Physics Letters A 377 (2013) 1885–1889

Contents lists available at SciVerse ScienceDirect

Physics Letters A

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Diffraction structures in high-energy electron–nucleus bremsstrahlung

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ARTICLE INFO

Article history:
Received 22 April 2013
Received in revised form 13 May 2013
Accepted 15 May 2013
Available online 17 May 2013
Communicated by V.M. Agranovich

ABSTRACT

The emission of hard bremsstrahlung during the collision of relativistic spin-polarized electrons with inert spin 0 and spin $\frac{1}{2}$ nuclei is calculated within the weak-potential approximation. Diffraction structures in the polarization correlations between the beam electron and the emitted photon are predicted for collision energies in the region 50–120 MeV if the photon is emitted at backward angles. The dynamical recoil plays a dominant role concerning the location and the shape of the structures. The target nuclei ^{19}F, ^{64}Zn and ^{89}Y are investigated.

The phenomenon of diffraction structures in the angular distribution of electrons scattered elastically from ions or atoms is well known. Such structures appear when the beam electrons are energetic enough to penetrate into the electronic cloud of the target, which requires energies of, say, 50 eV–5 keV. Then the scattering from the individual atomic electrons leads to an interference pattern, the so-called Ramsauer–Townsend effect [1–3] (for recent experiments on the scattering of quasifree electrons, see the review by Lucas et al. [4]). The occurrence of the structures not only in the angular distribution but also as a function of the beam energy is readily understood from inspecting the minimum momentum transfer q_{min} necessary for a given scattering event. In the case of elastic scattering with initial momentum $\boldsymbol{k}_i$ one has $q_{min} \approx 2|\boldsymbol{k}_i|\sin(\theta/2) \approx (\Delta R)^{-1}$ which in turn defines the average electron–nucleus distance ΔR which corresponds to the scattering angle θ at the given energy.

A more sensitive tool than the electron distribution for observing the structures in elastic scattering is the spin asymmetry in the case of polarized beam electrons, the so-called Sherman function S [5]. There is a multitude of early experimental polarization data which are compiled by Kessler [3]. When the electron is energetic enough to come close to the nucleus the diffraction structures disappear (around 50 keV). However, they reappear again at much higher energies, beyond, say, 40 MeV, in the scattering from heavy nuclei. At such energies the beam electron can distinguish the individual protons of the nucleus which affect the scattering of the electron in a similar way as do the atomic electrons in the low-energy regime. Such relativistic diffraction effects, modulating the cross sections and thus commonly used to analyze the nuclear structure (see e.g. [6,7]), have also been predicted for the Sherman function [8–11], but are still waiting for an experimental verification.

The close connection between the Sherman function in elastic electron scattering and the spin asymmetry A in bremsstrahlung by polarized electrons if only the emitted photon, but not the scattered electron, is observed, was pointed out quite early [12]. Even more, it was predicted for electrons scattering from point nuclei that all three polarization correlations for elastic scattering relating, respectively, to the three possible spin directions of the beam electron [13], will under certain conditions coincide in their angular distribution with the respective bremsstrahlung polarization correlations. These requirements are a high charge of the nucleus and a similar behavior of the two species of outgoing particles: they have both to be in helicity eigenstates, the photon has to be at the high-frequency end of the spectrum and the beam energy has to be high enough such that the electron rest mass m can be neglected [14]. As a consequence of this correspondence, it was expected that the high-energy diffraction structures should be visible not only in elastic scattering but also in hard bremsstrahlung.

In the present work target nuclei with charge $Z < 40$ and beam energies up to 120 MeV are considered, where a semirelativistic bremsstrahlung theory is still applicable. Our theory is based on the relativistic Born approximation which accounts for the influence of recoil [15,16] and nuclear structure [17,18]. Higher-order effects are included in an additional contribution to the radiative potential scattering amplitude by making use of the weak-potential Sommerfeld–Maue prescription introduced in [19]. The magnetic scattering, if present, is treated within the Born approximation.

For spin zero nuclei, to be considered first, only potential scattering takes place. There the recoil effects arise exclusively from energy and momentum conservation and are due to the finite mass of the target nucleus (the kinematical recoil). The doubly differential cross section for the emission of a bremsstrahlung photon with 4-momentum $k = (\omega/c, \boldsymbol{k})$ and polarization direction $\boldsymbol{e}_\lambda$ into the solid angle $d\Omega_k$ is given (in atomic units, $\hbar = m = e = 1$) by

0375-9601/$ – see front matter
http://dx.doi.org/10.1016/j.physleta.2013.05.031

$$\frac{d^2\sigma}{d\omega\, d\Omega_k}(\boldsymbol{\zeta}_i, \boldsymbol{e}_\lambda) = \frac{4\pi^2\omega}{c^3 v} \sum_{\sigma_f} \int d\Omega_f \, \frac{|\boldsymbol{k}_f| E_f}{f_{re}} \left| \frac{Z}{2\pi^2 c^2} W_{fi}^{el} \right|^2, \quad (1)$$

where $\boldsymbol{\zeta}_i$ is the spin vector of the beam electron and v the collision velocity. Since it is assumed that the scattered electron is not observed, a summation over its final spin polarization σ_f and an integral over the solid angle $d\Omega_f$ of emission has to be included. f_{re} is the recoil factor,

$$f_{re} = 1 - \frac{\boldsymbol{k}_f \boldsymbol{q} E_f}{\boldsymbol{k}_f^2 E_{nuc,f}}, \quad (2)$$

where $\boldsymbol{q} = \boldsymbol{k}_i - \boldsymbol{k}_f - \boldsymbol{k}$ is the momentum transfer to the nucleus and $k_i = (E_i/c, \boldsymbol{k}_i)$ and $k_f = (E_f/c, \boldsymbol{k}_f)$ are, respectively, the initial and final 4-momenta of the electron. $E_{nuc,f}$ is the final energy of the nucleus which is assumed to be initially at rest. The radiation matrix element for spin zero nuclei is given by

$$W_{fi}^{el} = \frac{2\pi^2 c^2}{Z} \int d\boldsymbol{r}\, \psi_f^{(\sigma_f)+}(\boldsymbol{r}) (\boldsymbol{\alpha} \boldsymbol{e}_\lambda^*) e^{-i\boldsymbol{kr}} \psi_i^{(\sigma_i)}(\boldsymbol{r}), \quad (3)$$

where $\psi_i^{(\sigma_i)}$ and $\psi_f^{(\sigma_f)}$ are the Dirac eigenstates of the scattering electron to a given nuclear potential, and $\boldsymbol{\alpha}$ is the vector of Dirac matrices. An exact evaluation of W_{fi}^{el} based on partial-wave expansions of the electronic wavefunctions (a powerful method at energies near and below 5 MeV [20,21]) is not possible in the energy regime considered here. We therefore use the weak-potential approximation (WPA) where W_{fi}^{el} is approximated by

$$W_{fi}^{el} \approx \left(u_{k_f}^{(\sigma_f)+} M_{fi,0} u_{k_i}^{(\sigma_i)}\right) F_1(q). \quad (4)$$

Here, $u_{k_i}^{(\sigma_i)}$, $u_{k_f}^{(\sigma_f)}$ are the electronic free 4-spinors and $F_1(q)$ is the Dirac form factor which accounts for the charge distribution of the nucleus [22,17]. The transition operator is taken as

$$M_{fi,0} = M_{fi}^{BH}(0) + \Delta M_{fi,0}, \quad (5)$$

where $M_{fi}^{BH}(0)$ is the recoil-modified (first-order) Bethe–Heitler operator [16],

$$M_{fi}^{BH}(\nu) = \frac{1}{\boldsymbol{q}^2 - (\Delta E_{nuc}/c)^2} \times \left\{ \boldsymbol{\alpha} \boldsymbol{e}_\lambda^* \frac{\boldsymbol{\alpha} c(\boldsymbol{k}_f + \boldsymbol{k}) + \beta c^2 + E_f + \omega}{(\boldsymbol{k}_f + \boldsymbol{k})^2 - (E_f + \omega)^2/c^2 + c^2} \gamma_0 \gamma^\nu + \gamma_0 \gamma^\nu \frac{\boldsymbol{\alpha} c(\boldsymbol{k}_i - \boldsymbol{k}) + \beta c^2 + E_i - \omega}{(\boldsymbol{k}_i - \boldsymbol{k})^2 - (E_i - \omega)^2/c^2 + c^2} \boldsymbol{\alpha} \boldsymbol{e}_\lambda^* \right\}, \quad (6)$$

for $\nu = 0$. ΔE_{nuc} is the energy transferred to the nucleus and γ^ν, $\nu = 1, 2, 3$, and $\gamma^0 = \beta$ (with $\gamma_0 \gamma^0 = 1$) are Dirac matrices. The higher-order effects are taken into consideration by $\Delta M_{fi,0}$ which is the difference between the respective radiation operators in the Sommerfeld–Maue (SM) theory (also termed Elwert–Haug theory [23], where the electrons are described in terms of semirelativistic Sommerfeld–Maue eigenfunctions to a fixed point nuclear potential) and in the Bethe–Heitler theory [24]. Recoil is disregarded in $\Delta M_{fi,0}$.

For the nucleus ^{64}Zn ($Z = 30$) a Fourier–Bessel expansion of the nuclear charge density $\varrho(r)$ is used [25], leading to the form factor

$$F_1(q) \approx \frac{4\pi}{Z} \int_0^\infty r^2\, dr\, \varrho(r)\, j_0(\tilde{q} r) = \frac{2\pi}{\tilde{q} Z} \sum_{k=1}^N \frac{a_k}{p_k} \left\{ \frac{\sin(\tilde{q} - p_k) R_0}{\tilde{q} - p_k} - \frac{\sin(\tilde{q} + p_k) R_0}{\tilde{q} + p_k} \right\}, \quad (7)$$

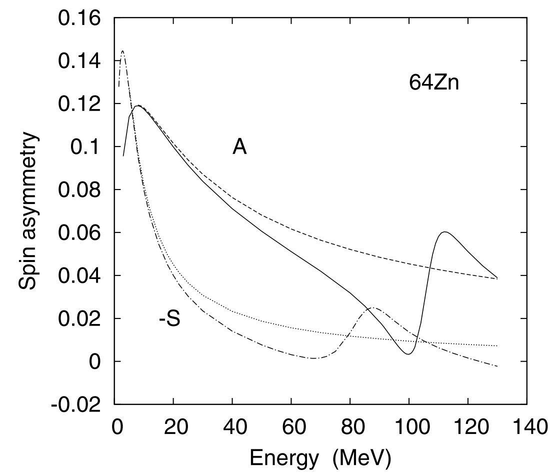

Fig. 1. Spin asymmetry as a function of beam energy $E_{i,kin}$ for electrons, spin-polarized perpendicular to the reaction plane, colliding with ^{64}Zn. The upper curves display A from bremsstrahlung at $\theta_k = 165°$ and $R = 0.8$ (——, WPA; — — —, SM theory). The lower curves display $-S$ from elastic scattering at the scattering angle $\theta = 165°$ (— · — · —, DW-0 theory using a Fourier–Bessel charge distribution; ······, DW-0 theory for a point-like nucleus [10,11]).

where $j_0(\tilde{q} r)$ is a spherical Bessel function, $\tilde{q} = \sqrt{\boldsymbol{q}^2 - (\Delta E_{nuc}/c)^2}$, $p_k = \frac{k\pi}{R_0}$ and a_k, N and R_0 are the parameters from the Fourier–Bessel expansion. $\varrho(r)$ is normalized such that $F_1(0) = 1$.

The spin asymmetry A is defined by [26,27]

$$A = \frac{d\sigma(\boldsymbol{\zeta}_i) - d\sigma(-\boldsymbol{\zeta}_i)}{d\sigma(\boldsymbol{\zeta}_i) + d\sigma(-\boldsymbol{\zeta}_i)}, \quad (8)$$

where $d\sigma(\boldsymbol{\zeta}_i) \equiv \sum_\lambda \frac{d^2\sigma}{d\omega\, d\Omega_k}(\boldsymbol{\zeta}_i, \boldsymbol{e}_\lambda)$ with $\boldsymbol{\zeta}_i$ in the direction of $\boldsymbol{k} \times \boldsymbol{k}_i$ (which is perpendicular to the reaction plane), including a sum over the two polarization directions $\boldsymbol{e}_\lambda$.

Fig. 1 shows the dependence of A on the beam energy $E_{i,kin} = E_i - mc^2$ for a fixed ratio $R = \omega/E_{i,kin} = 0.8$ and the angle $\theta_k = 165°$ between $\boldsymbol{k}$ and the z-axis along $\boldsymbol{k}_i$. Comparison is made with the SM theory [23] which is obtained from (1) by neglecting recoil and by replacing $\psi_i^{(\sigma_i)}$ and $\psi_f^{(\sigma_f)}$ in (3) by SM functions. Kinematical recoil effects being small in the displayed energy region, the deviations from the SM theory, which increase with E_i and become important above 40 MeV, are due to the finite nuclear size. Diffraction structures are visible above 80 MeV, having a similar shape as those appearing in the Sherman function S for the same collision geometry. Thereby S is obtained from the rhs of (8), now with $d\sigma(\boldsymbol{\zeta}_i) \equiv d\sigma/d\Omega(\boldsymbol{\zeta}_i)$ representing the elastic scattering cross section into a helicity (+) state at angle $\theta = \theta_k$. The elastic scattering cross section is calculated by means of a relativistic phase-shift analysis [8,10] with the help of the code RADIAL from Salvat et al. [28] for a given nuclear potential (called DW-0 theory). Note that the negative sign of S in the figure is due to opposite conventions for the electron spin orientation in the elastic scattering, respectively, bremsstrahlung literature. The bremsstrahlung interference structures decrease in width if $R \to 1$, but are nearly independent of photon angle (in the region beyond 160°). The onset of these structures at a higher energy as compared to elastic scattering results from the smaller average momentum transfer. In fact, since the integration over the electron scattering angles implies the sampling of a band width of momentum transfers, we may take $q_{ave} \approx |\boldsymbol{k}_i - \boldsymbol{k}|$ as an average momentum transfer. For a fixed ratio R and $|\boldsymbol{k}_i|/c \gg 1$ (i.e. $E_{i,kin} \gtrsim 5$ MeV) one obtains, using $|\boldsymbol{k}| = R E_{i,kin}/c$,