Physics Letters A 376 (2012) 1085–1089

Contents lists available at SciVerse ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Entropy, complexity and disequilibrium in compact stars

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ARTICLE INFO

Article history:
Received 27 October 2011
Received in revised form 6 February 2012
Accepted 7 February 2012
Available online 9 February 2012
Communicated by P.R. Holland

Keywords:
Information entropy
Complexity
Self-organization
Neutron stars
Strange stars
Equation of state

ABSTRACT

We used the statistical measurements of information entropy, disequilibrium and complexity to infer a hierarchy of equations of state for two types of compact stars from the broad class of neutron stars, namely, with hadronic composition and with strange quark composition. Our results show that, since order costs energy, Nature would favor the exotic strange stars even though the question of how to form the strange stars cannot be answered within this approach.

1. Introduction

In the recent past, scientists from different areas have looked at information theory to characterize physical and biological systems, their patterns and correlations. The idea is that a statistical measure of *complexity* (to be defined precisely below) encodes the self-organization of a system, and links the information stored in it (or the logic/information entropy) to its "distance" to the state of equilibrium probability distribution [1]. Recently, Sañudo and Pacheco [2] first related such measures to an astrophysical object, a white dwarf star, while Chatzisavvas et al. [3] applied these same concepts to another type of compact stars, collectively known as "neutron stars", where matter is in the densest form known in Universe and is under even more extreme physical conditions, namely supra-nuclear densities.

The importance of performing information theory studies on compact astrophysical objects results from the fact that the very nature of the matter in such extreme physical conditions is still uncertain, and these studies can shed a new light on this subject from a different point of view. In this Letter, we address "neutron stars" made of nuclear hadronic matter and made of free quarks (the self-bound strange stars, modeled in the context of MIT Bag Model).

The extension of these information concepts to astrophysical macroscopic objects is not straightforward, since the forces involved in the respective equilibrium configurations are very different from that ones in an atomic system. In atomic systems, the factor that determines the self-organization is mainly the Coulomb interaction and the Fermi exclusion principle. On the other hand, in a (ordinary) neutron star, gravity, strong and weak interactions all contribute. Finally, in a quark star the nuclear structure and the nucleons themselves have been bypassed, and the truly fundamental degrees of freedom show up to form a self-gravitating ball which is nevertheless bound by strong interactions, not gravity, although the latter is still very important for the overall structure. It is an open question whether the information quantities can be used for a gross description of these equilibrium configurations.

Motivated by these considerations, we compared the information and complexity stored in these two "neutron" stars of different microscopic composition, and found that these quantities are comparable in general, but sensitive to the composition, because the latter determines the behavior of the *radii* of the stars for the same mass. Since the value of the radius is also an important feature for an observational identification [4,5], information theory may link the formation and structure aspects.

2. Calculations and models

We used the statistical measure of complexity as defined by López-Ruiz, Mancini and Calbet [1], as modified by Catalán et al. [6]:

$C = H \times D,$

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0375-9601/$ – see front matter
doi:10.1016/j.physleta.2012.02.012

where $H = \exp(S)$ and S is the information entropy (or the information content of the system) in natural logarithmic units, D is the disequilibrium (identified with the distance of the system to its state of equiprobable probability distribution). In its original definition, the expressions for S and D are the following

$$S = -\int \rho(\mathbf{r}) \ln\bigl[\rho(\mathbf{r})\bigr] d\mathbf{r},$$
$$D = \int \rho^2(\mathbf{r})\, d\mathbf{r}.$$

The quantity $\rho(\mathbf{r})$ is the normalized probability distribution that describes the state of the system. S describes the uncertainty associated to that probability distribution while D stands for the information energy (as defined by Onicescu [7]), or the quadratic distance to the equiprobability.

In order to study our two types of compact objects in this way, we need an analogue to the probability distribution. Because the energy density distribution, $\epsilon(r)\,[\mathrm{erg/cm^3}]$, is related to the probability of finding a number of particles in a given location inside the star, we use the energy density profile as the quantity to enter in the integrals. However, in the case of the structure of our stars, the gravitation is non-Newtonian and we must solve the Tolman–Oppenheimer–Volkoff equations, or the equation of relativistic hydrostatic equilibrium of the star plus the mass integral, both complemented by the equation of state which describes the micro-physics (composition) of the stellar matter, to finally find $\epsilon(r) = c^2\rho(\mathbf{r})$, where $c = 3\times 10^{10}$ cm/s is the velocity of light and $\rho(r)$ $[\mathrm{g/cm^3}]$ is the matter density.

In this work we use the same approach of [3] to solve the TOV equation: first we define the barred quantities as the dimensionless variables scaling as follows

$$M(r) = \bar{M}(r)M_\odot \qquad \epsilon(r) = \bar{\epsilon}(r)\epsilon_0,$$
$$P(r) = \bar{P}(r)\epsilon_0 \quad \epsilon_0 = 1\ \mathrm{MeV/fm^3},$$

where $M(r)$ is the mass of the star in solar units ($M_\odot$), $P(r)$ is the pressure and ϵ_0 is an energy density scale, which in turn provide us the following form of the TOV equation and the mass equation

$$\frac{\bar{P}(r)}{dr} = -1.474\frac{\bar{M}(r)\bar{\epsilon}(r)}{r^2}\left(1+\frac{\bar{P}(r)}{\bar{\epsilon}(r)}\right) \times\left(1+11.2\times 10^{-6}r^3\frac{\bar{P}(r)}{\bar{M}(r)}\right)\left(1-2.948\frac{\bar{M}(r)}{r}\right)^{-1},$$
$$\frac{\bar{M}(r)}{dr} = 11.2\times 10^{-6} r^2\bar{\epsilon}(r).$$

Thus, the integrals to be evaluated are

$$S = -b_0\int \bar{\epsilon}(\mathbf{r})\ln\bigl[\bar{\epsilon}(\mathbf{r})\bigr] d\mathbf{r}, \tag{1}$$
$$D = b_0\int \bar{\epsilon}^2(\mathbf{r})\, d\mathbf{r}, \tag{2}$$

where $\bar{\epsilon}$ is the dimensionless energy density (which is just $c^2\rho/\epsilon_0$) and obtained from the solution of the TOV equation. The parameter $b_0 = 8.89\times 10^{-7}\ \mathrm{km^{-3}}$ is just a properly chosen quantity that makes S and D dimensionless. The integration is performed from 0 to the radius R [km]. We now refer separately to the specific cases of the hadronic star and the (strange) quark star, defined by different micro-physical descriptions.

A first treatment of the pure hadronic case has been given by [3], using a theoretically-motivated model equation of state. We instead use the so-called SLy4 equation of state in its analytic form [8] directly in the above form of the TOV equations, to obtain the energy density profiles for each initial value of the central pressure. Analytical representations of the equation of state are preferred over the tabulated ones, because they avoid two major problems of the latter: the ambiguity of the interpolation and impossibility of calculating the derivatives precisely. Furthermore, the analytical form is constructed obeying all the thermodynamic relations [8]. A suitable form of the equation of state SLy4 is

Table 1
Parameters of the fit.

i	a_i (SLy)	i	a_i (SLy)
1	6.22	10	11.4950
2	6.121	11	−22.775
3	0.005925	12	1.5707
4	0.16326	13	4.3
5	6.48	14	14.08
6	11.4971	15	27.80
7	19.105	16	−1.653
8	0.8938	17	1.50
9	6.54	18	14.67

$$\zeta = \frac{a_1 + a_2\xi + a_3\xi^3}{1+a_4\xi} f_0\bigl(a_5(\xi - a_6)\bigr) + (a_7 + a_8\xi) f_0\bigl(a_9(a_{10}-\xi)\bigr) + (a_{11}+a_{12}\xi) f_0\bigl(a_{13}(a_{14}-\xi)\bigr) + (a_{15}+a_{16}\xi) f_0\bigl(a_{17}(a_{18}-\xi)\bigr), \tag{3}$$

where the coefficients are given in Table 1 and $\xi = \log(\rho/\mathrm{g\,cm^{-3}})$, $\zeta = \log(P/\mathrm{dyn\,cm^{-2}})$.

This is currently a popular choice for detailed studies of dense matter and has all nuclear features of interest already built-in [9]. Another reason for choosing the *SLy*4 is that it allows maximum mass around $2M_\odot$, a minimum value similar to the quark equation of state discussed below.

The strange star models also need an equation of state describing the (self-bound) quark particles and their interactions. This is notoriously more involved than in the nuclear phase, since deconfinement is not yet properly understood. To calculate the information entropy, the disequilibrium and the complexity for our model of strange quark stars, we used one of the few analytical exact solution of the Einstein equations (which is of course a solution of the static TOV equation) for a spherically symmetric non-rotating perfect fluid. This solution is the anisotropic expression first obtained by Sharma and Maharaj [10] and studied by us in [11]. The advantage of this very accurate model is that in this way we have an analytical expression for the energy density that can be integrated easily, namely

$$\bar{\epsilon}(r) = \frac{1}{3}\frac{\rho_c c^2}{\epsilon_0}\frac{3+r^2/r_o^2}{(1+r^2/r_o^2)^2}. \tag{4}$$

In that expression, ρ_c is the central density and $r_o = r_o(\rho_c)$ is a parameter that controls the decay of the density profile. This analytical solution is obtained imposing the MIT Bag model for strange quark matter equation of state

$$p = \frac{1}{3}\bigl(c^2\rho - 4B\bigr), \tag{5}$$

where $B \simeq 57.5\ \mathrm{MeV/fm^3}$ is the energy density of vacuum. This simple expression (3) has been widely used because it readily captures the essential features of the deconfined phase. Crucial to our considerations of self-boundness (that is, a bound star even in the absence of gravitation [12]) is the existence and numerical value of the parametric vacuum energy density B, representing non-perturbative confining interactions. It is easily shown that for this massless quarks case