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Dynamical properties for an ensemble of classical particles moving in a driven potential well with different time perturbation

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ABSTRACT

We consider dynamical properties for an ensemble of classical particles confined to an infinite box of potential and containing a time-dependent potential well described by different nonlinear functions. For smooth functions, the phase space contains chaotic trajectories, periodic islands and invariant spanning curves preventing the unlimited particle diffusion along the energy axis. Average properties of the chaotic sea are characterised as a function of the control parameters and exponents describing their behaviour show no dependence on the perturbation functions. Given invariant spanning curves are present in the phase space, a sticky region was observed and show to modify locally the diffusion of the particles.

1. Introduction

After the seminal paper of Buttiker and Landauer [1] dealing with the tunnelling through a time-dependent barrier, many results on this thematic have been reported. Among them it is interesting to mention a drift of particles at a sequence of 1D time-dependent potential wells [2] and the investigation of the dwell time for a classical particle confined to move in a periodically time varying potential well [3,4]. Periodically modulated quantum channel was also used to study quantised ballistic conductance [5], transport properties for GaAs/AlGaAs were considered [6,7], while an intense electric field was used to characterise quantum transport in semiconductors super-lattices and multiple quantum wells [8]. More rigorous and analytical investigations were made for a two-level system [9,10] particularly using Laplace transform to obtain the probability of a particle to be transmitted from one and two barriers [11,12] while Fokker–Planck equation was applied to obtain the transition probability from one potential to the other and considering different scaling times as well as different noise lengths [13]. Applications of the formalism were also proved to be useful in a washboard potential [14] as well as in a double well potential [15]. The occurrence of phonon assisted resonance tunnelling was measured in [16] while escape time from a fluctuating barrier considering either dichotomic and Gaussian perturbations was obtained [17].

Recent applications involving Bose–Einstein Condensates (BEC) can also be discussed in this approach. A nonlinear effect which includes self-trapping in a double well potential was discussed in [18]. A modification of a double potential well which includes a barrier in the middle was used to describe a resonator [19]. Using the transfer-matrix technique, it was proved [20] that a time-dependence of the potential wells affect electron probability transmission. Heun confluent functions were used to represent the solutions of a family of double-well potential [21]. The effect of spin–orbit coupling in the tunnelling from wells in a BEC in a double well potential was discussed in [22] while a quantum phase transition in a BEC was also characterised [23]. The probability of finding a particle in a two-well periodic potential was obtained analytically in [24] while a power law was obtained in the characterisation of the survival probability of an ensemble of particles escaping from one well to the other one due to white noise [25].

In this Letter we use the same general procedure as made in [26,27] to obtain and characterise some dynamical properties for an ensemble of non-interacting (classical) particles confined to move inside of a periodically time-dependent potential well. We are seeking to understand the influence of the position of the lowest energy invariant spanning curve in the average properties of the chaotic sea. The Hamiltonian that describes the system is given by $H(x,p,t) = p^2/(2m) + V(x,t)$ where x, p and t correspond to the position, momentum coordinates and time respectively. As we

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shall see in the next section, the potential $V(x,t)$ is controlled by three relevant dimensionless control parameters. If one of the control parameters is equal to zero, the system is integrable. For the other situations, the phase space of the model is of mixed type. It contains periodic islands surrounded by a chaotic sea (which is characterised by a positive Lyapunov exponent). The *size* of the chaotic sea depends on the control parameters and is limited by a set of invariant spanning curves, which prevent the unlimited energy growth of the particle. Our main goal in this Letter is to understand and describe how the shape of the periodic potential influences the dynamics of an ensemble of particles moving along the chaotic sea by the position of the lowest energy invariant spanning curve. Considering different types of perturbations, we study the behaviour of the Lyapunov exponents and some average properties of the chaotic sea including the behaviour of the deviation around the average energy. We show that critical exponents used in the scaling theory, are independent of the perturbation function proposed. We also observe a region of strong trapping regime which affects the transport along the chaotic sea.

The Letter is organised as follows. In Section 2 we describe the model and obtain the equations of the mapping. The numerical results are considered in Section 3 and final remarks and conclusions are drawn in Section 4.

2. The model and the map

In this section we discuss all the details needed to construct the mapping that describes the dynamics of the model. The system under consideration has a dynamics governed by a Hamiltonian of the type $H(x,p,t) = p^2/(2m) + V(x,t)$ where x, p and t correspond to position, momentum coordinates and time respectively.

We emphasise that different kinds of potential shape lead to similar dynamics. For a chain of infinitely many and symmetric oscillating square wells with their bottoms moving periodically and synchronised in time (see Ref. [26] and Fig. 1(a)), the dynamics leads to diffusion in space [3,4]. One can also assume a single oscillating square well with periodic boundary conditions, as shown in Fig. 1(b). Finally, a step potential whose bottom moves periodically in time confined in an infinite box of potential (see Fig. 1(c)) leads to similar results in the phase space. This last shape is used in our simulations, where the potential $V(x,t)$ is given by

$$V(x,t) = \begin{cases} \infty, & \text{if } x \leqslant 0 \text{ or } x \geqslant (a+b)/2, \\ V_0, & \text{if } 0 < x < \frac{b}{2}, \\ V_1 f(t), & \text{if } \frac{b}{2} \leqslant x < \frac{a+b}{2}, \end{cases} \qquad (1)$$

where a, b, V_0, V_1 and w are the control parameters. The function $f = f(t)$ is time-dependent. Possible allusions of the time-dependent potential can be made as corresponding to the potential created by atoms localised in sequence along an infinite and symmetric chain while the oscillations may denote phonon effects or either the contact of the chain with a thermal bath. Hence the potential well is getting energy from a thermal bath and is transferring it to the particle. It is important to emphasise that the potential well considered in this Letter can indeed trap temporarily particles, see applications of trapping in a quantum dot in Ref. [28]. The shape of the phase space is strongly dependent on f. For a random perturbation, the particle exhibits unlimited diffusion in energy. On the other hand, for a smooth and periodic f the phase space is mixed. The existence of invariant spanning curves prevent the particle to exhibits unlimited energy growth. They indeed work as a physical barrier not letting the particle to pass through. The chaotic properties of the phase space are directly dependent on the location of the lowest energy invariant spanning curve and how it behaves as a function of the control parameters.

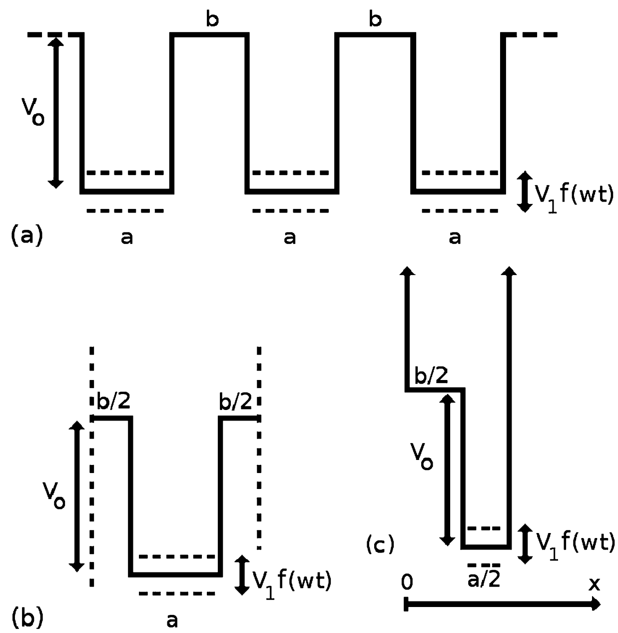

Fig. 1. Sketch of: (a) a chain of infinitely many and symmetric oscillating square wells with their bottoms moving periodically and synchronised in time. (b) A single oscillating square well. (c) A step potential whose bottom moves periodically in time.

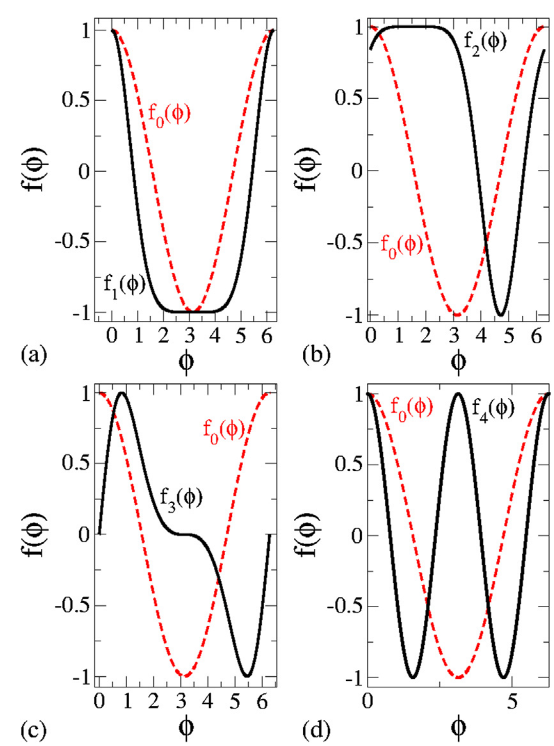

Fig. 2. (Colour online.) Plots of the functions: (a) $f_1(\phi)$; (b) $f_2(\phi)$; (c) $f_3(\phi)$; (d) $f_4(\phi)$ using $q = 2$. The dashed red lines correspond to plots for $f_0(\phi)$, used as comparison.

In this Letter we consider different expressions for $f(t)$ as shown in Fig. 2(a)–(d). The functions $f_1(\phi)$, $f_2(\phi)$, $f_3(\phi)$ and $f_4(\phi)$ are given by

$$f_1(\phi) = \cos[\phi + \sin(\phi)], \qquad f_2(\phi) = \sin[\phi + \cos(\phi)], \qquad (2)$$

and

$$f_3(\phi) = \sin[\phi + \sin(\phi)], \qquad f_4(\phi) = \cos(q\phi). \qquad (3)$$