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Kinklike structures in scalar field theories: From one-field to two-field models

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ABSTRACT

In this Letter we study the possibility of constructing two-field models from one-field models. The idea is to start with a given one-field model and use the deformation procedure to generate another one-field model, and then couple the two one-field models nontrivially, to get to a two-field model, together with some explicit topological solutions. We show with several distinct examples that the procedure works nicely and can be used generically.

1. Introduction

Topological solutions known as kinks, vortices and monopoles are of direct interest to several areas of nonlinear science; see, e.g., [1–5]. They appear in models describing spontaneous symmetry breaking, inducing phase transitions that could be used, for instance, to describe cosmic evolution in the early universe. In the simplest case of kinks, one usually requires a single real scalar field, which in the presence of spontaneous symmetry breaking can be used to mimic the Higgs field [1,2] or to map degrees of freedom in polymers [3] and in Bose–Einstein condensates [5].

The basic model described by a real scalar field can be further extended to the case of two real scalar fields, giving rise to more sophisticated models and topological structures, again of great interest to nonlinear science. However, the two-field models are much harder to be solved, and for this reason in the current work we investigate the presence of defect structures in models described by two real scalar fields, owing to construct new models, together with the respective topological solutions. We concentrate on kinks, which are classically stable static solutions that appear when the potential is a non-negative function of the scalar fields that define the model under consideration. The models that we consider admit Bogomolnyi–Prasad–Sommerfeld solutions [6], known as BPS states, which solve first-order differential equations, leading us with bosonic portions of more sophisticated supersymmetric theories. Also, the presence of two real scalar fields makes the investigation more realistic, enhancing the power for applications in a diversity of scenarios, as one can see, e.g., in Refs. [1,2,4–23] and in other works quoted therein.

A key issue concerning the presence of defect structures in models engendering two real scalar fields is that one has to solve the equations of motion, which are two coupled second-order ordinary nonlinear differential equations. To help dealing with this, the trial orbit method was proposed in [8], but there one faces an intrinsic difficulty, which concerns the presence of coupled second-order differential equations. This method was later shown to be very efficient, when adapted to first-order differential equations, which appear in the search of BPS states [6], valid when the potential V is non-negative and can be written as the derivative of another function, which we identify as W. This is explained in Ref. [20], and we also quote [24] for related investigations on this issue.

Our main motivation in the present work is to use the deformation procedure introduced in [25], taking it to construct models described by two real scalar fields, starting from a simpler model, described by a single real scalar field. As we are going to show below, it is possible to implement a general procedure, from which one starts with a single real scalar field, and use it to construct systems described by two real scalar fields. The approach relies on deforming the one-field model, to get another one-field model,

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and then coupling these two one-field model to end up with a two-field model, which we then solve easily.

An important issue related to the current work is that models described by two fields are more sophisticated and can describe junctions of defects [13–18]. Also, the procedure is of direct interest to generate braneworld solutions, in a five-dimensional AdS geometry with an extra dimension of infinite extent, and to produce bifurcation and pattern changing [26].

For pedagogical reasons, we organize the work as follows: we start the investigation with one and two real scalar field models, briefly reviewing the BPS approach and some general aspects about the deformation procedure in Sections 2 and 3, respectively. In Section 4 we introduce the method and we study several examples in Section 5. We end the work in Section 6, where we include some comments and conclusions.

2. Generalities

Let us first review some aspects relative to one and two real scalar fields in Minkowski spacetime. First, we introduce the Lagrangian density

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi), \tag{1}$$

with $\mu = 0, 1$, $\partial_\mu = \partial/\partial x^\mu$, $x^\mu = (x^0 = t, x^1 = x)$ and $\phi = \phi(x,t)$ stands for the real scalar field. We work with dimensionless fields and coordinates. By minimizing the action, we find the equation of motion

$$\ddot{\phi} - \phi'' = -\frac{\partial V}{\partial\phi}, \tag{2}$$

where we are using the standard notation, with dots representing derivatives with respect to time and primes standing for derivatives relative to the spatial coordinate. If we work with static solutions, we are led to

$$\phi'' = \frac{\partial V}{\partial\phi}. \tag{3}$$

Now, we use the function $W = W(\phi)$ to write $V(\phi)$ as

$$V(\phi) = \frac{1}{2}W_\phi^2, \tag{4}$$

with

$$W_\phi = \frac{dW}{d\phi}. \tag{5}$$

Here it is straightforward to derive that

$$\phi' = \pm W_\phi, \tag{6}$$

are first-order differential equations which solve the equation of motion.

The energy density for static solution can be written in the form

$$\begin{aligned}\varepsilon(x) &= \frac{1}{2}\phi'^2 + \frac{1}{2}W_\phi^2 \\ &= \frac{1}{2}(\phi' \mp W_\phi)^2 \pm \frac{dW}{dx}.\end{aligned} \tag{7}$$

Thus, the minimum energy configuration represents defect structure that solves the first-order equation (6) and has energy given by

$$E_{BPS} = \left|W(\phi(\infty)) - W(\phi(-\infty))\right|. \tag{8}$$

The same idea works for two scalar fields. In this case we introduce the model described by the two fields, $\phi(x,t)$ and $\chi(x,t)$, in the form

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi + \frac{1}{2}\partial^\mu\chi\partial_\mu\chi - V(\phi,\chi). \tag{9}$$

We deal with static fields, and the equations of motion become

$$\phi'' = \frac{\partial V}{\partial\phi}, \qquad \chi'' = \frac{\partial V}{\partial\chi}. \tag{10}$$

We consider the potential in the form

$$V(\phi,\chi) = \frac{1}{2}W_\phi^2 + \frac{1}{2}W_\chi^2, \tag{11}$$

and now the first-order equations can be written in the form

$$\phi' = \pm W_\phi, \qquad \chi' = \pm W_\chi. \tag{12}$$

Here the energy density is given by

$$\begin{aligned}\varepsilon(x) &= \frac{1}{2}\phi'^2 + \frac{1}{2}\chi'^2 + \frac{1}{2}W_\phi^2 + \frac{1}{2}W_\chi^2 \\ &= \frac{1}{2}(\phi' \mp W_\phi)^2 + \frac{1}{2}(\chi' \mp W_\chi)^2 \pm \frac{dW}{dx},\end{aligned} \tag{13}$$

and we see the energy is minimized for solutions to the first-order equations (12), attaining the value

$$E_{BPS} = \left|W(\phi(\infty),\chi(\infty)) - W(\phi(-\infty),\chi(-\infty))\right|. \tag{14}$$

An interesting aspect about the two-field model is that we can use the integrating factor to determine an analytical orbit equation, relating the two fields $\phi(x,t)$ and $\chi(x,t)$. In order to implement it, let us work with the first-order equations (12); we use them to write

$$\phi_\chi = \frac{d\phi}{d\chi} = \frac{W_\phi(\phi,\chi)}{W_\chi(\phi,\chi)}. \tag{15}$$

This is a central point in this work, which have inspired us to propose and solve the two-field models that we investigate in Sections 4 and 5.

3. Deformation procedure

Let us now review the main features of the deformation procedure, as given in Ref. [25]. We consider the model

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi), \tag{16}$$

where

$$V(\phi) = \frac{1}{2}W_\phi^2 \tag{17}$$

and

$$\phi' = W_\phi(\phi). \tag{18}$$

We introduce another one-field model, described by

$$\mathcal{L}_d = \frac{1}{2}\partial^\mu\chi\partial_\mu\chi - U(\chi), \tag{19}$$

where

$$U(\chi) = \frac{1}{2}W_\chi \tag{20}$$

and

$$\chi' = W_\chi(\chi). \tag{21}$$