

Fresnel coefficients for a layer of NIM



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ABSTRACT

Reflection and transmission of traveling and evanescent waves by a layer of material with a negative index of refraction (NIM) is studied by means of the Fresnel coefficients. We derive their values in the “NIM limit”, and we show that this limit is consistent with the exact solution. It is also indicated that simply substituting the negative values of the relative permittivity and permeability of the NIM material into the exact solution leads to incorrect results for evanescent waves.

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1. Introduction

The electromagnetic properties of a material are accounted for by the (relative) permittivity ε and the (relative) permeability μ , and both are complex-valued in general, with a positive imaginary part. The index of refraction n is a solution of $n^2 = \varepsilon\mu$, and we take the solution with $\text{Im} n > 0$. This leaves an ambiguity when n^2 is positive, and this can only happen when ε and μ are both positive or both negative. In that case, we consider ε and μ to have small positive imaginary parts, and we take the limit where the imaginary parts go to zero. For common dielectrics, ε and μ are positive, and the index of refraction is $n = \sqrt{\varepsilon\mu}$. For certain metamaterials, ε and μ are in the second quadrant of the complex plane, and the index of refraction is $n = -\sqrt{\varepsilon\mu}$ (we shall take the branch line for square roots just below the negative real axis). In the limit where both ε and μ are negative, the index of refraction n is negative.

A negative index of refraction material (NIM) is predicted to have peculiar properties [1–6]. Although NIM's are constructed from sub-wavelength metal structures [7–29], they are transparent at the frequency under consideration. When a plane wave propagates through the material, the energy flows against the direction of the wave vector, and this leads to unusual refraction at an interface with a NIM. When a ray refracts into a NIM, it appears at the opposite side of the surface normal, as compared to refraction into

a dielectric. Similarly, when a ray exits a layer of NIM, the transmitted ray also appears at the opposite side of the surface normal, as compared to transmission through a dielectric layer. As a result, a point-like object at one side of a layer of NIM has a real image at the other side, provided that the distance between the source and the interface (H) is less than the thickness (L) of the NIM layer. When H is larger than L , a virtual image is formed at a location in between the source and the second interface. Therefore, a layer of NIM acts as a lens.

This notion was taken a step further by Pendry [30], who showed that evanescent waves are amplified when passing through the layer, and they therefore contribute to the image. Evanescent waves die out over a short distance from the source, and so they generally do not contribute to the image. This limits image resolution in an optical system to spatial variations of about a wavelength. Since the evanescent waves are amplified by a layer of NIM, they will contribute to the image, and this may provide a mechanism for obtaining images with sub-wavelength resolution. Such a superlens has been studied theoretically and experimentally, with mixed results [31–35].

It has been realized for a long time that the assumption of a perfect (theoretical) NIM leads to inconsistencies [36–38]. One of the problems is that the Fresnel transmission coefficients seem to grow exponentially with decreasing wavelength of the parallel component of the incident wave vector. As a result, for $H < L$, as in the superlens configuration, the electric and magnetic fields diverge in a region around the second interface (over a distance of $L - H$ at both sides). We shall show in this Letter that this is the result of an erroneous computation of the Fresnel coefficients. Also,

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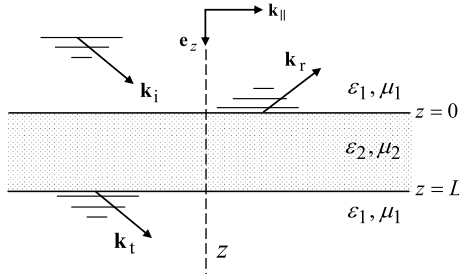


Fig. 1. The figure shows the setup considered. A wave can either be traveling or evanescent. A traveling wave is represented by a wave vector. If the wave is evanescent, it travels in the $\mathbf{k}_{\parallel}$ direction and it decays in the z direction, as indicated by the thin lines. The a wave and the b wave in the layer are not shown.

the common notion [30,32,39] that a layer of NIM does not reflect any radiation appears to be false.

2. Fresnel coefficients for a layer

The electromagnetic field, emitted by a source, can be represented by an angular spectrum, which is a superposition of traveling and evanescent plane waves. Therefore, transmission of radiation through a layer of NIM is determined by the Fresnel transmission coefficients (s and p polarization) of these waves. We shall consider the setup shown in Fig. 1. A layer of material has permittivity ε_2 and permeability μ_2 , and is embedded in an ε_1, μ_1 medium (both positive). The incident wave has wave vector $\mathbf{k}_i$, and $\mathbf{k}_r$ and $\mathbf{k}_t$ are the wave vectors of the reflected and transmitted waves, respectively. Inside the layer, there are two waves, due to multiple reflections, and their wave vectors will be indicated by $\mathbf{k}_a$ and $\mathbf{k}_b$. Due to boundary conditions, all wave vectors must have the same parallel component $\mathbf{k}_{\parallel}$, and we shall assume that this vector is real. In an angular spectrum, vector $\mathbf{k}_{\parallel}$ is the integration variable, so here we consider this vector as given. For its magnitude we shall write $k_{\parallel} = \alpha k_0$, with $k_0 = \omega/c$ and ω the angular frequency of the radiation. The z component of $\mathbf{k}_i$ then follows from the dispersion relation $\mathbf{k}_i \cdot \mathbf{k}_i = n_1^2 k_0^2$, which only leaves the sign of k_z undetermined. From causality it follows that we need to take $k_{i,z} = k_0 v_1$, in terms of the dimensionless parameter

$$v_1 = \sqrt{n_1^2 - \alpha^2}. \quad (1)$$

The variable α , representing the magnitude of $\mathbf{k}_{\parallel}$, is in the range $0 \leq \alpha < \infty$. For $\alpha < n_1$, the parameter v_1 is real and the incident wave is traveling. For $\alpha > n_1$, v_1 is positive imaginary, and the incident wave is evanescent. Similar considerations hold for the other wave vectors. The parameter representing the z components of the wave vectors in the layer is v_2 , defined as the solution of

$$v_2^2 = n_2^2 - \alpha^2, \quad \text{Im } v_2 > 0. \quad (2)$$

We shall consider s polarized light, with $\mathbf{e}_s = \mathbf{e}_z \times \mathbf{k}_{\parallel}/k_{\parallel}$. The complex amplitudes of the electric fields in the three regions are then

$$\mathbf{E}(\mathbf{r}) = E_0 \mathbf{e}_s e^{i\mathbf{k}_{\parallel} \cdot \mathbf{r}} (e^{ik_0 v_1 z} + R e^{-ik_0 v_1 z}), \quad z < 0, \quad (3)$$

$$\mathbf{E}(\mathbf{r}) = E_0 \mathbf{e}_s e^{i\mathbf{k}_{\parallel} \cdot \mathbf{r}} (D_a e^{ik_0 v_2 z} + D_b e^{ik_0 v_2 (L-z)}), \quad 0 < z < L, \quad (4)$$

$$\mathbf{E}(\mathbf{r}) = E_0 \mathbf{e}_s e^{i\mathbf{k}_{\parallel} \cdot \mathbf{r}} T e^{ik_0 v_1 z}, \quad z > L, \quad (5)$$

with E_0 an overall amplitude. The corresponding complex amplitudes of the magnetic fields follow from $\mathbf{B}(\mathbf{r}) = (-i/\omega) \nabla \times \mathbf{E}(\mathbf{r})$, and the Fresnel coefficients are determined by requiring that $\varepsilon \mathbf{E}_{\perp}$, $\mathbf{E}_{\parallel}$, $\mathbf{B}_{\parallel}$ and $\mathbf{B}_{\perp}/\mu$ are continuous at $z=0$ and at $z=L$. This yields

$$R = \frac{1}{\Lambda} [(\mu_2 v_1)^2 - (\mu_1 v_2)^2] (1 - e^{2iv_2 \ell}), \quad (6)$$

$$D_a = \frac{2}{\Lambda} \mu_2 v_1 (\mu_1 v_2 + \mu_2 v_1), \quad (7)$$

$$D_b = \frac{2}{\Lambda} \mu_2 v_1 (\mu_1 v_2 - \mu_2 v_1) e^{iv_2 \ell}, \quad (8)$$

$$T = \frac{4}{\Lambda} \mu_1 \mu_2 v_1 v_2 e^{iv_2 \ell}, \quad (9)$$

with

$$\Lambda = (\mu_1 v_2 + \mu_2 v_1)^2 - (\mu_1 v_2 - \mu_2 v_1)^2 e^{2iv_2 \ell}, \quad (10)$$

and here we have set $\ell = k_0 L$ for the dimensionless layer thickness.

The free parameters are $\varepsilon_1, \mu_1, \varepsilon_2, \mu_2$ and ℓ , and we see α as the variable. Of particular interest is the behavior of the Fresnel coefficients for large α . Then we have $v_1 \approx v_2 \approx i\alpha$, and for $\alpha \rightarrow \infty$ we obtain the limits

$$D_b = T = 0, \quad (11)$$

$$D_a = \frac{2\mu_2}{\mu_2 + \mu_1}, \quad (12)$$

$$R = \frac{\mu_2 - \mu_1}{\mu_2 + \mu_1}. \quad (13)$$

When the incident wave is on the border line between traveling and evanescent, we have $\alpha = n_1$, and therefore $v_1 = 0$. Then $D_a = D_b = T = 0$ and $R = -1$. We see from Eq. (3) that the reflected wave extinguishes the incident wave, and we have $\mathbf{E}(\mathbf{r}) = 0$ everywhere.

3. Layer of perfect NIM

The expressions from the previous section hold for arbitrary ε_2 and μ_2 . For a perfect NIM we would have $\varepsilon_2 \equiv -\varepsilon_1$, $\mu_2 \equiv -\mu_1$ and $n_2 \equiv -n_1$. From Eq. (2) it follows that $v_2^2 = v_1^2$. Just like for the index of refraction n_2 , we need to consider the solution for v_2 as a limit. It follows easily that for traveling waves we need to take $v_2 = -v_1$, and for evanescent waves we have $v_2 = v_1$. We then find $R = 0$ and $T = \exp(-iv_1 \ell)$. For $\alpha < n_1$ we have $D_a = 1$ and $D_b = 0$, and for $\alpha > n_1$ we find $D_a = 0$ and $D_b = \exp(-iv_1 \ell)$.

Since $R = 0$ for all α , it seems that a perfect NIM does not reflect any radiation. However, if we let $\mu_2 \rightarrow -\mu_1$ in Eq. (13), we see that on general grounds the value of R becomes very large in this limit. Similarly, T should vanish for α large (Eq. (11)), whereas $T = \exp(-iv_1 \ell)$ diverges as $\exp(\alpha \ell)$. The Fresnel coefficient D_b has the same problem. From the fact that D_b and T grow as $\exp(\alpha \ell)$ for α large, it can be shown that the electric and magnetic fields diverge in the neighborhood of the $z=L$ interface (both above and below) when the distance between a source and the interface is smaller than the layer thickness L . Therefore, if such a perfect NIM would be constructed, a single radiating atom in its neighborhood would make it self destruct.

Fig. 2 shows $|R|$ and $|T|$ for $\varepsilon_1 = \mu_1 = 1$, $\varepsilon_2 = -1$, $\mu_2 = -1 + i/5$ and $\ell = 2$, computed with the exact results of Section 2. For traveling waves, $\alpha < n_1$, we have $|R| \approx 0$ and $|T| \approx 1$, as expected for a layer of NIM. For evanescent waves, however, the Fresnel reflection coefficient is large. The transmission coefficient has a peak, just above the index of refraction, and it vanishes for α large. Apparently, setting $\varepsilon_2 \equiv -\varepsilon_1$ and $\mu_2 \equiv -\mu_1$ in the exact expressions for the Fresnel coefficients for a layer does not give the correct results for a layer of NIM.

4. The NIM limit

Expressions (6)–(9) for the Fresnel coefficients of a layer hold for any ε_2 and μ_2 . However, care should be exercised when

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