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Non-Markovian dynamics of spin squeezing

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ABSTRACT

We evaluate the spin squeezing dynamics of N independent spin-1/2 particles with exchange symmetry. Each particle couples to an individual and identical reservoir. We study the time evolution of spin squeezing under the influence of different decoherence. The spin squeezing property vanishes with evolution time under Markovian decoherence, while it collapses quickly and revives under non-Markovian decoherence. As spin squeezing can be regarded as a witness of multipartite entanglement, our scheme shows the collapses and revivals of multipartite entanglement under the influence of non-Markovian decoherence.

1. Introduction

Quantum correlation has been playing a central role in quantum information science and has also found many promising applications such as achieving interferometric [1–4] and enhancing the signal-to-noise ratio in spectroscopy [5,6] beyond the standard quantum noise limit. The spin squeezed state is one kind of quantum correlated states [7,8] with reduced fluctuations in one of the collective spin components, which can be used to improve the precision of atomic interferometers and atomic clocks. As an important quantum correlation, entanglement is based on the superposition principle combined with the Hilbert space structure, while spin squeezing is originated from another fundamental principle of quantum mechanics—the uncertainty principle. It has been proved that the spin squeezing is closely related to and implies quantum entanglement [3,5,9–11]. As a measure of multipartite entanglement spin squeezing is relative easy to be operated and measured.

To evaluate the potential application of quantum correlations such as spin squeezing and entanglement, it is therefore essential to include a realistic description of noise in experiments of interests [12]. The dynamics of entanglement in open systems has been broadly studied [13]. A peculiar aspect of the entanglement dynamics is the well-known "entanglement sudden death" phenomenon [14–16] and recently the "sudden death" of spin squeezing during a Markovian process has been investigated [17,18]. The unidirectional flow of information in which the decoherence and noise act consistently, characterizes a Markovian process. However, there are some systems such as condensed-matter systems which are strongly coupled to the environment and the coupling leads to a different regime where information also flows back into the system from the surroundings, which characterizes a non-Markovian process. Memory effects caused by the information flowing back to the system during a non-Markovian process can temporarily interrupt the monotonic increases or decreases of distinguishability such as spin squeezing parameter. In this Letter we study the spin squeezing dynamics of N independent spin-1/2 particles with exchange symmetry which are coupled to individual and identical non-Markovian decoherence channels and show the collapses and revivals of spin squeezing.

2. Spin squeezing definitions

We consider an ensemble of N two-level particles with lower (upper) state $|\downarrow\rangle$ ($|\uparrow\rangle$). Adopting the nomenclature of spin-1/2 particles, we introduce the total angular momentum

$$\vec{J} = \sum_{j=1}^{N} \vec{S}_j, \tag{2.1}$$

where $\vec{S}_z^j = \frac{1}{2}\hat{\sigma}_z^j = \frac{1}{2}(|\uparrow\rangle_j\langle\uparrow| - |\downarrow\rangle_j\langle\downarrow|)$. At this point, it is convenient to introduce the following definition of spin squeezing parameter [5,19]

$$\xi^2 = \frac{N(\Delta J_{\vec{n}_\perp})^2_{min}}{\langle\vec{J}\rangle^2}. \tag{2.2}$$

Here the minimization is over all directions denoted by $\vec{n}_\perp$, perpendicular to the mean spin direction $\vec{n} = \langle\vec{J}\rangle/|\langle\vec{J}\rangle|$. If $\xi^2 < 1$ is satisfied, the spin squeezing occurs and the N-qubit state is entangled.

There are also other definitions for spin squeezing parameters which might show different sensitivities to the decoherence channels. We introduce another parameter defined by Tóth et al. [11]

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$$\xi'^2 = \frac{\lambda_{min}}{\langle \vec{J}^2 \rangle - N/2}, \tag{2.3}$$

where λ_{min} is minimum eigenvalue of the matrix $\Gamma = (N-1)\Upsilon + C$ with $\Upsilon_{kl} = C_{kl} - \langle J_k \rangle \langle J_l \rangle$ for $k, l \in \{x, y, z\}$ the covariance matrix and $C = [C_{kl}]$ with $C_{kl} = \langle J_l J_k + J_k J_l \rangle / 2$ is the global correlation matrix.

3. One-axis twisted spin squeezed states

Now we introduce one kind of spin squeezed states—one-axis twisted spin squeezed states. Consider an ensemble of N spin-1/2 particles with exchange symmetry and assume that the dynamical properties of the system can be described by collective operators J_α, $\alpha = x, y, z$. The one-axis twisting Hamiltonian [20–22] is an Ising-type Hamiltonian

$$\hat{H} = \sum_{j \neq k} \frac{1}{4} f(j,k) \big(\mathbb{I} - \hat{\sigma}_z^j\big) \otimes \big(\mathbb{I} - \hat{\sigma}_z^k\big), \tag{3.1}$$

which involves all pairwise interactions with coupling constant $f(j,k)$.

The one-axis twisted spin squeezed state [23–25] can be prepared by the evolution of the above Hamiltonian

$$|\psi_t\rangle = \exp(-i\hat{H}t)|+\rangle^{\otimes N} = \prod_{j \neq k} \exp\left[-\frac{i}{4} f(j,k) t \hat{\sigma}_z^j \hat{\sigma}_z^k\right] |+\rangle^{\otimes N}, \tag{3.2}$$

where $|+\rangle = (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$. If we choose the evolution time to satisfy $f(j,k)t = m\pi$ with m an integer, the state $|\psi_t\rangle$ is a product state. If $f(j,k)t = (2m+1)\pi/2$, $|\psi_t\rangle$ becomes a graph state. For $0 < f(j,k)t < \pi/2$, $|\psi_t\rangle$ is a one-axis twisted spin squeezed state characterized by spin squeezing parameter ξ (ξ').

The spin squeezing parameter ξ of the one-axis twisted spin squeezed state with all coupling coefficients satisfying $f(j,k)t = \alpha$ takes this form

$$\xi^2 = \frac{1 - (N-1)[\sqrt{A^2 + B^2} - A]/4}{\cos^{2N-2}\alpha}, \tag{3.3}$$

where

$$A = 1 - \cos^{N-2}(2\alpha), \qquad B = 4\sin\alpha \cos^{N-2}\alpha. \tag{3.4}$$

The mean spin direction for the one-axis twisted spin squeezed state is

$$\vec{n} = \big(\cos(N\alpha), \sin(-N\alpha), 0\big), \tag{3.5}$$

and the orthogonal direction is

$$\vec{n}_\perp = \big(-\cos\phi \sin(-N\alpha), \cos\phi \cos(N\alpha), \sin\phi\big). \tag{3.6}$$

The minimum spin squeezing parameter with respect to α is obtained $\xi \propto 1/N^{1/3}$ shown in Fig. 1.

The spin squeezing parameter with another definition ξ' of the one-axis twisted spin squeezed state above takes the form

$$\xi'^2 = \frac{\min(a,b)}{(1 - 1/N)(1 + \cos^{N-2} 2\alpha)/2 + 1/N}, \tag{3.7}$$

where

$$a = 1 - (N-1)\big(\sqrt{A^2 + B^2} - A\big)/4,$$
$$b = 1 + (N-1)\big[\big(1 + \cos^{N-2} 2\alpha\big)/2 - \cos^{2N-2}\alpha\big]. \tag{3.8}$$

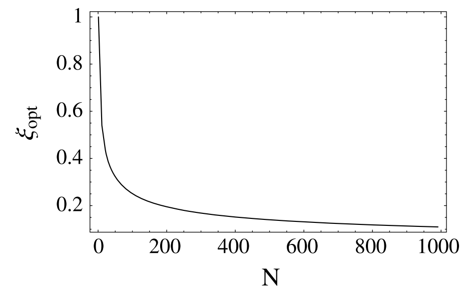

Fig. 1. The plot of the spin squeezing parameter ξ for a one-axis twisted spin squeezed state vs. the number of the qubits N optimized with respect to α.

4. Evolution of spin squeezing in the presence of decoherence

For a single qubit coupled to a noisy channel which is described by a thermal reservoir, the evolution of this qubit is governed by a general master equation of a Lindblad form

$$\frac{d}{dt}\chi = i[\hat{H}_r, \chi] + \mathcal{L}\chi, \tag{4.1}$$

where the reference system is

$$\hat{H}_r = \frac{\Delta}{2} \sum_{j=1}^{N} \hat{\sigma}_z^j \tag{4.2}$$

with Δ is the strength of the external field. The optimal spin squeezed states are eigenstates of the Hamiltonian [26]. Whereas, the incoherent processes are described by the superoperator $\mathcal{L}$:

$$\begin{aligned}\mathcal{L}\chi = &-\frac{b}{2}(1-s)[\hat{\sigma}_+\hat{\sigma}_-\chi + \chi\hat{\sigma}_+\hat{\sigma}_- - 2\hat{\sigma}_-\chi\hat{\sigma}_+] \\ &- \frac{b}{2}s[\hat{\sigma}_-\hat{\sigma}_+\chi + \chi\hat{\sigma}_-\hat{\sigma}_+ - 2\hat{\sigma}_+\chi\hat{\sigma}_-] \\ &- \frac{2c-b}{8}[2\chi - 2\hat{\sigma}_z\chi\hat{\sigma}_z],\end{aligned} \tag{4.3}$$

with $\hat{\sigma}_\pm = (\hat{\sigma}_x \pm i\hat{\sigma}_y)/2$. For $b = 0$, $c = \gamma$ and an arbitrary s, the generator Eq. (4.3) describes the coupling between the qubit and a dephasing channel. For $s = 1/2$ and $b = c = \gamma$, the qubit is coupled to a depolarizing channel. Whereas, for $s = 1$ and $b = 2c = \gamma$, that is coupled to a decay channel (pure damping channel).

Equivalently, one can use the resulting completely positive map $\mathcal{E}$ with $\chi' = \mathcal{E}\chi$ to describe the decoherence channels

$$\mathcal{E}\chi = \sum_{j=0}^{3} p_j \hat{\sigma}_j \chi \hat{\sigma}_j, \tag{4.4}$$

with χ a density matrix for a single-qubit state and $\sum_{j=0}^{3} p_j = 1$. These decoherence channels are of practical interests in quantum information science. This class contains for example: (i) for $p_0 = (1 + 3\kappa^2)/4$ and $p_1 = p_2 = p_3 = (1 - \kappa^2)/4$ with $\kappa = e^{-\gamma t}$ $\mathcal{E}$ describing a depolarizing channel; (ii) for $p_0 = (1 + \kappa^2)/2$, $p_1 = p_2 = 0$ and $p_3 = (1 - \kappa^2)/2$ a dephasing channel. Finally, the decay channel is described

$$\mathcal{E}\chi = E_0 \chi E_0^\dagger + E_1 \chi E_2^\dagger, \tag{4.5}$$

with the Kraus operators $E_0 = \begin{pmatrix} 1 & 0 \\ 0 & \kappa \end{pmatrix}$ and $E_1 = \begin{pmatrix} 0 & \sqrt{1-\kappa^2} \\ 0 & 0 \end{pmatrix}$ [27].

The quantum master equations with the time-local structures are also very useful for the description of non-Markovian processes. Suppose we have a time-local master equation of the form

$$\frac{d}{dt}\chi = i[\hat{H}_r, \chi] + \mathcal{K}(t)\chi, \tag{4.6}$$