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A model for capillary rise in micro-tube restrained by a sticky layer

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ABSTRACT

Fluid transport in a microscopic capillary under the effects of a sticky layer was theoretically investigated. A model based on the classical Lucas-Washburn (LW) model is proposed for the meniscus rise with the sticky layer present. The sticky layer consists of two parts: a fixed (located at the wall) and a movable part (located on the inside of the capillary), affecting the micro-capillary flow in different ways. Within our model, the movable layer is defined by the capillary radius and pressure gradient. From the model it follows that the fixed sticky layer leads to a decrease of capillary radius, while the movable sticky layer increases flow resistance. The movable layer thickness varies with the pressure gradient, which in turn varies with the rising of the meniscus. The results of our theoretical calculation also prove that the capillary radius has a greater effect on the meniscus height, rather than the additional resistance caused by the movable layer. Moreover, the fixed sticky layer, which affects the capillary radius, has a greater influence than the movable sticky layer. We conclude that the sticky layer causes a lower imbibition height than the LW model predicts.

Introduction

The spontaneous capillary filling phenomenon is a fundamental mechanism for spontaneous imbibition in porous media with wide applications in many fields. In the petroleum industry, it is used extensively for oil and gas exploration during the flooding process of porous media such as sandstone or various carbonates such as the fractured reservoir, especially in cases with low permeability. The oil saturated matrix imbibes water in and pushes oil to the surface via the capillary force, which is known as imbibition displacement. A recent paper presented a review of the fundamental mechanisms of spontaneous imbibition for oil production from matrix systems, including the microscopic mechanism, with experimental method and influence factors [1]. Moreover, in the development of unconventional reservoirs, which contain micro- and nanoscale capillaries, the effect of imbibition is enhanced. Thus, the theory of capillary rise is essential for in-depth understanding oil exploration.

The porous medium is often described as "a bundle of capillary tubes" and further simplified as a vertical tube with a small radius. The basic capillary rise theory has been studied extensively. Lucas [2] and Washburn [3] first found the analytic solution for the height of capillary rise, ignoring gravity and inertia, known as the Lucas-Washburn equation (LW equation). Depending on the fluid stress situation, various regimes of imbibition have been studied separately, such as early regime and later regime [4,5]. In the early regime of capillary rise, the viscous resistance and gravity can be ignored due to the very low imbibition height, with only the inertia and capillary taken into account. The formula is known as the Bosanque equation, and shows the rising height as a function of the square root of time [6]. D. Quere confirmed the law in the case of early regime experimentally in a straight capillary in 1997, and also discovered the oscillations around the equilibrium height due to inertia [4]. The capillary rise time exponent is not always equal to 0.5 vs. meniscus height, confirmed by certain experimental results, particularly with the effect of tortuosity taken into account [7–9]. It is well known that the capillary in porous media such as natural sandstone is always tortuous. For a more accurate description of capillary rise in porous media, Cai [10] proposed the analytical expression for capillary rise height as a function of time, by introducing the tortuosity and fractal dimension for a tortuous capillary in the early stage of rising. The model of a single capillary with tortuosity was extended to porous media, and its predictions exhibited better agreement with experimental data for various porosities [11]. Fries and Dreyer proposed that the gravity can't be ignored in the later regime of capillary rise, and published an analytic solution for capillary rise with Lamber W function taking gravity into account [12]. Siddhartha gives an overall analysis of different regimes in vertical capillary filling [13,14].

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In addition to the kinetics, other aspects of capillary rise are studied. Ichikawa [15] investigated the dimensionless model of capillary rise by analyzing the dynamic contact angle. Shen [16] studied the effect of inherent roughness of capillary on meniscus height, with his results in better agreement with the experiments. With the permeability of porous media decreasing, the flow scale of capillary also decreases and the study of flow characteristics has to take more factors into account since the flow shows micro-scale features. Presently, the study of micro-scale flow in the petroleum industry mainly focuses on the analysis of three key factors: surface roughness, intermolecular forces and the sticky layer. It is commonly believed that flow in micro- and nanocapillaries shows non-linear characteristics due to these factors. A great number of experiments also found its nonlinear characteristics [17–19]. Among all those three factors, the sticky layer is particularly important and commonly found in reservoirs, since crude oil contains a large number of colloidals and asphaltene, which easily adsorb onto the solid capillary wall to form a sticky layer. As mentioned above, the sticky layer affects the flow particularly in the micro- and nanoscale, resulting in distinct characteristics with respect to macroscale flow. Thus, the study of sticky layer in capillary rise is of great interest for petroleum engineering. It has been found experimentally that the thickness of sticky layer is not fixed [20,21]. It consists of two parts, a movable sticky layer, the thickness of which decreases with increasing pressure gradient, and a fixed sticky layer, which is adsorbed on the capillary wall, reducing the effective capillary radius. Albeit there is a multitude of experimental research of capillary rise and the micro-flow affected by the sticky layer, the theory is still lacking.

Our studies focus on the imbibition phenomenon and utilize available sticky layer experimental data. The aim of this paper is to present a theoretical analysis of the capillary rise under the influence of the sticky layer. A capillary rise kinetic model, based on the basic vertical capillary model, with the dynamic sticky layer taken into account is obtained.

Resistance calculation of the sticky layer

The profile diagram of capillary with a sticky layer is shown in Fig. 1. It can be seen that:

$$r_0 = r_e + \delta_f, \tag{1}$$

where r_0, r_e and δ_f are the capillary radius, effective capillary radius, and thickness of fixed sticky layer, respectively. The movable sticky layer thickness δ_m is defined by capillary radius and pressure gradient [22] and can be written as:

$$\delta_m = r_e e^{-c' gradp}, \tag{2}$$

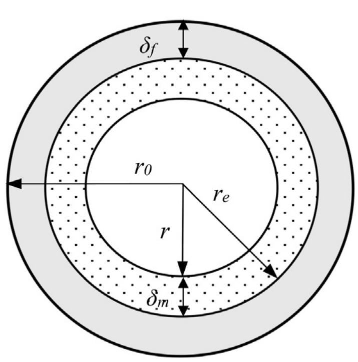

Fig. 1. Profile diagram of the capillary with the sticky layer.

where *gradp* denotes the pressure gradient. During the process of capillary rise, as the meniscus rises, and gravity and flow velocity vary, thereby inducing the change of pressure gradient as well as δ_m. c' is obtained experimentally and represents the reduction of movable sticky layer thickness with the increase of pressure.

Thus, the variable capillary radius r is:

$$r = r_e - \delta_m = r_e\left(1 - e^{-c' gradp}\right) \tag{3}$$

$e^{-c' gradp}$ is differentiable, thus it can be rewritten using the Maclaurin series:

$$e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \cdots\cdots - \frac{x^{2n-1}}{(2n-1)!} + \frac{x^{2n}}{(2n)!} \tag{4}$$

Taking only the first two terms into account, we obtain:

$$r = r_e c' gradp \tag{5}$$

Based on Poiseuille and Darcy's Law, the apparent permeability of the capillary k is related to effective capillary radius r_e as:

$$k = \frac{r^2}{8} \tag{6}$$

Therefore, the variable permeability induced by the boundary layer is:

$$k = \frac{\left(r_e c' gradp\right)^2}{8} \tag{7}$$

From Eq. (7) and Darcy's Law, we obtain flow velocity of the bulk fluid, v:

$$v = -\frac{k}{\mu} gradp = -\frac{\left(c' r_e\right)^2}{8\mu} gradp^3, \tag{8}$$

where μ is the viscosity of the fluid. If we define Δp as the pressure differential, according to Darcy's Law, Δp also represents the pressure drop along the tube. Denoting the flow distance by ΔL, Eq. (9) can be written as:

$$gradp = \frac{\Delta p}{\Delta L} = \left(-\frac{8\mu v}{\left(c' r_e\right)^2}\right)^{\frac{1}{3}}. \tag{9}$$

Further, we can obtain the additional resistance of the movable sticky layer:

$$\Delta p = \left(-\frac{8\mu v}{\left(c' r_e\right)^2}\right)^{\frac{1}{3}} \cdot \Delta L \tag{10}$$

Capillary rise with the sticky layer

If the flow rate is low, the inertial force is negligible and the vertical cylindrical capillary with a viscous, non-compressible liquid is obeys Newton's second law (see Fig. 2)

$$F_c - F_{vis} - F_g = 0, \tag{11}$$

where F_c, F_{vis} and F_g are the capillary force, viscous force, and gravity, respectively. The right-hand side of the equation is equal to the inertial force. Eq. (11) can be written as:

$$\frac{2\sigma \cos\theta_e}{R} - \frac{8\mu}{R^2}\frac{dh}{dt}h - \rho g h = 0, \tag{12}$$

where σ, θ_e, and μ are the surface tension, equilibrium contact angle between water and the capillary surface on a flat surface, and viscosity of the liquid, respectively. h refers to the height of capillary rise, R is the radius of the tube and ρ is the density of the fluid. Effects of the sticky layer effects are not considered in Eq. (12).