Physics Letters A 376 (2012) 1096–1101

Contents lists available at SciVerse ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Color path-integral Monte Carlo simulations of quark–gluon plasma

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ARTICLE INFO

Article history:
Received 10 January 2012
Received in revised form 3 February 2012
Accepted 3 February 2012
Available online 8 February 2012
Communicated by V.M. Agranovich

ABSTRACT

Thermodynamic properties of a strongly coupled quark–gluon plasma (QGP) of constituent quasiparticles are studied by a color path-integral Monte Carlo simulations (CPIMC). For our simulations we have presented QGP partition function in the form of color path integral with new relativistic measure instead of Gaussian one used in Feynman and Wiener path integrals. For integration over color variable we have also developed procedure of sampling color variables according to the group SU(3) Haar measure. It is shown that this method is able to reproduce the available quantum lattice chromodynamics (QCD) data.

1. Introduction

Investigation of properties of the QGP is one of the main challenges of strong-interaction physics, both theoretically and experimentally. Many features of this matter were experimentally discovered at the Relativistic Heavy Ion Collider (RHIC) at Brookhaven. The most striking result, obtained from analysis of these experimental data [1], is that the deconfined quark–gluon matter behaves as an almost perfect fluid rather than a perfect gas, as it could be expected from the asymptotic freedom. There are various theoretical approaches to studying QGP. Each approach has its advantages and disadvantages. The most fundamental way to compute properties of the strongly interacting matter is provided by the lattice QCD [2,3]. Interpretation of these very complicated Monte Carlo computations requires application of various QCD motivated, albeit schematic, models simulating various aspects of the full theory. Moreover, such models are needed in cases when the lattice QCD fails, e.g. at large baryon chemical potentials and out of equilibrium. While some progress has been achieved in the recent years, we are still far away from having a satisfactory understanding of the QGP dynamics.

A semi-classical approximation, based on a point like quasiparticle picture, has been introduced in [4–8]. It is expected that the main features of non-Abelian plasmas can be understood in simple semi-classical terms without the difficulties inherent to a full quantum field theoretical analysis. Independently the same ideas were implemented in terms of molecular dynamics (MD) [9]. Recently this MD approach was further developed in a series of works [10,11]. The MD allowed one to treat soft processes in the QGP which are not accessible by perturbative means.

A strongly correlated behavior of the QGP is expected to show up in long-ranged spatial correlations of quarks and gluons which, in fact, may give rise to liquid-like and, possibly, solid-like structures. This expectation is based on a very similar behavior observed in electrodynamic plasmas [12,13]. This similarity was exploited to formulate a classical nonrelativistic model of a color Coulomb interacting QGP [10] which was numerically analyzed by classical MD simulations. Quantum effects were either neglected or included phenomenologically via a short-range repulsive correction to the pair potential. Such a rough model may become a critical issue at high densities, where quantum effects strongly affect properties of the QGP. To account for the quantum effects we follow an idea of Kelbg [14] rigorously allowing for quantum corrections to the pair potential.[1] To extend the method of quantum effective potentials to a stronger coupling, we use the original Kelbg potential in the path-integral approach, which effectively maps the problem onto a high-temperature weakly coupled and weakly degenerated one. This allows one to extend the analysis to strong couplings and is, therefore, a relevant choice for the present purpose.

In this Letter we extend previous classical nonrelativistic simulations [10] based on a color Coulomb interaction to the quantum regime. For quantum Monte Carlo simulations of the thermodynamic properties of QGP we have to rewrite the partition function of this system in the form of color path integrals with a *new relativistic measure* instead of Gaussian one used in Feynman and Wiener path integrals. For integration partition function over color variables we have developed procedure of sampling color

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[1] The idea to use a Kelbg-type effective potential also for quark matter was independently proposed by K. Dusling and C. Young [15]. However, their potentials are limited to weakly nonideal systems.

0375-9601/$ – see front matter
doi:10.1016/j.physleta.2012.02.005

quasiparticle variables according to *the group SU(3) Haar measure* with the quadratic and cubic Casimir conditions. The developed approach self-consistently takes into account the Fermi (Bose) statistics of quarks (gluons). The main goal of this Letter is to test the developed approach for ability to reproduce known lattice data [2] and to predict other properties of the QGP, which are still unavailable for the lattice calculations. First results of applications of the path-integral approach to study of thermodynamic properties of the nonideal QGP have already been briefly reported in [16,17]. In this Letter we have shown that CPIMC is able to reproduce the QCD lattice equation of state and related thermodynamic functions and also yields valuable insight into the internal structure of the QGP.

Theoretical treatment and hydrodynamic simulations of the experimentally observed expansion of the fireball consisting of quarks and gluons at relativistic heavy-ion collisions suggest knowledge not only thermodynamic but also transport properties of the QGP. Unfortunately the CPIMC method itself is not able to yield transport properties. To simulate quantum QGP transport and thermodynamic properties within unified approach we are going to combine path-integral and Wigner (in phase space) formulations of quantum mechanics. The canonically averaged quantum operator time correlation functions and related kinetic coefficients will be calculated according to the Kubo formulas. In this approach CPIMC is used not only for calculation thermodynamic functions but also to generate initial conditions (equilibrium spatial, momentum, spin, flavor and color quasiparticle configurations) for generation of the color trajectories being the solutions of related differential dynamic equations. Correlation functions and kinetic coefficients are calculated as averages of Weyl's symbols of dynamic operators along these trajectories. The basic ideas of this approach have been published in [18]. Using this approach we are going to calculate diffusion coefficient and viscosity of the strongly coupled QGP. Work on these problems is in progress. In this dynamic approach energy of the quasiparticle system is conserved. In contrast known attempt to use quantum discreet color variable in classical simulations combining dynamic differential equations for quasiparticle coordinates and momenta and discrete color variables results in violation of energy conservation low due to unphysical increase of kinetic energy [4].

2. Thermodynamics of QGP

2.1. Basics of the model

Our model is based on a resummation technique and lattice simulations for dressed quarks, antiquarks and gluons interacting via the color Coulomb potential. The assumptions of the model are similar to those of [10]:

I. Quasiparticle masses (m) are of order or higher than the mean kinetic energy per particle. This assumption is based on the analysis of lattice data [19,20]. For instance, at zero net-baryon density it amounts to $m \geqslant T$, where T is a temperature.
II. In view of the first assumption, interparticle interaction is dominated by a color-electric Coulomb potential, see Eq. (1). Magnetic effects are neglected as subleading ones.
III. Relying on the fact that the color representations are large, the color operators are substituted by their average values, i.e. by Wong's classical color vectors (8D in SU(3)) with the quadratic and cubic Casimir conditions [21].
IV. We consider the 3-flavor quark model. Since the masses of the 'up', 'down' and 'strange' quarks extracted from lattice data are still indistinguishable, we assume these masses to be equal. As for the gluon quasiparticles, we allow their mass to be different (heavier) from that of quarks.

The quality of these approximations and their limitations were discussed in [10]. Thus, this model requires the following quantities as a function of temperature and chemical potential as an input:

1. the quasiparticle masses, m, and
2. the coupling constant g^2.

All the input quantities should be deduced from the lattice data or from an appropriate model simulating these data.

2.2. Color path integrals

Thus, we consider a multi-component QGP consisting of N color quasiparticles representing N_g dressed gluon and of various flavors N_q dressed quarks and $\bar{N}_q$ antiquarks. The Hamiltonian of this system is $\hat{H} = \hat{K} + \hat{U}^C$ with the kinetic and color Coulomb interaction parts

$$\hat{K} = \sum_i \sqrt{p_i^2 + m_i^2(T,\mu_q)},$$
$$\hat{U}^C = \frac{1}{2}\sum_{i\neq j} \frac{g^2(T,\mu_q)\langle Q_i|Q_j\rangle}{4\pi|r_i - r_j|}. \quad (1)$$

Here $i = 1,\ldots,N$ is the quasiparticle index, $N = N_q + \bar{N}_q + N_g$, i and j summations run over quark and gluon quasiparticles, 3D vectors r_i are quasiparticle spatial coordinates, the Q_i denotes the Wong's color variable (8D-vector in the SU(3) group), T is the temperature, and μ_q is the quark chemical potential, $\langle Q_i|Q_j\rangle$ denotes the scalar product of color vectors. We use relativistic kinematics, as seen from Eq. (1). Nonrelativistic approximation for potential energy is used to disregard magnetic interaction and retardation in the Coulomb interaction. In fact, the quasiparticle mass and the coupling constant, as deduced from the lattice data, are functions of T and, in general, μ_q.

The thermodynamic properties in the grand canonical ensemble with given temperature T, baryon (μ_q) and strange (μ_s) chemical potentials, and fixed volume V are fully described by the grand partition function[2]

$$Z(\mu_q,\mu_s,\beta,V) = \sum_{N_u,N_d,N_s,\bar{N}_u,\bar{N}_d,\bar{N}_s,N_g} \frac{\exp\{\mu_q(N_q-\bar{N}_q)/T\}\exp\{\mu_s(N_s-\bar{N}_s)/T\}}{N_u!N_d!N_s!\bar{N}_u!\bar{N}_d!\bar{N}_s!N_g!} \times Z(N,V,\beta),$$
$$Z(N,V,\beta) = \sum_\sigma \int_V dr\, d\mu_Q\, \rho(r,Q,\sigma;N_u,N_d,N_s,\bar{N}_u,\bar{N}_d,\bar{N}_s,N_g;\beta), \quad (2)$$

where $N_q = N_u + N_d + N_s$ and $\bar{N}_q = \bar{N}_u + \bar{N}_d + \bar{N}_s$ are total numbers of quarks and antiquarks of all flavors, respectively, N_g is the number of gluon quasiparticles, $\rho(r,Q,\sigma;N_u,N_d,N_s,\bar{N}_u,\bar{N}_d,\bar{N}_s,N_g;\beta)$ denotes the diagonal matrix elements of the density matrix operator $\hat{\rho} = \exp(-\beta\hat{H})$ and $\beta = 1/T$. Here σ, r and Q denote the multi-dimensional vectors related to spin, spatial and color degrees of freedom of all quarks, antiquarks and gluons. The σ summation and integration $dr\,d\mu_Q$ run over all individual degrees of freedom of the particles, $d\mu_Q$ means differential of the group SU(3) Haar measure. Usual choice of the strange chemical potential is $\mu_s = 0$ (nonstrange matter). Therefore, below we omit μ_s from the list of variables.

[2] Here we explicitly write sum over different quark flavors (u,d,s). Below we will assume that the sum quark degrees of freedom is understood in the same way.