Results in Physics 8 (2018) 793–798

Contents lists available at ScienceDirect

Results in Physics

journal homepage: www.journals.elsevier.com/results-in-physics

Propagation phenomena in a visco-thermo-micropolar elastic medium under the effect of micro-temperature

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ARTICLE INFO

Article history:
Received 3 October 2017
Received in revised form 30 November 2017
Accepted 20 December 2017
Available online 12 January 2018

Keywords:
Visco-elasticity
Green Naghdi theory
Micro-temperature
Reflection
Micropolar

ABSTRACT

The analysis is made on reflection of waves in thermoelastic micropolar medium. The medium is having an additional property of viscosity, while studying waves the effect of micro-temperature is also been considered. It is found that after reflection three longitudinal and three transverse waves propagate through the medium. Reflected coefficients are calculated for each wave to examine deviation of reflected waves. Results obtained theoretically are shown graphically against angle of incidence. It is analyzed that effect of viscosity and micro-temperature reaches to its maximum level during intermediate values of angle of incident.

Introduction

The linear theory of linear viscoelasticity is considered as very important branch of Elastodynamics. It was observed by Freudenthal [1] that, most of the solids when subjected to dynamic loading exhibit a viscous effect. Because of this viscous effect internal friction produces attenuation and dispersion. Initially, Biot [2,3] and Bland [4] linked the solution of linear viscoelastic problems with corresponding linear elastic solutions. A notable works in this field were the work of Gurtin and Sternberg [6], and Ilioushin [7] offered an approximation method for the linear thermal viscoelastic problems. Problems related with micropolar viscoelastic waves was initiated by McCarthy and Eringen [5]. They discussed the propagation conditions and growth equations which govern the wave propagation of waves in micropolar viscoelasticity. Some sources are considered on study of viscoelastic materials are, Othman and Fekry [8], they studied the effect of initial stress on generalized thermo-viscoelastic medium with voids and temperature-dependent properties under Green-Naghdi theory. Kumar and Choudhary [9] analyzed different wave problems in micropolar visco-elastic thermo elastic solid. Effect of rotation on generalized thermo-viscoelastic Rayleigh Lamb waves was discussed by Sharma and Othman [10].

The theory of micro temperatures deals with the propagation of the temperature wave in a rigid heat conductor which allows the variation of thermal properties at a microstructure level. The theory of thermodynamics for elastic material with inner structures was developed by Grot [11] according to which the molecules possess micro-temperatures along with macro-deformation of the body the micro temperatures depend homogeneously on the micro-coordinates of the microelement. The experimental data for the silicone rubber containing spherical aluminum particles and for human blood presented by Riha [12] conform closely to the predicted theoretical model of thermoelasticity for micro-temperatures. Some authors recently invested some results related with wave propagation [13–15].

Green and Naghdi developed three models for generalized thermoelasticity of homogeneous isotropic materials, which are labeled as model I, II and III [16–18]. The nature of these theories is such that when the respective theories are linearized, model I [16] reduces to the classical heat conduction based on Fourier's law. The linearized versions of model II and III permit propagation of thermal waves at finite speed. Model II, in particular, exhibits a feature that is not present in the other established thermoelastic

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https://doi.org/10.1016/j.rinp.2017.12.064
2211-3797/

models as it does not sustain dissipation of thermal energy [18]. In this model, the constitutive equations are derived by starting with the reduced energy equation and by including the thermal displacement gradient among other constitutive variables. Green and Naghdi's third model [17] admits the dissipation of energy. In this model, the constitutive equations are derived by starting with the reduced energy equation, where the thermal displacement gradient and temperature gradient are among the constitutive variables. The uniqueness of the solution of governing equations for the GN type II model was established in [19]. Chandra Sekharaiah [20] studied the one dimensional thermal wave propagation in a half-plane based on the GN model. Some works on reflection waves in a half-space is discusser (see, Refs. [21–25]). Researchers as Othman and Song [26], Gupta and Rani [27] and Bayones and Abd-Alla [28] studied different type of waves propagating under different external influences. Kumar et al. [29] explained plane waves propagation in microstretch thermoelastic medium with micrtemperature. New features on waves reflection with an external parameters as magnetic field, initial stress and rotation has been investigated in (Refs. [30–36]).

In this article the authors are interested in the study of seismic waves and their reflection from a surface of thermoelastic medium. It is of great practical importance in geophysical investigations. Seismic signals carry a lot of information about the internal structure of the earth and this information is of great help in exploration of variable materials. We basically study the reflection of plane waves at the free surface of the micropolar generalized thermoelastic half space solid. The medium is naturally viscoelastic and the effect of micro temperature is also been considered while analyzing the amplitude of reflected waves. Green Naghdi theory of type III is considered to represent the heat waves conducting through the medium. Reflected coefficients for both transverse and longitudinal waves are obtained theoretically, analyzed and finally represented graphically against the incident angle.

Formulation of the problem

Cartesian Coordinates (x, y, z) are being selected to represent the system of problem. Origin is on surface $y = 0$ and z-axis directed along depth of solid. Basic governing equations for the problem are,

$$\rho\ddot{u}_i = (\lambda_I + \mu_I)u_{j,ji} + (\mu_I + k_I)u_{i,jj} + k_I\varepsilon_{ijk}\varphi_{k,j} - \beta T_{,i} \tag{1}$$

$$(\alpha_I + \beta_I' + \gamma_I)\,\vec{\nabla}(\vec{\nabla}.\vec{\varphi}) - \gamma_I\vec{\nabla}\times(\vec{\nabla}\times\vec{\varphi}) + k_I(\vec{\nabla}\times\vec{u}) - 2k_I\vec{\varphi} - \mu_1(\vec{\nabla}\times w) = j\rho\frac{\partial^2\vec{\varphi}}{\partial t^2}, \tag{2}$$

$$k_6\nabla^2 w + (k_4 + k_5)\nabla(\nabla.w) + \mu_1(\vec{\nabla}\times\vec{\varphi}) - b\frac{\partial w}{\partial t} - k_2 w - k_3\nabla T = 0, \tag{3}$$

$$K^*\nabla^2 T + K\nabla^2\dot{T} + k_1(\vec{\nabla}.\vec{w}) = \rho C_E\dot{T} + \beta T_0\ddot{u}_{i,i}, \tag{4}$$

GN-II can be obtained by adjusting $K = 0$ in Eq. (4), the constitutive equations are

$$\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij} + k(u_{j,i} - \varepsilon_{ijk}\phi_k) - \beta T\delta_{ij} \tag{5}$$

$$m_{ij} = \alpha\phi_{l,l}\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i}, \quad q_{ij} = -k_4 w_{l,l}\delta_{ij} - k_5 w_{i,j} - k_6 w_{j,i} \quad j, i, l, k = 1, 2, 3. \tag{6}$$

Where $\alpha_I, \beta_I', \gamma_I, \mu_I, \lambda_I, k_i (i = 1, 2, \ldots, 6)$ are constitutive coefficients u_i, σ_{ij}, e_{ij}, m_{ij} are the components of displacement vector, of stress tensor, strain tensor and couple stress tensor respectively, j is the micro inertia moment, the mass density is ρ, the specific heat at constant strain is C_E, K^*, K are the thermal conductivity and the material characteristic respectively of the theory. T_0 is the reference temperature, $\beta = (3\lambda_I + 2\mu_I)\alpha_t$ where α_t are the coefficients of linear thermal expansion for the material.

Assuming the viscoelastic nature of the material [10],

$$\lambda_I = \lambda + \frac{\partial}{\partial t}\lambda_v, \quad \mu_I = \mu\left(1 + \frac{\partial}{\partial t}\tau_v\right), \quad k_I = k(1 + \frac{\partial}{\partial t}\tau_v),$$
$$\alpha_I = \alpha\left(1 + \frac{\partial}{\partial t}\tau_v\right), \beta_I = \beta(1 + \frac{\partial}{\partial t}\tau_v), \quad \gamma_I = \gamma\left(1 + \frac{\partial}{\partial t}\tau_v\right),$$

where, τ_v is the sensitive part representing the viscosity.

Displacement and microrotation components are taken as,

$$\vec{u} = (u_1(x_1, x_3), 0, u_3(x_1, x_3)) \quad \text{and} \quad \vec{w} = (w_1(x_1, x_3), 0, w_3(x_1, x_3)) \tag{7}$$

Following are the non-dimensional parameters introduced for the problem,

$$(x_i', u_i') = \frac{w^*}{c_1}(x_i, u_i), \quad t' = w^* t, \quad \phi_2' = \frac{w^{*2} j}{c_1^2}\phi_2, w^{*\prime} = \frac{K_I}{\rho j},$$
$$m_{ij}' = \frac{w^*\lambda_i^*}{c_1\lambda_I}, \quad w_i' = \frac{c_1}{w^*}w_i \tag{8}$$

The component of displacement functions $(u_1, 0, u_3)$ and micro temperature $(w_1, 0, w_3)$ are connected with potential functions R, ψ and G, H respectively, by the relation [24],

$$u_1 = \frac{\partial R}{\partial x} - \frac{\partial\psi}{\partial z}, \quad u_3 = \frac{\partial R}{\partial z} + \frac{\partial\psi}{\partial x} \quad \text{and} \quad w_1 = \frac{\partial G}{\partial x} - \frac{\partial H}{\partial z},$$
$$w_3 = \frac{\partial G}{\partial z} + \frac{\partial H}{\partial x} \tag{9}$$

Making use of Eqs. (7)–(9) in (1)–(4) we obtained the following set of equations,

$$\left((\delta_1 + \delta_2)\nabla^2 - \frac{\partial^2}{\partial t^2}\right)R - \delta_4 T = 0, \tag{10}$$

$$\left(\delta_2\nabla^2 - \frac{\partial^2}{\partial t^2}\right)\psi + \delta_3\phi_2 = 0, \tag{11}$$

$$\left(\delta_5\nabla^2 - 2\delta_3 - \frac{\partial^2}{\partial t^2}\right)\phi_2 + \delta_6\nabla^2\psi + \delta_7\nabla^2 H = 0, \tag{12}$$

$$\left(K_6\nabla^2 + \delta_8\frac{\partial}{\partial t} + \delta_{10}\right)H - \delta_9\phi_2 = 0, \tag{13}$$

$$\left((K_4 + K_5 + K_6)\nabla^2 - \delta_8\frac{\partial}{\partial t} - \delta_{10}\right)G - \delta_{11}T = 0, \tag{14}$$

$$\left(\varepsilon_2 + \varepsilon_3\frac{\partial}{\partial t} - \frac{\partial^2}{\partial t^2}\right)T + \delta_{12}\nabla^2 G - \varepsilon_1\nabla^2\ddot{R} = 0. \tag{15}$$

where,

$$\delta_1 = \frac{\lambda + \mu}{\rho c_1^2}, \quad \delta_2 = \frac{\mu + k}{\rho c_1^2}, \quad \delta_3 = \frac{k}{\rho\omega^{*2} j}, \quad \delta_4 = \frac{\beta T_0}{\rho c_1^2},$$
$$\delta_5 = \frac{\gamma}{j\rho c_1^2}, \quad \delta_6 = \frac{k}{\rho c_1^2}, \delta_7 = \frac{\mu_1\omega^{*2}}{\rho c_1^4}, \quad \delta_8 = \frac{bc_1^2}{\omega^*}, \quad \delta_9 = \frac{\mu_1 c_1^4}{\omega^{*4} j},$$
$$\delta_{10} = \frac{K_2 c_1^2}{\omega^{*2}}, \quad \delta_{11} = \frac{K_3 c_1^2 T_0}{\omega^{*2}}, \quad \delta_{12} = \frac{K_1}{\rho C_E c_1^2 T_0}, \quad \varepsilon_1 = \frac{\beta}{\rho C_E},$$
$$\varepsilon_2 = \frac{K^*}{\rho C_E c_1^2} \quad \text{and} \quad \varepsilon_3 = \frac{K\omega^*}{\rho C_E c_1^2}.$$