which is developed for two-dimensional phononic crystals [14–16]. The acoustic wave propagating in the media can be written as

$$-\omega^2 C_{11}^{-1} p = \frac{\partial \sigma_x}{\partial x} + \frac{\partial \sigma_z}{\partial z},$$
$$\sigma_{x_i} = \rho^{-1} \frac{\partial p}{\partial x_i} \quad (x_i = x, z), \tag{1}$$

where p and C_{11}, ρ are the pressure field and the elastic stiffness, mass density of the media, respectively. For each layer, because of the periodicity of the system along the x-direction, the elastic parameters and wave solution can be expanded as a Fourier summation along x-direction, and thus Eqs. (1) can be expressed as

$$\sum_{G'} \left[\omega^2 C_{11}^{-1} - (k+G')(k+G)\rho_{G-G'}^{-1}\right] p_{k+G'} = \beta \sum_{G'} \rho_{G-G'}^{-1} p_{k+G'}. \tag{2}$$

The above equation in each layer can be described by a superposition of a set of perpendicular modes $e^{i\alpha_n x}$,

$$\begin{bmatrix} \mathbf{p} \\ j\boldsymbol{\sigma} \end{bmatrix} = \sum_{n=-M}^{M} e^{i\alpha_n x} \left[\sum_{m=1}^{N} A_{mL} e^{i\beta_{mL} z} \begin{pmatrix} \mathbf{p}_{mL}^n \\ \boldsymbol{\sigma}_{mL}^n \end{pmatrix} + \sum_{m=1}^{N} A_{mU} e^{i\beta_{mU} z} \begin{pmatrix} \mathbf{p}_{mU}^n \\ \boldsymbol{\sigma}_{mU}^n \end{pmatrix} \right], \tag{3}$$

where $j = \sqrt{-1}$, $\alpha_n = \alpha_0 + nG_x$ and $N = 2M+1$, and $(\mathbf{p}_m^n, \boldsymbol{\sigma}_m^n)^t$ are the eigen-vectors corresponding to the eigen-values β_m; A_{mL} and A_{mU} are the amplitude of the lower- and upper-forward waves. In Eq. (3), the first term on the right-hand side denotes the lower-forward waves and the second term corresponds to the upper-forward waves. Once the wave solutions are obtained, we can construct the S-matrices for each layer and for the unit cell with the help of the boundary condition

$$\begin{pmatrix} \mathbf{p}^{Li} \\ j\boldsymbol{\sigma}^{Li} \end{pmatrix} = \begin{pmatrix} \mathbf{p}^{U(i+1)} \\ j\boldsymbol{\sigma}^{U(i+1)} \end{pmatrix}, \tag{4}$$

where superscript Li ($U(i+1)$) denotes the lower (upper) boundary of the ith ($(i+1)$th) layer. Eq. (4) establishes the relationship between the i and the $i+1$ layers. We use the S-matrix of interface between the sliced layers as well as the S-matrix of intralayer through a simple iteration algorithm to obtain the S-matrix of unit cell [17]. After the S-matrix of the unit cell is obtained, the relation can be expressed as

$$\begin{pmatrix} A_L^N \\ A_U^0 \end{pmatrix} = S \begin{pmatrix} A_L^0 \\ A_U^N \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} A_L^0 \\ A_U^N \end{pmatrix}, \tag{5}$$

where $A_N(A_0)$ is the amplitude of the acoustic wave on the bottom (top) side of the whole structure. In the bottom side of the structure, there exists only the outgoing wave, i.e. $A_L^N = 0$, and

$$\begin{pmatrix} A_L^N \\ A_U^0 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} A_L^0 \\ 0 \end{pmatrix}, \tag{6}$$

which gives

$$A_L^N = S_{11} A_L^0, \qquad A_U^0 = S_{21} A_L^0. \tag{7}$$

From Eqs. (3), (6), and (7), the reflection, transmission and incident waves can be, respectively, written as

$$\begin{pmatrix} \mathbf{p}^{ref} \\ j\boldsymbol{\sigma}^{ref} \end{pmatrix} = \begin{pmatrix} \mathbf{p}_U^0 \\ \boldsymbol{\sigma}_U^0 \end{pmatrix} S_{21} A_L^0, \tag{8}$$

$$\begin{pmatrix} \mathbf{p}^{tra} \\ j\boldsymbol{\sigma}^{tra} \end{pmatrix} = \begin{pmatrix} \mathbf{p}_L^N \\ \boldsymbol{\sigma}_L^N \end{pmatrix} S_{11} A_U^0, \tag{9}$$

$$\begin{pmatrix} \mathbf{p}^{in} \\ j\boldsymbol{\sigma}^{in} \end{pmatrix} = \begin{pmatrix} \mathbf{p}_L^0 \\ \boldsymbol{\sigma}_L^0 \end{pmatrix} A_L^0. \tag{10}$$

The reflection coefficient and transmission coefficient can be expressed as

$$R = \left| \frac{\sum_i^N \mathrm{Re}(p_i^{0-})^* \sigma_i^{0-}}{\sum_{i=1}^N \mathrm{Re}(p_i^{0+})^* \sigma_i^{0+}} \right|, \tag{11}$$

$$T = \left| \frac{\sum_i^N \mathrm{Re}(p_i^{N+})^* \sigma_i^{N+}}{\sum_{i=1}^N \mathrm{Re}(p_i^{0+})^* \sigma_i^{0+}} \right|, \tag{12}$$

where $(p_i)^*$ represents the conjugate of the ith component of the vector $\mathbf{p}_i$, and Re denotes an operation deriving the real part of a complex value. Energy conservation condition $T + R = 1$ should be kept in the calculation process. In our numerical work, in the straight case, there is no need to slice the layer into a number of smaller layers. But for the tapered case, the unit cell must be sliced into a number of smaller layers. In our numerical practice, the unit cell is sliced into 100 thinner layers to get convergent results.

In order to test the acoustic transmission spectra calculated by EMMT, and further investigate the property of transmission by plotting pressure fields, the FE simulation is also presented. In the FE simulation, one primitive cell is considered and the periodic boundary conditions are imposed on the x-direction and the perfectly matched layers are imposed on the input and output sides [18].

3. Numerical results and discussions

To test our methods, we consider a standard geometry consisted of 4 mm-wide square steel rods with the period of $d = 4.5$ mm, thus the aperture size is 0.5 mm and the thickness of the grating is 4 mm. The same system has been investigated by Lu et al. in Ref. [4]. We can see from Fig. 2 that the acoustic transmission in FE method are in good agreement with those