Physics Letters A 376 (2012) 798–802

Contents lists available at SciVerse ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Estimation of frequency noise in semiconductor lasers due to mechanical thermal noise

Kenji Numata [a,b,*], Jordan Camp [b]

[a] Department of Astronomy, University of Maryland, College Park, MD 20742, USA
[b] NASA Goddard Space Flight Center, Code 663, 8800 Greenbelt Rd., Greenbelt, MD 20771, USA

ARTICLE INFO

Article history:
Received 7 December 2011
Accepted 9 January 2012
Available online 12 January 2012
Communicated by P.R. Holland

Keywords:
Thermal noise
Semiconductor laser

ABSTRACT

We evaluate mechanical thermal noise in semiconductor lasers, applying a methodology developed for fixed-spacer cavities for laser frequency stabilization. Our simple model determines an underlying fundamental limit for the frequency noise of free-running semiconductor laser, and provides a framework where the noise may be potentially reduced with improved design.

1. Introduction

Single longitudinal-mode semiconductor lasers are used in variety of applications, including interferometric sensing, spectroscopy, and coherent communication. In order to achieve higher performance, pursuit of low frequency noise (i.e. narrow linewidth) semiconductor laser has been an important subject in these areas for many years. In the meantime, mechanical thermal fluctuation (Brownian motion) is recently realized to be one of the fundamental noise sources in precision optical measurements. The mechanical thermal noise is a direct sensitivity limiting factor especially in optical cavity experiments (see for example, [1,2]). Since the cavity length fluctuation due to the thermal noise can determine the minimum linewidth of free-running laser, as the Schawlow–Townes limit can [3,4], it is important to know its potential thermal noise level. Especially in semiconductor lasers, the thermal noise contribution is expected to be larger due to the laser small mechanical size (to be explained later in detail). However, to our knowledge, there have been no calculation of thermal noise in such miniaturized optical components, except for a primitive calculation on optical fibers [5].

In this Letter, we evaluate mechanical thermal noise in semiconductor lasers using a simple model, which is similar to the one used to evaluate thermal noise in fixed-spacer cavities for laser frequency stabilization [6]. We only deal with statistical mechanical thermal fluctuation associated with an intrinsic loss, namely, all forms of background dissipation that are homogeneously distributed within a material. (The noise caused by this type of thermal fluctuation is often called "Brownian noise" in a narrow sense [7], and we call it "thermal noise" throughout this Letter. All other types of "thermal" noise sources, such as the noise coupled from classical temperature variation to optical path length and other types of statistical thermal noise coupled to the heat conduction equation, are out of the scope of this Letter.) According to our estimations, the thermal noise may not be the direct limiting factor of observed frequency noise of semiconductor laser. However, our insight provides a direction to achieve lower frequency noise with semiconductor lasers in the long run as all sources of noise become better quantified.

2. Formulation of Brownian thermal noise in cavity

2.1. Fluctuation dissipation theorem

The intrinsic mechanical loss ϕ has relevance to estimate the thermal noise level. ϕ is defined as the ratio of the imaginary to the real part of an elastic modulus and is often constant with frequency [8]. It is responsible for energy loss (dissipation), and is measured as the inverse of the quality factor at a mechanical resonance, or from the imaginary part of the mechanical response function. The fluctuation dissipation theorem (FDT) [9] relates the power spectral density of observable quantity (displacement) $X(t)$, $G_X(f)$, with the average dissipated strain energy, $W_{diss}(f)$, under a gedanken conjugate force, $F_0\cos(2\pi ft)\mathbf{P}(\mathbf{r})$, as [10]

$$G_X(f) = \frac{2k_B T}{\pi^2 f^2}\frac{W_{diss}(f)}{F_0^2}. \tag{1}$$

* Corresponding author at: NASA Goddard Space Flight Center, Code 663, 8800 Greenbelt Rd., Greenbelt, MD 20771, USA.
E-mail address: kenji.numata@nasa.gov (K. Numata).

0375-9601/$ – see front matter
doi:10.1016/j.physleta.2012.01.013

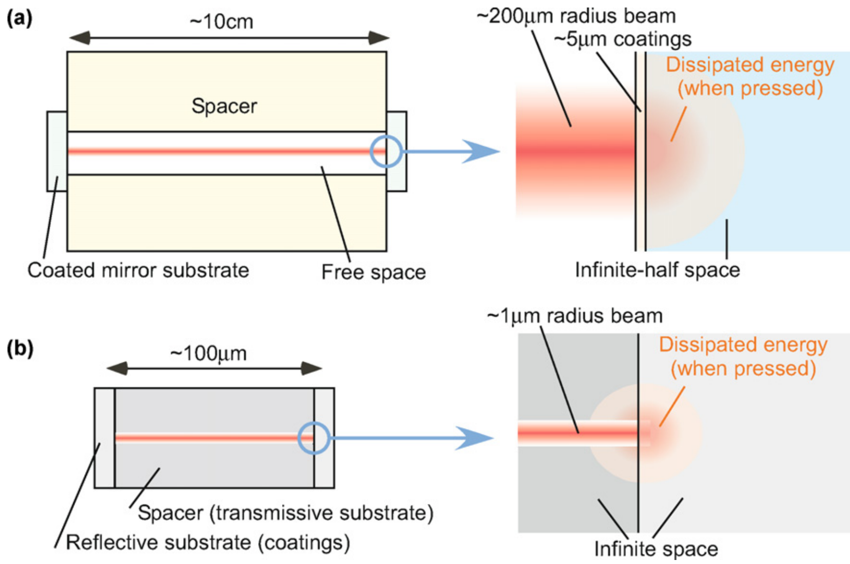

Fig. 1. Typical structure and situation around the beam spot in (a) fixed-spacer cavity for laser frequency stabilization and (b) semiconductor laser cavity. In the fixed-spacer cavity, the mirror substrate can be treated as an infinite-half space. In contrast, the semiconductor cavity should be treated as an infinite space, since the elastic energy is distributed and dissipated more evenly between the two substrates.

Here, k_B is the Boltzmann constant, T is the temperature, f is the frequency, and $\mathbf{P}(\mathbf{r})$ is the weighing function. Using the weighting function, we can write $X(t) = \int \mathbf{u}(\mathbf{r},t)\cdot\mathbf{P}(\mathbf{r})\,dV$. Here, $\mathbf{u}(\mathbf{r},t)$ is the displacement of the system at position $\mathbf{r}$. $W_{\mathrm{diss}}(f)$ depends on the losses (ϕ) of the materials and their distribution (not to be confused with the absorbed optical power and the noise associated with it [7]). Since the strain energy is stored and dissipated effectively around the force-applied area, the loss close to the beam spot is emphasized in optical cavities.

2.2. Fixed-spacer cavity

Before calculating thermal noise in semiconductor lasers, we briefly revisit an important and instructive application of the FDT. The FDT was used to indicate thermal noise as a limit to the frequency stability of a laser locked to an optical cavity comprised of a fixed-spacer supporting two mirrors (Fig. 1(a)). The frequency noise spectrum (square-root of power spectral density), $\sqrt{G_\nu(f)}$, is modeled as [6]

$$\sqrt{G_\nu(f)} = \frac{c}{L\lambda}\left\{\frac{4k_BT}{\pi f}\left[\frac{L}{3AE_{\mathrm{sp}}}\phi_{\mathrm{sp}} + \frac{1-\sigma^2}{\sqrt{\pi}w_0E_{\mathrm{sub}}}\phi_{\mathrm{sub}}\left(1+\frac{2}{\sqrt{\pi}}\frac{1-2\sigma}{1-\sigma}\frac{\phi_{\mathrm{coat}}}{\phi_{\mathrm{sub}}}\frac{d}{w_0}\right)\right]\right\}^{\frac{1}{2}}, \tag{2}$$

below the mechanical resonance frequency of the system. Here, c is the speed of light, λ is the wavelength, L is the cavity (spacer) length, $E_{\mathrm{sp}}/E_{\mathrm{sub}}$ are the Young's modulus of the spacer/mirror substrate, A is the cross sectional area of the spacer, σ is the Poisson's ratio of the mirror substrate, w_0 is the $1/e^2$ Gaussian beam radius, d is the thickness of the coatings, and $\phi_{\mathrm{sp}}/\phi_{\mathrm{sub}}/\phi_{\mathrm{coat}}$ are the mechanical loss of the spacer/substrate/coatings. The term with parenthesis $\{\}$ represents the displacement noise, and $c/(L\lambda)$ represents the length-to-frequency conversion factor obtained from the resonance condition of laser within the cavity. The first term inside the parenthesis $\{\}$ represents spacer contribution, and the second term represents mirror contribution, corrected by the factor inside $()$ to treat coatings. Smaller beam radius w_0 enhances the mirror contribution, since it does not average any longer-scale fluctuations within the beam illuminated area. In this model, contributions from two mirrors and two ends of the spacer are simply added up, assuming everything is uncorrelated. The spacer contribution is usually negligible compared to the mirror contribution [6]. The mirror *surface* fluctuation is analytically calculated, assuming Gaussian weighing and very thin coatings on the substrate that occupies infinite-half space [11–13]. Comparison of the model predictions with the lowest observed frequency noise levels in cavities was very good [14–17].

2.3. Semiconductor laser cavity

In semiconductor lasers, the geometry involved is a very small beam directed to a reflective substrate (or thicker coatings) through a transmissive substrate (Fig. 1(b)). Thus we consider the thermal noise *inside* the substrate with the following expression, by assuming the two substrates have same mechanical properties, by treating the system as infinite space, and by neglecting other effects that alter the resonance condition, such as detuning (difference between the Bragg reflection peak and the lasing wavelength),

$$\sqrt{G_\nu(f)} = \frac{c}{L\lambda}\left\{\frac{4k_BT}{\pi f}\left[\frac{L}{3AE_{\mathrm{sp}}}\phi_{\mathrm{sp}} + \frac{1}{8}\frac{(3-4\sigma)(1+\sigma)}{\sqrt{\pi}w_0E_{\mathrm{sub}}(1-\sigma)}\phi_{\mathrm{sub}}\right]\right\}^{\frac{1}{2}}. \tag{3}$$

The factor difference in the mirror term, $(1-\sigma^2)$ and $(3-4\sigma)(1+\sigma)/(1-\sigma)/8$, comes from the difference in the Green's tensors for the equations of equilibrium of infinite-half and infinite spaces [18]. (The rest of the factor in this term can be derived in the same manner as in [7,19].) In semiconductor lasers, both the small beam radius w_0 and the short cavity length L will enhance the thermal noise. Since the spacer contribution is not dependent on the factors w_0 and L, the internal fluctuation is expected to have larger contribution to the frequency noise.

3. Estimation of thermal noise level

3.1. Calculation method and common assumptions

We now apply the formalism to evaluate thermal noise in specific semiconductor lasers. As typical values, we adopt $\lambda = 1550$ nm and $T = 300$ K. For semiconductor materials we use