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Characterisation of fibre entanglement in nonwoven fabrics based on knot theory

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A B S T R A C T

Owing to the structural complexity, the entanglement of fibres within nonwoven fabrics is generally characterised in indirect way by determining fabric mechanical properties using destructive testing and empirical modelling. Using elements of knot theory, a new approach is presented in this paper to characterise degree of entanglement in webs and nonwoven fabrics by topological representation of the fibrous assembly. The identification of fibre crossings followed by calculation of splitting number gives a numerical estimate of the degree of entanglement that can be potentially linked to the mechanical properties of the fibrous assembly.

1. Introduction

Nonwoven fabrics are industrially applied in composite production as flow media in resin transfer moulding (RTM), vacuum-assist and press moulding as well as reinforcements in bio-composites and as nanofibrous scaffolds for tissue engineering constructs. The entanglement of fibres within nonwovens influences the mechanics and attrition of the bulk structure. Intuitively, the degree of fibre entanglement relates to the general physical integrity of a nonwoven material in a way that, up to a limit, more entangled structures exhibit greater mechanical strength. Nonwovens are porous assemblies consisting of entangled and overlaying microscale or nanoscale fibres that have a substantially *X–Y* planar arrangement, Fig. 1. Depending on the method of web formation and bonding used to produce the fabric, a small fraction of fibre segments may be aligned approximately in the *Z*-direction. The integrity of the structure and its mechanical properties both in-plane and out of plane are heavily influenced by the physical interaction of the constituent fibres and particularly, the degree of fibre entanglement.

Despite its influence on the mechanical properties, structural characterisation of fibre entanglement mostly relies on indirect methods since the complexity of real nonwoven fabrics makes direct characterisation problematic. Previous attempts to characterise fibre entanglement have therefore included indirect methods based on the measurement of fabric tensile properties and fibre dimensions [1,2]. In early attempts to characterise mechanically bonded nonwovens such as those produced by hydroentangling, the terms "entanglement completeness" and "entanglement frequency" were introduced. Entanglement completeness is the proportion of fibres that break in relation to fibres that slip during fabric strength testing, while entanglement frequency relates to the number of entanglement sites along individual lengths of fibre [3]. Watanabe et al. [4] associated entanglement to a parameter described as the "contribution coefficient of strength". Ghassemieh et al. [5] used image analysis techniques to characterise hydroentangled fabric structure by measuring the fibre segment length. As entanglement increased, a reduction in the straight fibre segment length was noted due to fibres curling and moving from one plane to another. The resulting fibre segment length distribution was therefore suggested as a measure of fibre entanglement.

In the process of hydroentanglement, fibres are intensively entangled by forces in the fluid medium and the average vorticity in a fibrous web has been found to be correlated to fabric tensile strength [6]. Mao and Russell [7] devised a fundamental expression for determining the hydroentanglement intensity based on applied water jet energy and fibre physical properties in the web, however this method was limited only to the characterisation of two-dimensional fibre deflections.

There is a need to develop improved methods of general structural analysis since understanding of nonwoven fabric microstructure is fundamental to identifying methods of engineering higher performance fabrics and composite products. In terms of fibre arrangement textile materials can be classified as having regular (periodic) or irregular (non-periodic) structure or a combination of both. Regular textiles materials, such as those that are woven or knitted can be described by a unit cell approach but for irregular structures a large volume of the structure (or a large number of small samples) needs to be considered to obtain a reasonable

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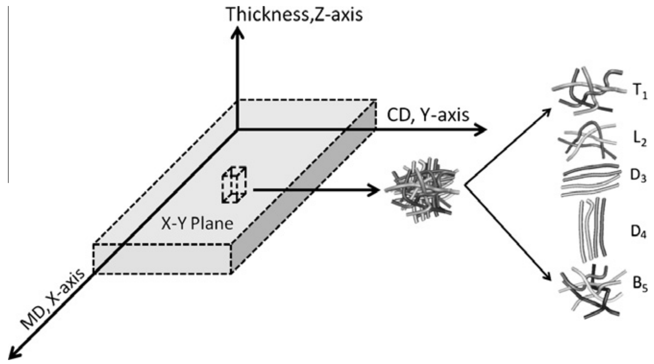

Fig. 1. Fabric structure in Cartesian coordinates. MD – machine direction, CD – cross direction. $\boldsymbol{T}_1$, $\boldsymbol{L}_2$, $\boldsymbol{D}_3$, $\boldsymbol{D}_4$ and $\boldsymbol{B}_5$ – entangled and disentangled fibre subsets.

understanding about the properties with a desirable degree of accuracy. Simple textile structures can be represented using binary [8–10] and non-binary coding [11–13] but as the complexity of the structure increases, it becomes difficult to represent it all by a single method. At the microscale, nonwovens are non-repeating structures and it would be an oversimplification to describe their internal structure on the basis of a unit cell because the placement of individual fibres cannot be controlled during the manufacturing process. Experimental results can normally be related to the properties of a specific structure produced by a specific method therefore the development of a universal method of structural analysis is much more convenient in the sense that the same approach can be applied to the study of various fabric structures. At present, such a universal method has not yet been developed.

The degree of entanglement is a purely structural characteristic of the material and therefore it is convenient to use a topological method in order to approach the structural characterisation. In recent years many attempts have been made to use topological methods to explain the relationship between the structure and properties of various objects. In particular, topological methods have led to useful outcomes when applied to the dynamics of DNA supercoiling and recombination properties [14,15]. The entanglement of polymer chain arrangements has been characterised by Arteca [16,17] using geometrical measures since the open chains are trivial in topology. The overcrossing number and the number of bond-to-bond crossings in a regular two-dimensional projection of the chain were calculated and averaged over all possible projections. Using this approach, a quantitative characteristic of the self-entanglement in a polymer network was obtained. However, an assembly such as a nonwoven may have a large number of fibre crossings but this does not necessarily mean that the fibres are entangled. As an example, a structure where all fibres are positioned in separate planes without any z-directional displacement has no entanglement as shown in Fig. 2 and it remains the same through all projections of the structure. Rogen and Bohr [18] constructed a family of global geometrical measures based on Vassiliev knot invariants [19] that provided better discrimination between many known protein chain structures than simple measures of their secondary structure.

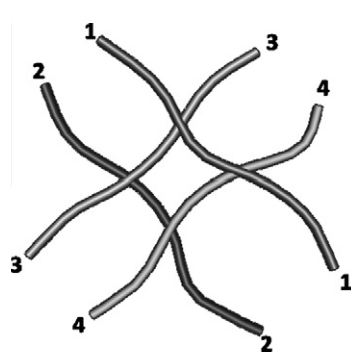

Fig. 2. A fibre assembly with $S = 0$.

Morton and Grishanov [20] applied multi-variable Alexander polynomial to identify whether or not the fabric has a layered structure. Grishanov and Vassiliev [21] used finite-type invariants to recognise if the fabric structure is split into independent layers. In both cases the textile structures in question were regular woven and knitted fabrics with a pattern repeating in two orthogonal directions and the methods used did not provide means for numerical characterisation of the degree of entanglement of the structures considered here.

An approach introduced by Grishanov et al. [22,23] is further developed in this paper as a step towards a generally applicable model for nonwoven and related web structures. In the context of this study the term ‘fibre’ is equivalent to ‘string’, ‘thread’ and ‘rod’ where all refer to a thin, flexible and stretchable object. Nonwoven fabrics, webs and related structures that are mechanically bonded as a result of the friction between fibres and cohesive forces can be modelled using topological objects. Their structural integrity is maintained by mechanical constraints only and therefore fibres can be assumed to be thin, flexible and elastic strings. The fibres are arranged in a three dimensional fashion which is simplified to two dimensions by planar projections of fibres. Based on knot theory, a combination of topology and geometry is presented herein to characterise the fibre interactions and degree of entanglement within a nonwoven fibre assembly in order to provide a basis for a new analytical method.

2. Topological objects – knots, braids, links and tangles

Knot theory is the study of the topological properties of idealized objects embedded in space, which are knots, links, braids and tangles. These objects are assumed to be made of infinitely thin, extensible, flexible and frictionless strings. A brief description of each of these objects is now given.

2.1. Knot

A mathematical knot, $K \subset R^3$, is a subset of points homeomorphic to a circle, it is a smooth non-self-intersecting closed curve which is impossible to untie without cutting (Fig. 3a). A knot equivalent to a circle is called an “unknot” or a trivial knot [19,24,25].

2.2. Link

Links are collections of knots. Whereas a knot is an embedding of a single circle S^1 into space R^3, a link is a finite disjoint union of knots in R^3: $L = K_1 \cup K_2 \cup \ldots \cup K_n$ where each knot K_i is a component of the link (Fig. 3b) [19,24,25]. According to [26], a link can be imagined as several entangled strings.

2.3. Braid

A braid is a set of ascending simple non-intersecting strings with connecting points $A_1;:::;A_n$ on a line with points $B_1;:::;B_n$ on a parallel line, Fig. 3c [19,25].

2.4. Tangle

A tangle is a generalisation of knots, links and braids. It is an arbitrary collection of closed and opened strings with their ends