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Nonlinear radiative peristaltic flow of hydromagnetic fluid through porous medium

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ABSTRACT

The radiative heat and mass transfer in wall induced flow of hydromagnetic fluid through porous medium in an asymmetric channel is analyzed. The fluid viscosity is considered temperature dependent. In the theory of peristalsis, the radiation effects are either ignored or taken as linear approximation of radiative heat flux. Such approximation is only possible when there is sufficiently small temperature differences in the flow field; however, nonlinear radiation effects are valid for large temperature differences as well (the new feature added in the present study). Mathematical modeling of the problems include the complicated system of highly nonlinear differential equations. Semi-analytical solutions are established in the wave reference frame. Results are displayed graphically and discussed in detail for the variation of various physical parameters with the special attention to viscosity, radiation, and temperature ratio parameters.

Introduction

Peristalsis is a phenomenon which allows the fluid transport due to traveling waves along the walls of channel or tube. This phenomenon can be observed in biological organs like esophagus, intestines, ureters, bile duct, and arteries. Locomotion of some worms is also based on the peristaltic principle. Many industrial fluids such as corrosive fluids, toxic fluids, and sanitary fluids are transported by means of peristaltic motion. After the revolutionary investigation of Latham [1] and Shapiro et al. [2], several investigators have been involved in discussing the peristaltic mechanism under different situations [3–7].

The flow of an electrically conducting fluid is significantly influenced by the presence of magnetic field. Such property of a fluid can be used to control the flow of fluid by applying magnetic field on it. For instance, the blood is a biomagnetic fluid due to the presence of hemoglobin, so the application of magnetic field is helpful to control the blood pressure and bleeding during surgeries. Giant magnetic resistive (GMR) sensors are used to observe the peristaltic activity of ureter in vivo. Study of peristaltic motion under the influence of magnetic field played a key role in the development of blood pumps which may be utilized for cardiac operations specifically for a disease in the stenosed artery. It is also used in the treatment of other diseases: cancer, tumour, ulcer, and several diseases of uterus and intestines. Few recent studies [8–11] on the topic are mentioned in the list of references.

Peristaltic flow through porous medium is very common in various biological processes like transport processes of urine through ureter with stones, human lungs, and arterial systems of mammals. It is observed that the arteries in humans/animals arterial systems narrow down because of intra-vascular plagues. Such stenosis may arise various major complications: the ingrowth of tissues, bulging of the artery downstream, the development of a coronary thrombosis, and the weakening, which badly effect the normal pattern of blood flow through the arteries. In certain pathological situations, the porous medium can be seen when the arteries are clogged due to blood clots or fatty cholesterol. Thus, the analysis of peristaltically induced blood flow through permeable medium is helpful for diagnosis of aforesaid diseases. Eldesoky and Mousa [12] analyzed the peristalsis of compressible Maxwellian in an axisymmetric cylindrical tube with homogeneous porous medium. A mathematical model which deal with the transient peristaltic heat flow in finite length porous channel is given by Tripathi [13]. In another study, Vajravelu et al. [14] discussed the peristaltic flow of Jeffrey fluid in a vertical channel with porous space. Mekheimer et al. [15] examined the peristaltic mechanism for the flow of compressible Maxwell fluid through porous medium in micro-channel. Peristaltic flow through porous saturated

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channel with convective boundary conditions is studied by Hussain et. al. [16]. Last but not least, Ramesh [17] documented the peristaltic analysis for the flow of hydromagnetic couple stress fluid in channel with porous medium.

The processes of heat and mass transfer can be discussed separately or jointly; however, the efficient approach is to discuss them collectively. Mass transfer is the transport of chemical species from one location to another by diffusion and/or convection due to concentration gradients. Cross diffusion effects can be observed due to simultaneous occurrence of heat and mass transfer. The energy flux generated by concentration gradient is known as diffusion-thermo (or Dufour) effect. Similarly temperature gradients can also cause mass transfer, and this effect is termed as thermo-diffusion (or Soret) effect [18,19]. There are many examples in nature and industry in which mass transfer and heat transfer are coupled together such as fractional distillation and evaporative cooling. Examples include the respiration in lungs and at cell level, sweating, secretion, diffusion of nutrients from blood to tissues, distillation process, humidifying and dispersion of contaminants. The cumulative Soret and Dufour effects have been investigated by many researchers. One such detailed analysis has been presented by Eldabe et al. [20]. In continuation, Hayat et al. [21] studied heat mass transfer effects on peristaltic flow of Casson fluid with convective conditions and chemical reaction. Mustafa et al. [22] constructed the mathematical model for the peristaltic transport of fourth grade fluid with heat and mass transfer effects. Srinivas et al. [23] discussed the influence of heat and mass transfer on the peristaltic flow of a viscous fluid in a vertical asymmetric channel. Noreen et al. [24] examined the Soret and Dufour effects on peristaltic flow with heat and mass transfer when the flow is subjected to the inclined magnetic field.

Radiation in its nonlinear form is also substantial improvement in the study of peristaltic flow with heat transfer. It is observed that peristaltic mechanisms in human body involve all of three heat transfer modes: conduction, convection, and radiation. For example, heat energy that is produced inside the tissues is conducted to adjacent tissues, convected by the blood stream, or stored inside the tissues. For healthy body functions, the human body must maintain thermal equilibrium; therefore, the excess heat is radiated in form of infrared waves to prevent the body from overheating. The radiative heat transfer analysis in peristaltically induced flows hold an essential importance vis-a-vis the applications in enterprise, nuclear plants, and medical procedures. Few investigations [25–28] have been added in the theory of peristalsis. In these studies, the radiation term has been linearized using Rosseland approximation [29], which is only valid for small temperature differences in the flow field, but according to Stefan-Boltzmann law, the thermal energy radiated by a body is proportional to the fourth power of absolute temperature. This shows the nonlinear nature of the radiative heat flux which holds for large temperature differences within the flow as well. In order to improvise the results, we have incorporated the nonlinear radiative heat flux in the energy equation.

It is known fact that viscosity is a vital feature affecting the ability of propulsion in peristalsis. It is an essential parameter when flow measurements of fluids are made. For many fluids in nature, viscosity variation due to temperature is not negligible especially when viscous dissipation effects are taken into account. In coal slurries, for instance, the viscosity of fluid varies with temperature. In general, the coefficients of viscosity for real fluids are functions of temperature. In many thermal transport processes, the temperature distribution within the flow field is not uniform, i.e., the fluid viscosity may be changed noticeably if large temperature differences exist in the system. Thus taking viscosity to be constant is but a strong approximation that is contrary to the reality and needs more realistic consideration of variable viscosity, so taking viscosity as a function of temperature will provide a profound understanding of peristaltic rheology in more realistic situations. A reference to the existing literature reveals that very little has been said for peristaltic flow with temperature-dependent viscosity [20,30,31] and nothing for nonlinear thermal radiation. Thus, it is of great interest to establish a heat mass transfer analysis in peristaltic flow of hydromagnetic fluid through porous medium in an asymmetric channel when the fluid viscosity is temperature dependent and the effects of nonlinear thermal radiation are present in the flow field- the crux of present investigation.

The organization of the paper is as follows:

The equations that governs the flow with heat and mass transfer are modeled in Section “Mathematical modeling”. Solution methodology is presented in Section “Method of solution”. Section “Results and discussion” includes the graphical illustrations and discussion. Section “Conclusion” contains the concluding remarks.

Mathematical modeling

Consider the two-dimensional asymmetric channel of uniform thickness d_1+d_2. An incompressible electrical conducting viscous fluid fills the porous space inside the channel. The fluid motion in the channel is produced when sinusoidal waves of small amplitudes, b_1 and b_2, propagate along the channel walls. The channel length is considered to be infinite in the direction of $\bar{X}$-axis; whereas, the channel width is parallel to $\bar{Y}$-axis. The geometries of the traveling waves on upper and lower walls can be described mathematically as [10,26]:

$$\bar{Y}=\bar{H}_1(\bar{X},\bar{t})=d_1+b_1\cos\left(\frac{2\pi}{\lambda}(\bar{X}-c\bar{t})\right),\qquad \text{(upper wall)} \tag{1}$$

$$\begin{aligned}\bar{Y}&=\bar{H}_2(\bar{X},\bar{t})\\ &=-d_2-b_2\cos\left(\frac{2\pi}{\lambda}(\bar{X}-c\bar{t})+\phi\right),\qquad \text{(lower wall)}\end{aligned} \tag{2}$$

with the condition

$$b_1^2+b_2^2+2b_1b_2\cos\phi\leqslant(d_1+d_2)^2, \tag{3}$$

where λ is the wavelength, c is the speed of waves, $\bar{t}$ is the time, and $\phi\in[0,\pi]$ is the phase difference. The upper wall is kept at temperature T_0 and concentration C_0; whereas, the temperature and concentration at lower walls is assumed to be $T_1(>T_0)$ and $C_1(>C_0)$ respectively. It is further assumed that an external magnetic field $\bar{\mathbf{B}}=(0,B_0,0)$ is applied in the transverse direction of the flow, and the fluid viscosity varies with the change in temperature. The effects due to the change in thermal dissipation, and nonlinear radiation are taken into account. Thus, we have:

Ohm’s law [32]

$$\bar{\mathbf{J}}=\sigma\left[\bar{\mathbf{E}}+\bar{\mathbf{V}}\times\bar{\mathbf{B}}\right] \tag{4}$$

The Reynolds law for viscosity [33]

$$\bar{\mu}(\bar{T})=\mu_0(1-\gamma(\bar{T}-T_0)). \tag{5}$$

In above equations, σ as electrical conductivity of the fluid, $\bar{\mathbf{J}}$ is the current density vector, $\bar{\mathbf{E}}$ is the electric field, $\bar{\mathbf{V}}$ is the velocity field vector, γ is the viscosity variation parameter, and μ_0 is the constant viscosity at the reference temperature T_0.

The Rosseland approximation gives the net radiative heat flux $\bar{q}$ by the expression

$$\bar{q}=-\frac{4}{3k^*}\bar{\nabla}(e_b), \tag{6}$$

in which k^* is the mean spectral absorption coefficient. If we take the black body emissive power e_b in terms of the absolute