Results in Physics 10 (2018) 145–149

Contents lists available at ScienceDirect

Results in Physics

journal homepage: www.elsevier.com/locate/rinp

# Would an alternative gravity theory developed from an improved gravitational action approach includes negative kinetic energy dynamic degrees of freedom?

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ARTICLE INFO

*Keywords:*
General relativity
Alternative models
Ghost states

ABSTRACT

The observed unexpected accelerating expansion of the universe, by Riess and his collaborators in 1998, has become one of the most important problems of the contemporary physics. A considerable effort has been spent by theoretical physicists to explain this observation for a while. When one looks at these attempts more closely, two of approaches attract attention: (i) Multi-dimensional alternative gravity models, (ii) Approaches which takes the more general and complex action than it is original Einstein-Hilbert form, which had been given as Ricci scalar $R$. The second type of these approaches must be examined carefully, because they could be generically involved dynamical degrees of freedom which possess negative kinetic energy (shortly called as 'ghost states' or simply 'ghosts'). In this work, an alternative theory has been studied to understand if it contains ghosts or not. This alternative approach belongs to the second type of the approaches which mentioned above, and it is given as: $S_{gravity} = \int d^4x\sqrt{-g}f(R, R_{\mu\nu}R^{\mu\lambda\nu\rho}R_{\lambda\rho})$ where $S_{total} = S_{gravity} + S_{matter}$. And this model has been examined by this way to see if this specific alternative model could be used to explain the present time acceleration of the universe or not.

## Introduction

The general relativity theory is expressed by the Einstein equation given in the following [1]:

$$R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} \tag{1.1}$$

where $R_{\mu\nu}$, $R$, $g_{\mu\nu}$, $T_{\mu\nu}$, $c$ and $G$ denotes the Ricci tensor, Ricci scalar, metric tensor, pressure-energy-momentum tensor, speed of light in vacuum and the universal gravitational constant, respectively. However, when it was understood that the equation did not allow a stationary universe in this way, in 1917 the Einstein equation was added a parameter (famous cosmological constant) that would allow it to be stationary [2]. It is understandable that Einstein does this, because in those years there is no clue as to the idea that the universe is not static. Therefore, if the equations lead to dynamic universe models, it is perceived as a problem to be corrected. Einstein's equations were transformed into the following new form

$$R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}. \tag{1.2}$$

The cosmological constant works as a parameter that balances the recalling effect of gravitational force created by matter-energy in the universe, creating a repulsive effect [2]. In 1929, the famous astronomer Edwin Hubble discovered that the universe was expanding [3]. Therefore, there is no need for correction in Eq. (1.2), i.e., cosmological constant, and Einstein re-extracted it from the Eq. (1.2). However, although Einstein excluded the cosmological constant from the Eq. (1.2), the discussions on the cosmological constant have grown by day-to-day until now. Cosmological constant problem is one of the biggest problems of today's physics.

However, in 1998 the observation [3] that the rate of expansion of the universe increased, i.e., accelerated, led to a fundamental change in this situation [3–10]. If the big bang is considered as the first acceleration, this observation, pointing to the second known acceleration, has become a major focus of theoretical physicists. This has been a very unexpected development. Because the theoretical expectation during the universe's enlargement process is not a rapid increase in the rate of expansion because of the recalling attraction of matter-energy in the spread of time. Attention has been drawn to the theory of general relativity to explain this after the observation. It is not possible to explain this observation using the original form of the General Relativity equations. Hence, it has begun to intensify the agenda of the theoretical physicists to question how it can be modified to account for this

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https://doi.org/10.1016/j.rinp.2018.05.021
Received 20 March 2018; Received in revised form 9 May 2018; Accepted 10 May 2018
Available online 17 May 2018

surprising development, at the same time, without disturbing the integrity and coherence of the equations, by continuing to account for events which can be well explained with at least the same degree of truth. The goal is to obtain an alternative theory of gravity that can explain Einstein's equation and explain how the universe accelerated in time. As far as we can understand [10], only about 4.9% of our universe consists of ordinary matter (baryonic matter) we know. The remaining part consists of 27.8% dark matter and 67.3% dark energy, which we do not know much about. As it is understood, dark matter-energy problem and cosmological constant problem are closely related problems. Thence, the earliest theoretical candidate of the dark energy is the famous cosmological constant. Although, the cosmological constant with cold dark matter ($\Lambda$CDM) scenario yields excellent conclusions for the accelerated expansion phenomenon suffers from some issues such as the fine-tuning and cosmological coincidence. From this point of view, many physicists have tried to obtain different proposals in order to explain the speedy expansion phase theoretically: scalar field minimally coupled with gravity [11–14], unified dark matter-energy expressions [15–20], assuming the existence of extra dimensions [21–26] and modified gravity models [27–38].

While these efforts have brought about various approaches, two approaches have come to the forefront:

1) Approaches with extra dimensions,
2) Approaches that use more complicated expressions instead of curvature scalar R contained in the 4-dimensional action expression.

One of the main differences between the two approaches is the infinite number of degrees of freedom of the extra dimensions in the effective 4-dimensional theorem [39]. Kaluza-Klein theory and DGP model are examples of this approach [40]. The second group approach, on the other hand, is the total action expression given in the original form of the general theory of relativity, as below.

$$S_{total} = S_{gravity} + S_{matter} \tag{1.4}$$

where

$$S_{gravity} = \int d^4x\sqrt{-g}R. \tag{1.5}$$

As given and in the act of gravity, also known as the Einstein-Hilbert action, instead of the Ricci scalar chosen as $R$ as the simplest possible form, are approaches based on more complex functions of R that can be expressed by $f(R)$. In this case, the gravitational action is expressed in the following manner:

$$S_{gravity} = \int d^4x\sqrt{-g}f(R) \tag{1.6}$$

As is known, it is possible to obtain Einstein field equations given by (1.1) by finding the extremum points of the obtained expression by taking the variation of the action according to the metric tensor with the Palatini approach, starting from the original action expression given by Eq. (1.5) [2]. Moreover, when a more complex function of $R$, such as $f(R)$, is taken instead of the Ricci scale $R$ in the action expression, the field equations obtained by variation management in general will be different from the Einstein motion equations given by the Eq. (1.1). Already the goal of the approach is to solve the problem of accelerating expansion using this difference. However, it has recently been shown that such a modification corresponds to or is reduced to Einstein's original theory by adding an extra scalar [41–43]. In this case, instead of the Ricci scalar $R$ in the original action expression, a more general variation like $f(R,R_{\mu\nu}R^{\mu\nu},R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta},...)$ involving combinations of Ricci and Riemann tensors has been introduced [43,44]. Studies on the subject showed that; making such modifications in the expression of action may lead to situations that have negative normality and therefore negative probability, negative energy in the term kinetic energy, and move forward in time. The existence of these conditions, which correspond to physically unacceptable incoherent and unstable solutions, removes the coherence of the theory that includes them. Therefore, in alternative gravitational theory, which is a candidate for solving the problem of accelerating expansion of the universe, which will be obtained by modifying the original action expression, a consistency control must first be made whether such situations are included or not. These conditions are also referred to briefly as ghosts in the literature.

Two scientists from New York University, A. Nunez and S. Solganik [43], prove that an alternative gravitation theory, which can be obtained by choosing a function $f(R,R_{\mu\nu}R^{\mu\nu},R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta})$ instead of $R$, is the most common case and therefore not useful. Moreover, even if this can be removed with special fine parameter adjustments, they show that the theory to be obtained by this adjustment is reduced to scalar-tensor gravitational theories, and so it is again insufficient [45].

The theme of this work is to check whether an original alternative gravitational theory which is a candidate for solving the problem of the accelerating expansion of the universe involves primarily negative kinetic energetic conditions that lead to inconsistency. If the theory seems consistent in this respect, then it is possible to discuss how the parameters of the alternative action function can be adjusted so that the universe can account for the acceleration problem in terms of observation values [46].

The alternative theory to be investigated deals with a function chosen as $f(R,R_{\mu\nu}R^{\mu\nu\alpha\beta}R_{\alpha\beta})$ instead of $R$ in the original action expression.

## The model

The action for the theory of gravity is generally expressed as below:

$$S = S_{gravity} + S_{matter} \tag{3.1}$$

Here $S_{gravity}$ stands for the gravitational part of the total action while $S_{matter}$ indicates the matter part including radiation. In the general theory of relativity, the action of gravity is given by the following relation

$$S_{gravity} = \int d^4x\sqrt{-g}R \tag{3.2}$$

which is also known as the Einstein-Hilbert action. The main task of this study is to make a consistency check by investigating whether the alternative action approach, originally to be considered as follows, has led to dynamic freedom degrees with negative kinetic energy; this alternative approach is to show if there is a candidate approach to explain the problem of the accelerating expansion of the universe. Hence, we focus on the following extended form of the gravitational action

$$S_{gravity} = \int d^4x\sqrt{-g}f(R,R_{\mu\nu}R^{\mu\lambda\nu\rho}R_{\lambda\rho}). \tag{3.4}$$

In this significant approach, the total action transforms the following new version:

$$S_{total} = \int d^4x\sqrt{-g}f(R,R_{\mu\nu}R^{\mu\lambda\nu\rho}R_{\lambda\rho}) + S_{matter}. \tag{3.5}$$

To control the presence of negative kinetic energetic states, equations of motion will first be obtained using the Palatini approach. For this purpose, we need to take the variation according to the metric tensor of the action expression and equal to zero. If the variation of action is taken, then it is found that

$$\delta S = \delta\int d^4x\sqrt{-g}f(R,R_{\mu\nu}R^{\mu\lambda\nu\rho}R_{\lambda\rho}) + \delta S_{matter} \tag{3.6}$$

The first term on the right-hand side of the equation is calculated as $R_{\mu\nu}R^{\mu\lambda\nu\rho}R_{\lambda\rho} = P$. Consequently, one can write

$$\delta\int d^4x\sqrt{-g}f(R,R_{\mu\nu}R^{\mu\lambda\nu\rho}R_{\lambda\rho}) = \int d^4xf(R,P)\delta\sqrt{-g} + \int d^4x\sqrt{-g}\delta f(R,P) \tag{3.7}$$

The following expression can be written for the first term on the right side of the equation