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Non-equilibrium thermodynamics, heat transport and thermal waves in laminar and turbulent superfluid helium

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ABSTRACT

This review paper puts together some results concerning non equilibrium thermodynamics and heat transport properties of superfluid He II. A one-fluid extended model of superfluid helium, which considers heat flux as an additional independent variable, is presented, its microscopic bases are analyzed, and compared with the well known two-fluid model. In laminar situations, the fundamental fields are density, velocity, absolute temperature, and heat flux. Such a theory is able to describe the thermomechanical phenomena, the propagation of two sounds in liquid helium, and of fourth sound in superleak. It also leads in a natural way to a two-fluid model on purely macroscopical grounds and allows a small amount of entropy associated with the superfluid component.

Other important features of liquid He II arise in rotating situations and in superfluid turbulence, both characterized by the presence of quantized vortices (thin vortex lines whose circulation is restricted by a quantum condition). Such vortices have a deep influence on the transport properties of superfluid helium, as they increase very much its thermal resistance. Thus, heat flux influences the vortices which, in turn, modify the heat flux. The dynamics of vortex lines is the central topic in turbulent superfluid helium.

The model is generalized to take into account the vortices in different cases of physical interest: rotating superfluids, counterflow superfluid turbulence, combined counterflow and rotation, and mass flow in addition to heat flow. To do this, the averaged vortex line density per unit volume L, is introduced and its dynamical equations are considered. Linear and non-linear evolution equations for L are written for homogeneous and inhomogeneous, isotropic and anisotropic situations. Several physical experiments are analyzed and the influence of vortices on the effective thermal conductivity of turbulent superfluid helium is found.

Transitions from laminar to turbulent flows, from diffusive to ballistic regimes, from isotropic to anisotropic situations, are analyzed, thus providing a wide range of practical applications. Besides the steady-state effective thermal conductivity, the propagation of harmonic waves is also studied, motivated by the fact that vortex line density is experimentally detected via the attenuation of second sound and because it provides

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dynamical information on heat transport and thermal waves which complement the static information of the thermal conductivity.

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