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Corrugated walls analysis in microchannels through porous medium under Electromagnetohydrodynamic (EMHD) effects

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ABSTRACT

This study looks for corrugated walls analysis in microchannels through porous medium under the impact of Electromagnetohydrodynamic (EMHD) effects. The incompressible and electrically conducting second grade fluid is considered between the two slit microparallel plates. The periodic sinusoidal waves are described for the small amplitude either in phase or out of phase for the corrugations of two wavy walls. By employing mathematical computation, we evaluated the corrugation effects on velocity for EMHD flow. By using perturbation technique, we investigated the analytical solutions of the velocity and volume flow rate. The influence of all parameters on velocity and the mean velocity profiles have been analyzed through graphs. The important conclusion from the analysis is that the small value of amplitude ratio parameter reduces the unobvious wave effect on the velocity.

Introduction

Microfluidics is most important in micro-electro-mechanical system. It is use for mixing, flow control, separation, detection and studying fundamental biochemical and physical processes. Microfluidics play important role in technological processes involving efficient design of the heat and mass transfer. In past few years many microfluidic devices developed [1], such as the electro-osmosis micropumps [2], and electromagnetohydrodynamic (EMHD) micropumps [3,4]. The important microfluidic system is EMHD micropump which generates the continuous flow pattern. In EMHD micropumps the pumping source is Lorentz force, as a result of the interaction between electric and magnetic fields. The EMHD micropump in microfluidic systems has important research such as flow control in the fluidic networks, pumping, stirring and mixing, thermal reactors and microcoolers [5–7]. Applications of EMHD devices, such as fluid pumping, in fluidic networks the flow control and fluid mixing, liquid chromatography and stirring [8]. In microchannels a great amount of the attention received analytical and numerical models of EMHD flow [9,10]. Tso and Sundaravadivelu [11] investigated the effect of electromagnetic fields on the surface tension between parallel plate microchannel. In the parallel plate microchannel, Chakraborty and Paul [12] study the EMHD forces effect on fluid flow. For more details see Refs. [13–19].

The previous studies based on smooth channels. Roughness always exists on the surface of channel due to adsorption of other species. Roughness applied in mechanical manufacturing and biomedical areas [20,21]. Buren et al. [22] study the EMHD Newtonian fluid flow in micro parallel channel by corrugated walls and roughness effect on the velocity. Liu et al. [23] study the electroosmotic flow between the slit of micro channel by considering the Jeffrey fluids. Dongqing et al. [24] investigated the EMHD effect of Jeffrey fluids on corrugated walls in microchannel under the effect of the electric and the magnetic fields. Thien et al. [25] investigated the Stokes problem of the viscous fluid with corrugated pipes. Wang [26] studied the result of the roughness between corrugated plates on stokes flow. Ng at al. [27] focued on the result of surface roughness of slip flow in circular microtube with the corrugated walls by method of perturbation. The corrugations effects on the Darcy-Brinkman flow examined Chiu-On and Wang [28]. Wavy roughness effects inside circular microtube were studied as the effect of slip and no-slip cases in [29–31]. The flow not only depends on the orientation of corrugations, but also the phase difference of corrugations.

In the current study, we investigate the effect of EMHD flow of viscoelastic fluid by the corrugated microparallel plates. The system is considered under the effect of Lorenz force which is generated electrical and magnetic field interaction. The remaining paper is presented as follows. The EMHD equations of Second grade fluid under the wavy conditions are derived and by applying the method of perturbation, we investigated the analytical solutions of the velocity and volume flow rate. The influence of all parameters on

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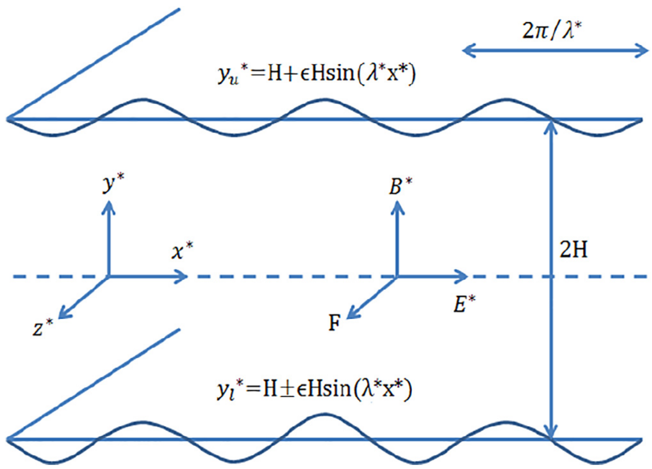

Fig. 1. Geometrical sketch.

velocity and the mean velocity profiles are analyzed through graphs. With the help of graphs, we analyzed the effect of the Hartmann number, wave number, Reynolds number and non dimensional parameter γ on velocity profile.

Formulation of the problem

We considered, EMHD flow of viscous, incompressible and electrically conducting second grade fluid with electrical conductivity σ and density ρ between two corrugated walls with the height $2H$. The microchannel height is taken 100 µm and corrugated wall amplitude is set to be $0.1H$. At middle of microchannel, we have taken the Cartesian coordinate system with a origin fixed. The width of channel is assumed to be W along x^* direction while the length L is taken along z^* direction are much larger than the layer thickness i.e. W, L $\gg$ 2H. The equations of upper and lower wavy walls are [17] (Fig. 1)

$$y_l^* = -H \pm \varepsilon H \sin(\lambda^* x^*) \quad \text{and} \quad y_u^* = H + \varepsilon H \sin(\lambda^* x^*)$$

where λ^* is wave number and ε is small amplitude. We applied along the x^* direction electric field $\vec{\mathbf{E}}^*$ while magnetic field $\vec{\mathbf{B}}^*$ along y^* direction. The Lorenz force $\vec{\mathbf{J}} \times \vec{\mathbf{B}}^*$ is taken along z^* direction and it is generated by interaction between the electric field $\vec{\mathbf{E}}^*$ and the magnetic field $\vec{\mathbf{B}}^*$, where $\vec{\mathbf{J}} = \sigma(\vec{\mathbf{E}}^* + \vec{\mathbf{u}}^* \times \vec{\mathbf{B}}^*)$ indicates the current density.

The governing equations for conservation of mass and momentum can be written as,

$$\nabla^* \cdot \mathbf{u}^* = 0. \tag{1}$$

$$\rho \frac{d\mathbf{u}^*}{dt^*} = \nabla^* \cdot \mathbf{S}^* + \rho \mathbf{b}^* - \frac{\mu}{k}\mathbf{u}^*, \tag{2}$$

where

$$\mathbf{S}^* = -\rho \mathbf{I}^* + \boldsymbol{\tau}^*, \tag{3}$$

Then momentum equation given us

$$\rho \frac{\partial \mathbf{u}^*}{\partial t^*} + \rho(\mathbf{u}^* \cdot \nabla^*)\mathbf{u}^* = -\nabla^* p + \nabla^* \cdot \boldsymbol{\tau}^* + \vec{\mathbf{J}} \times \vec{\mathbf{B}}^* - \frac{\mu}{k}\mathbf{u}^*, \tag{4}$$

For second grade fluid the stress tensor is defined as [19]

$$\boldsymbol{\tau}^* = \mu \mathbf{A}_1^* + \alpha_1^* \mathbf{A}_2^* + \alpha_2^* \mathbf{A}_1^{*2}, \tag{5}$$

in which $\boldsymbol{\tau}^*$ is the stress tensor, α_1^* and α_2^* are material constants, μ is the coefficient of viscosity and $\mathbf{A}_1^*$, $\mathbf{A}_2^*$ are the kinematic tensors.

Assume that flow in z^*-axis and independent of z^* due to continuity equation. The Eq. (4) can be simplified as,

$$\rho \frac{\partial w^*}{\partial t^*} = -\frac{\partial \hat{p}}{\partial z^*} + \mu\left(\frac{\partial^2 w^*}{\partial x^{*2}} + \frac{\partial^2 w^*}{\partial y^{*2}}\right) + \alpha_1^* \frac{\partial}{\partial t^*}\left(\frac{\partial^2 w^*}{\partial x^{*2}} + \frac{\partial^2 w^*}{\partial y^{*2}}\right) + \sigma B^* E^* - \sigma B^{*2} w^* - \frac{\mu}{k} w^*. \tag{6}$$

The Clausius-Duhem inequality and the condition that the Helmholtz free energy is minimum in equilibrium provide the following restrictions [19]

$$\mu \geqslant 0, \quad \alpha_1^* \geqslant 0 \quad \text{and} \quad (\alpha_1^* + \alpha_2^*) = 0. \tag{7}$$

Then a general conversation on the limitations for the μ, α_1^* and α_2^* can be initiated in work of Dunn and Rajagopal. The issue of a sign of material constant α_1^* and α_2^* is much controversy.

The no-slip boundary conditions can be stated as

$$w^*(x^*, y_u^*) = w^*(x^*, y_l^*) = 0 \tag{8}$$

In the microchannel, we assumed liquid is incompressible fluid and only taken along the z^* direction. Suppose channel is open along z^* direction so the pressure gradient $\partial p/\partial z^*$ can be ignored along the microchannel [2,7] and the velocity $w^*(x^*, y^*, t^*)$ is satisfies

$$\rho \frac{\partial w^*}{\partial t^*} = \mu\left(\frac{\partial^2 w^*}{\partial x^{*2}} + \frac{\partial^2 w^*}{\partial y^{*2}}\right) + \alpha_1^* \frac{\partial}{\partial t^*}\left(\frac{\partial^2 w^*}{\partial x^{*2}} + \frac{\partial^2 w^*}{\partial y^{*2}}\right) + \sigma B^*(E^* - B^* w^*) - \frac{\mu}{k} w^*. \tag{9}$$

In present study of EMHD flow, we suppose that velocity and electric field are periodical and in the complex forms can be written as

$$\begin{aligned} w^* &= R\{\tilde{w}(x^*, y^*) e^{i\omega t^*}\}, \\ E^* &= R\{E_0 e^{i\omega t^*}\}, \end{aligned} \tag{10}$$

where R represents the real part of function, ω is the angular frequency, $\tilde{w}$ and E_0 are the amplitude of velocity and electric field and i is the imaginary part.

Using Eq. (10) into Eq. (9), we get

$$i\rho\omega\tilde{w} = (\mu + \alpha_1^*\omega i)\left(\frac{\partial^2 \tilde{w}}{\partial x^{*2}} + \frac{\partial^2 \tilde{w}}{\partial y^{*2}}\right) + \sigma B^*(E_0 + B^*\tilde{w}) - \frac{\mu}{k}\tilde{w}. \tag{11}$$

We defined the dimensionless variables as

$$(x, y) = \frac{(x^*, y^*)}{H}, \quad w = \frac{\tilde{w}}{H\omega}, \tag{12}$$

Plugging Eq. (12) into Eq. (11), the dimensionless momentum equation can be expressed as

$$(1 + \gamma i)\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) - \left(Ha^2 + \mathrm{Re}\, i + \frac{1}{D_a}\right) w + Ha\beta = 0, \tag{13}$$

where

$$\mathrm{Re} = \frac{\rho\omega H^2}{\mu}, \quad Ha = B^* H\left(\frac{\sigma}{\mu}\right)^{\frac{1}{2}}, \quad \beta = E_0\left(\frac{\sigma}{\mu}\right)^{\frac{1}{2}} \Big/ \omega, \quad \gamma = \frac{\alpha_1^*\omega}{\mu}, \tag{14}$$

where Ha, Re and β represents the Hartmann number, Reynolds number and non-dimensional parameter respectively.

The corresponding non-dimensional boundary conditions are

$$w = 0, \quad y_u = 1 + \varepsilon \sin(\lambda x), \quad y_l = -1 \pm \varepsilon \sin(\lambda x). \tag{15}$$

In Eq. (15), $y_u = y_u^*/H$, $y_l = y_l^*/H$, the '+'symbol means the two wavy walls corrugation is in phase and the '−' symbol means the half period out of phase.