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Transport properties of graphene under periodic and quasiperiodic magnetic superlattices

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ABSTRACT

We study the transmission of Dirac electrons through the one-dimensional periodic, Fibonacci, and Thue–Morse magnetic superlattices (MS), which can be realized by two different magnetic blocks arranged in certain sequences in graphene. The numerical results show that the transmission as a function of incident energy presents regular resonance splitting effect in periodic MS due to the split energy spectrum. For the quasiperiodic MS with more layers, they exhibit rich transmission patterns. In particular, the transmission in Fibonacci MS presents scaling property and fragmented behavior with self-similarity, while the transmission in Thue–Morse MS presents more perfect resonant peaks which are related to the completely transparent states. Furthermore, these interesting properties are robust against the profile of MS, but dependent on the magnetic structure parameters and the transverse wave vector.

1. Introduction

The physical properties of graphene in the presence of inhomogeneous perpendicular magnetic fields have attracted considerable attention, since its realization. The transport and bound states of Dirac electrons in graphene were reported in various magnetic structures involving single barrier [1], several barriers [2,3], and quantum dots [4]. There also exist many theoretical works on periodic magnetic superlattices (MS) in graphene. Dell'Anna found that the Fermi velocity at Dirac points is isotropically renormalized in MS, in contrast to the case of electric superlattices, and the spectrum and the nature of the states strongly depend on the conserved longitudinal momentum and the barrier width of MS [5]. The low-energy electronic structure of graphene under a one-dimensional MS could be mapped into that of graphene under an electric superlattice or vice versa [6]. The gapped states were studied analytically in graphene under periodic magnetic and electric fields [7]. We found that the transport has a general splitting rule through MS in graphene, of which the corresponding vector potential is a periodic field, and the splitting is independent of the MS profile [8]. In Refs. [9] and [10], the electronic properties of a magnetic Kronig–Penney superlattice with δ-function barriers have been discussed in graphene, where electron transport could be understood in terms similar to light propagation in periodic stratified media.

On the other hand, many works on the quasiperiodic system [11] have been performed, which is an intermediate case between periodic and disordered ones. The Fibonacci superlattices and Thue–Morse superlattices are two typical quasiperiodic systems discussed widely. Those studies have shown that the quasiperiodic systems have a highly fragmented energy spectrum, and their eigenstates can be critical states with self-similar pattern which are neither extended nor localized [12,13]. Many theoretical works on optical transmission, energy spectrum, and density of states in various quasiperiodic systems have been reported [12–17].

Recently, the transport properties through quasiperiodic electric superlattices in graphene have been investigated [18–20]. Biswas discussed the resonant tunneling through a Fibonacci superlattice in bilayer graphene [18]. Sena et al. found that the spectrum of quasibound states in Fibonacci graphene superlattice distributes as a Cantor-like set by virtue of transverse wave vector [19]. However, the results on transport of Dirac electrons in quasiperiodic MS are still lacking, and the magnetic field greatly affects the physical properties of Dirac electrons in comparison to the pure electric field, especially for the Klein tunneling. In this work, the quasiperiodic MS in graphene are considered, involving Fibonacci MS and Thue–Morse MS. The required magnetic profile can be produced by ferromagnetic stripes located on top of the graphene layer, or by virtue of other means [21]. The transmission of Dirac electron through the quasiperiodic MS is discussed theoretically based on

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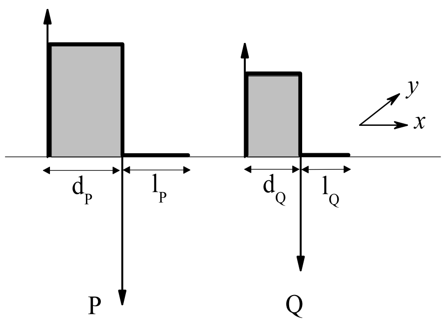

Fig. 1. The schematics of magnetic fields of building blocks P and Q indicated by the black arrows, and the corresponding vector potentials indicated by shaded areas.

numerical results and compared with the ones in periodic MS. It is found that the transmission as a function of incident energy is fragmented and has common structure in quasiperiodic MS, and the features are quite different from the results for quasiperiodic electric superlattices in graphene [18–20]. The nature of these features originates from the quasiperiodicity of the systems, analogous to the optical transmission spectrum in quasiperiodic photonic structures.

The Letter is organized as follows. In Section 2, we introduce the periodic MS, Fibonacci MS, and Thue–Morse MS, and the transfer-matrix method is used. We show the numerical results and discussions in Section 3. Finally, we draw conclusions in Section 4.

2. Model and method

The Fibonacci structure can be realized by juxtaposing the two basic building blocks P and Q in Fibonacci sequence, and the nth generation of the process S_n is given by the recursive rule $S_n = S_{n-1}S_{n-2}$, for $n \geqslant 1$, starting with $S_0 = P$ and $S_{-1} = Q$. The Fibonacci generations are $S_1 = PQ$, $S_2 = PQP$, $S_3 = PQPPQ$, etc. The total number of building blocks P and Q in each sequence is equal to the Fibonacci number $F_n = F_{n-1} + F_{n-2}$ with $F_0 = F_{-1} = 1$. The Thue–Morse structure based on Thue–Morse sequence can be defined by the recursive relation $U_n = U_{n-1}\overline{U}_{n-1}$, for $n \geqslant 1$, with $U_0 = PQ$ and $\overline{U}_0 = QP$, where $\overline{U}_n$ is the complement of U_n. The Thue–Morse generations are $U_1 = PQQP$, $U_2 = PQQPQPPQ$, etc., and the total number of building blocks in each sequence is equal to 2^{n+1}.

We shall consider the one-dimensional periodic and quasiperiodic MS perpendicular to the plane of graphene. The magnetic field is assumed to be uniform along the y-direction and to vary along the x-direction. The quasiperiodic Fibonacci MS and Thue–Morse MS can be realized by two magnetic blocks P and Q arranged in Fibonacci and Thue–Morse sequences, respectively. In addition, three periodic MS are considered, i.e., $(P)^m$, $(PQ)^m$, and $(PQQ)^m$ with the period number m, where P, PQ, and PQQ are the unit cells of the three periodic MS, respectively. Fig. 1 depicts the profiles of magnetic blocks P and Q, each of which is made up of two opposite magnetic δ-function barriers. In the Landau gauge, both corresponding vector potentials A_P and A_Q have rectangular shapes with barrier widths $d_{P/Q}$ and well widths $l_{P/Q}$. Thus, all the vector potential fields are superlattices, and their structures are the same as their corresponding MS, which play a key role to the following transport properties.

At low energy, the electron in graphene could be described by an effective massless Dirac equation with a linear energy dispersion. In the presence of a magnetic field perpendicular to the plane, the equation reads as

$$\left[v_f \sigma \cdot \left(\mathbf{p} + e\mathbf{A}(x)\right)\right]\Psi = E\Psi, \tag{1}$$

where the Fermi velocity $v_f \approx 0.86 \times 10^6$ m/s, the pseudospin matrix $\sigma = (\sigma_x, \sigma_y)$ is the Pauli matrix, $\mathbf{p} = (p_x, p_y)$ is the momentum operator, and $\mathbf{A}(x)$ is the vector potential. For convenience, the dimensionless units are introduced: $l_B = \sqrt{\hbar/eB_0}$, $E_0 = \hbar v_f/l_B$, $B(x) \to B_0 B(x)$, $A(x) \to B_0 l_B A(x)$, $\vec{r} \to l_B\vec{r}$, $k \to k/l_B$, and $E \to E_0 E$. For a typical value $B_0 = 0.1$ T, we have $l_B = 81$ nm and $E_0 = 7.0$ meV. The magnetic field and the corresponding vector potential are infinite and homogeneous along the y-direction, resulting in the conservation of the transverse wave vector k_y. The vector potential A is constant in each region of the models. For given incident energy E and transverse wave vector k_y, the solution in jth region of Eq. (1) could be written as $\Psi_j(x, y) = \psi_j(x)e^{ik_y y}$ with

$$\psi_j(x) = a_j \begin{pmatrix} 1 \\ \frac{q_j + ik_j}{E} \end{pmatrix} e^{iq_j x} + b_j \begin{pmatrix} 1 \\ \frac{-q_j + ik_j}{E} \end{pmatrix} e^{-iq_j x}. \tag{2}$$

Here, $k_j = k_y + A_j$, q_j is the longitudinal wave vector satisfying

$$q_j^2 + (k_y + A_j)^2 = E^2. \tag{3}$$

In order to calculate the transmission probability and the energy levels of quasibound states for periodic and quasiperiodic MS, the transfer-matrix method is employed. Eq. (2) can be rewritten as $\psi_j(x) = G_j H_j \binom{a_j}{b_j}$, where $G_j = \begin{pmatrix} 1 & 1 \\ (q_j+ik_j)/E & (-q_j+ik_j)/E \end{pmatrix}$ and $H_j = \begin{pmatrix} e^{iq_j x} & 0 \\ 0 & e^{-iq_j x} \end{pmatrix}$. Based on the continuity condition of the wave functions at the interface $x = x_j$ between jth and $(j+1)$th regions, one can get: $\binom{a_{j+1}}{b_{j+1}} = M_j \binom{a_j}{b_j}$, and $M_j = H_{j+1}^{-1}(x_j)G_{j+1}^{-1}G_j H_j(x_j)$. Thus, the total transfer-matrix for MS with n regions can be written as $M = M_{n-1}\cdots M_j \cdots M_1$. Then the transmission probability can be obtained from $T = 1 - |M_{21}|^2/|M_{22}|^2$, and M_{ij} is the matrix element of M. Assuming that the eigenstates decay exponentially in the vector potential barriers at both extremities of the MS, one may get the condition for quasibound states [22]:

$$M_{22} = 0, \tag{4}$$

in the energy region $k_y < E < (A + k_y)$ where the eigenstates are evanescent inside the barriers and propagating inside the wells.

3. Results and discussions

In this section, the transport properties of Dirac electron through periodic MS, Fibonacci MS, and Thue–Morse MS in graphene are studied numerically. The widths of barriers and wells for both magnetic blocks are the same and fixed as $d_{P/Q} = l_{P/Q} = 0.5$ in units of l_B in the following results.

First, the transmission probability as a function of incident energy for periodic MS $(P)^5$, $(PQ)^5$, and $(PQQ)^5$ is shown in Figs. 2(a)–(c) at $A_P = 3.0$ and $A_Q = 1.0$. Obviously, the transmission exhibits a new kind of resonance, which is not the Klein tunneling due to the suppression of the Klein tunneling in this energy region. From Fig. 2(a) we can see that the transmission exhibits 4-fold resonance splitting in $(P)^5$, and the splitting is $(m-1)$-fold in $(P)^m$ [8]. Quite differently, for periodic MS $(PQ)^5$ of which the unit cell is arranged with two magnetic blocks, the transmission presents two resonant domains in the considered energy region, and there are 4-fold resonant peaks in each domain, as shown in Fig. 2(b). Compared with $(P)^5$, the resonant domains in $(PQ)^5$ become narrow, due to the narrowed energy band of quasibound states which will be shown later. Fig. 2(c) shows that the transmission presents three resonant domains in $(PQQ)^5$ of which the unit cell is arranged with three magnetic blocks. Thus, it can be concluded that the transmission would present n resonant domains and $(m-1)$-fold resonant peaks in each domain, for the periodic MS of which the period number is m and the unit cell is arranged with n magnetic blocks. Furthermore, the position of