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Three dimensional third grade nanofluid flow in a rotating system between parallel plates with Brownian motion and thermophoresis effects

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ABSTRACT

The heat and mass transmission effects of non-Newtonian electrically conducting nanofluid flows, considering (third grad fluid) between two parallel plates with rotating system has been analyzed. The problem is demonstrated in such a way that the upper plate is static and the lower plate is stretched. The nanofluid flows of third grade fluid under the influence of thermophoresis and Brownian motion in a rotating system is the main goal of this study. The basic governing equations are transformed by the use of suitable similarity variables of differential equations which are nonlinear and coupled. An optimal approach has been used to acquire the solution of the modeled problems. The convergence of the method has been shown numerically. The variation of the heat flux, mass flux and their effects on the temperature and concentration profiles have been analysed numerically. Furthermore, for comprehension the physical presentation of the embedded parameters that is, Prandtl number Pr, Deborah number β, viscosity parameter R, rotation parameter Kr, Brownian motion parameter Nb, thermophoretic parameter Nt, magnetic parameter M, third grade fluid parameters α_1,α_2,β, dimensional thickness λ and Schmidt number Sc are plotted graphically and discussed.

Introduction

Fluid flows in a rotating system are a natural phenomenon. During the rotation of fluid, the molecules of the fluid strike with each other which effect on velocity, density, volume, etc. In fact, this rotation exists between the fluid particles internally and increases when the fluid starts flowing. The rotation can be reduced, but cannot be ignored completely.

Flow in a rotating system has abundant uses in the field of industry and technology. In the existing literature, most of the related study of such type of phenomena exists on Newtonian fluids and very little study exists for non-Newtonian nanofluid. Taylor and Geoffrey [1] was the first to give the experimental concept of the viscous fluid motion in a rotating system. Howard and Greespan [2,3] have investigated the basic and detailed theory of fluid movement towards a rotating system. Attia et al. [4] have inspected viscous flow between parallel plates with the effect of magneto hydrodynamics. Borkakoti et al. [5] have investigated magneto hydrodynamics viscous fluid flow in horizontal plates. Vajravelua and Kumar [6] has examined magneto hydrodynamics viscous fluid flow in horizontal plates in a rotating system, in which one plate was stretched and the other was porous. They obtained numerical solution and studied the effect of physical parameters. Their work was extended by Mehmood and Asif [7]. Recently Reddy [8] investigated MHD flow among binary rotating plates under the effects of thermal radiation and homogeneous-heterogeneous reactions. Recently, Shah et al. [9] have studied the effects of Hall current on steady three dimensional non-Newtonian nanofluid in a rotating frame with Brownian motion and thermophoresis effects.

Due to eccentric features of nanoliquids, that makes them proficient in various applications, nanofluids are used in pharmaceutical procedures, hybrid powered engines, fuel cells, microelectronics and currently it is mostly used in the field of nanotechnologies [10,11]. Sheikholeslami et al. [12–17] had done lots of work in the field of viscous fluids with rotating systems in two and three dimensions with different

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effects using nanofluids. For solution of problems, they used numerical techniques and described the effects of achieving parameters in detailed. Mahmoodi and Kandelousi [18,19] has investigated the hydro magnetic effects of Kerosene–alumina nanofluid flow in the occurrence of heat transfer analysis. Tauseef et al. [20] and Rokni et al. [21] have examined the magneto hydrodynamics and temperature effects on nanofluids flow in parallel plates in a rotating system. Non-Newtonian fluids flows with rotation are investigated by Hayat et al. [22–24] using different fluid models and extend their work in two and three dimensional flow. Nadeem et al. [25] have investigated micropolar nanofluid in two horizontals and parallel plates with rotation. They obtained the analytical solution to the problem and discussed the embedded parameters. Jena et al. [26] have investigated viscoelastic fluid with the effect of MHD and internal heat in porous channel with rotating system. Hameed et al. [27] investigated the combined magnetohydrodynamic and electric field effect on an unsteady Maxwell nanofluid flow over a stretching surface under the influence of variable heat and thermal radiation. Recently, Shah et al. [28,29] studied the effects of hall current on three dimensional non-Newtonian nanofluids and micropolar nanofluids in a rotating frame.

Third grade fluid is one of the significant sub types of non-Newtonian fluids, which were obtained from the research of Rivlin, and Ericksen [30]. Hayat et al. [31] examined third grade fluid with the effects of Soret and Dufour using three dimensional coordinate system. Mustafa [32] have studied the variety cases of stretching discs. Shehzad et al. [33] have examined nanofluid flow of third grade fluids with the effect of viscous dissipation and Newtonian heating. Sajid et al. [34] has used the finite element technique in their work taking third grade fluid flow, which was investigated in porous horizontal plate. Rashidi et al. [35] have obtained solutions of time dependent squeezing flow between two parallel plates. Recently Shehzad et al. [36] have studied third order fluid flows above, exponentially extending surface. Hussnain et al. [37] have investigates second grade fluid in three-dimensional rotating frame. Serdar Baris [38] has investigated second grade fluid at a moving flat plate. Recently Shoaib et al. [39] have examined second grade fluid in three-dimension an infinite horizontal plane wall with episodic suction. Ramzan et al. [40] have studied second grade nanofluid with radioactive effects. Mustafa et al. [41] have studied second grade nanofluid over an extending sheet.

In the fields of science and engineering most of mathematical problems are complex in their nature and the exact solution is almost very difficult or even not possible, so for the solution of such problems, Numerical and Analytical methods are used to find the approximate solution. One of important and popular techniques for the solution of such types of problems is the Homotopy analysis method. It is a substitute method and its main advantage is applying to the nonlinear differential equations without discretization and linearization. Liao [42–45] was the first one who derived this method for the solution of nonlinear problems and generally proved that this method is rapidly convergent to the approximated solutions. This method also provides the series solutions in the form of functions of a single variable. Solution with this method is important because it involves all the physical parameters of the problem and we can easily discuss its behavior. Abbasbandy et al. [46,47] have also used this technique to solve highly nonlinear and coupled equations due to its fast convergence. Gul et al. [48,49] have used the HAM method and have compared their results with the existing work.

As it is already mentioned that most of the published work related to rotation is in viscous fluids and very little literature available on the non-Newtonian nanofluids in a rotating system. The advancement of research in the field of nanofluids and its application in industry and technology is the need of the current era. Therefore, the determination of present research is based on the study of third grade nanofluid in a rotating system. So, for this aim study of third grade nanofluid flow is carried out. HAM is used for the solution modelled equations which are nonlinear and coupled. The effect of modelled parameters has been studied graphically.

Problem formulation

Consider the flows of non-Newtonian third grade electrically conducting nanofluid between two parallel plates. The distance between the upper and lower plates is designated with h. The plates are rotated with a constant angular velocity Ω around the y-axis. It should be noted that $\Omega < 0$ implies both plates rotate in the opposite directions, $\Omega > 0$ means that both plates rotate in the same direction, $\Omega = 0$ stands for the static case. In this study the $|\Omega| < 1$ has been considered in which the lower plate rotation is faster than the upper plate [29]. A coordinate system (x,y,z) is selected in such a method the x-axis is along the plate; the y-axis is perpendicular to it and the z-axis is normal to the xy plane. The plates are located at $y = 0$ & $y = h$. The lower plate is being stretched by the two forces has the same magnitude but opposite in direction so that the position (0,0,0) remains unchanged. The flow of the fluid and heat transfer is assumed in steady state which is incompressible, laminar and stable. A magnetic field B_0 is substituted along the y direction with which the fluid is rotating. Keeping in view the above deliberation, the basic equations of continuity, velocity, concentration and energy are as stated [12–17,28]:

$$u_x + v_y + w_z = 0, \tag{1}$$

$$\begin{aligned}\rho(uu_x + vu_y + 2\Omega w) = {}& -\widehat{p_x} + \mu(u_{xx} + u_{yy}) + \widehat{\alpha_1}\begin{pmatrix} 2w_y w_{xy} + u_x u_{yy} + 13u_{xx} + 3u_y(u_{xy} + v_{xx}) \\ + 2v_x(u_{xy} + 2v_{xx}) + v(v_{yyy} + u_{yxx}) \\ + 2w_x(w_{yy} + 2w_{xx}) + u(u_{xyy} + u_{yyy}) \end{pmatrix} \\ & + \widehat{\alpha_2}[w_y w_{xy} + 8u_x u_{yy} + 2(u_y + v_x)(u_{xy} + v_{xx}) + w_x(w_{yy} + w_{xx})] \\ & + \widehat{\beta}\begin{bmatrix} 2v_x w_x w_{xy} + u_{yy}\{4(u_x)^2 + 3(v_x)^2 + (w_x)^2 + 3(u_y)^2 + (w_y)^2\} \\ + u_x w_x w_{xx} + 2w_y(w_{yy}v_x + 2w_{xx}u_x w_{xy}) + 4v_x(u_x v_{xx} + 3u_x u_{xy}) \\ + 2u_y\{w_y w_{yy} + v_x(3u_{yy} - u_{xx}) + u_x(3u_{xy} + v_{xx})\} \\ + u_{xx}\{20(u_x)^2 - (v_x)^2 + (w_x)^2 - (u_y)^2 + (w_y)^2\} \end{bmatrix} \\ & - \sigma B_0^2 u,\end{aligned} \tag{2}$$

$$\begin{aligned}\rho(uv_x - vu_x) = {}& -\widehat{p_y} + \mu(u_{xy} + u_{yy}) + \widehat{\alpha_1}\begin{bmatrix} u_y(4u_{yy} - 2u_{xx}) - u_{yy}u_x + 13u_{xy}u_x \\ + 2w_y(w_{yy} + w_{xx}) + 3v_x(v_{yy} - u_{xx}) \\ - v(u_{xyy} + u_{yyy}) + u(v_{xxx} - u_{xyy}) + 2w_x u_{xy} \end{bmatrix} \\ & + \widehat{\alpha_2}[8u_x u_{xy} + w_x(w_{yy} + 2w_{xx}) + 2(u_y + v_x)(u_{yy} - u_{xx}) + 2w_x w_{xy}] \\ & + 2\widehat{\beta}\begin{bmatrix} -(w_x)^2(u_{xy} - v_{xx}) + u_x\{v_x(3u_{xx} - u_{yy}) + u_x(v_{xx} - 5u_{xy}) + 3v_y u_{xx} - w_x w_{xy}\} \\ + v_x\{v_x(u_{xy} + 3u_{yy}) + w_x w_{xx} + w_{xy}w_y + 6u_y v_{xx}\} - u_x w_y w_{yy} \\ + u_y\{w_x w_{xx} + v(u_{xy} + 3v_{xx} - 4u_x u_{yy} + 2u_{xy}v_x) + w_y w_{xy} - (w_y)^2(u_{xy} - v_{xx})\} \end{bmatrix},\end{aligned} \tag{3}$$

$$\begin{aligned}\rho(uw_x + vw_y - 2\Omega u) = {}& \mu(\partial_x^2 w_{xx} + \partial_y^2 w_{yy}) + \widehat{\alpha_1}\begin{bmatrix} w_z(u_{xx} + u_{yy}) + w_y(v_{xx} - u_{xy}) \\ + 2w_{xy}(u_y + v_x) + u_x(w_{xx} - w_{yy}) \\ + w_{y_{xx}}(u + u) + vw_{yyy} + uw_{xxx} \end{bmatrix} \\ & + \widehat{\alpha_2}[2(u_y + v_x)vw_{xx} + w_x(w_{yy} + w_{xx}) + w_y(v_{xx} - \partial_x u_{xy}) + 2u_x w_{xx} \\ & - 2u_x w_{yy}] - \sigma B_0^2 w + 2\widehat{\beta}\,[(u_x)^2 w_{yy} + w_{yy}\{(v_x)^2 + (w_x)^2\} + 2v_x w_x u_{xy} \\ & + 2w_y\{4u_x u_{xy} + 2w_x w_{xy} + v_x(-u_{xx} + u_{yy})\}],\end{aligned} \tag{4}$$

$$uT_x + vT_y + wT_z = \alpha^*(T_{xx} + T_{yy} + T_{zz}) + \tau\begin{bmatrix} D_B\{C_x T_x + C_y T_y + C_z T_z\} \\ + \frac{D_T}{T_c}\{(T_x)^2 + (T_y)^2 + (T_z)^2\} \end{bmatrix}, \tag{5}$$

$$uC_x + vC_y + wC_z = D_B(C_{xx} + C_{yy} + C_{zz}) + \frac{D_T}{T_0} \times \{T_{xx} + T_{yy} + T_{zz}\}. \tag{6}$$

In Eqs. (1)–(6) u, v and w represents the velocity components, υ, μ represents the coefficient of kinematic and dynamic viscosities, ρ is the density of the fluid, and σ denotes electrical conductivity, $\widehat{\alpha_1}, \widehat{\alpha_2}$ are the second grade parameters and $\widehat{\beta}$ is the third grade parameter and Ω is the angular velocity. In Eqs. (5) and (6), T represents temperature, α^*, is the thermal diffusivity, c_p represents specific heat, thermal conductivity of fluid is represented by k, D_B denotes the coefficient of Brownian diffusion and D_T represents thermophoretic diffusion. The term $\tau = \frac{(\rho c)_p}{(\rho c)_f}$ is the ratio of nanoparticles and effective heat capacity, ρ_f denotes the base fluid density and ρ_p represents the density of the particles, C is the coefficient concentration of the fluid particles.

The boundary conditions are defined as [14,15]: