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Is there a relationship between curvature and inductance in the Josephson junction?

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ABSTRACT

A Josephson junction is a device made of two superconducting electrodes separated by a very thin layer of isolator or normal metal. This relatively simple device has found a variety of technical applications in the form of Superconducting Quantum Interference Devices (SQUIDs) and Single Electron Transistors (SETs). One can expect that in the near future the Josephson junction will find applications in digital electronics technology RSFQ (Rapid Single Flux Quantum) and in the more distant future in construction of quantum computers. Here we concentrate on the relation of the curvature of the Josephson junction with its inductance. We apply a simple Capacitively Shunted Junction (CSJ) model in order to find condition which guarantees consistency of this model with prediction based on the Maxwell and London equations with Landau-Ginzburg current of Cooper pairs. This condition can find direct experimental verification.

Introduction

A Josephson junction is a device that usually is made of two superconducting electrodes separated by very thin layer of non-superconducting material. The barrier which separates the electrodes must be very thin. If the layer is made of isolator (S-I-S junction) then its thickness is of order of several Angstroms. In case of nonsuperconducting metal (S-N-S junction) this thickness can be on the level of several microns. The junction can also be made as a constriction that weakens the superconductivity at the point of contact (S-s-S junction). Until a critical current is reached, a current of electron pairs can flow across the barrier without any resistance. First time, theoretical prediction that Cooper pairs can tunnel from one electrode to another was given by Brian Josephson in 1962 [1]. In his paper Josephson predicted relationships for the current and voltage across the weak link. This prediction was confirmed by Philip Anderson and John Rowell [2].

Depending on the number of dimensions the Josephson device can be considered as a two (large area Josephson junction), one (long Josephson junction) and even zero dimensional system (point Josephson junction). In particular, the junction can be considered as the long Josephson junction if its transverse dimension is smaller than the Josephson length. Josephson effect, similarly like superconductivity, is an example of a macroscopic quantum phenomenon. The leading dynamical variable that describes behavior of this system is a scalar field ϕ. This variable is a gauge invariant difference of phases of the macroscopic wave functions that describe superconducting electrodes. The behavior of this dynamical variable is determined by the sine-Gordon model. The solutions of this nonlinear model are studied for years [3,4].

The effective equation that describes dynamical processes in the junction can be obtained in many ways [5,6]. In one of simplified approaches the point junction is replaced by a circuit that contains the supercurrent (Josephson current) and some capacitive element (see Fig. 1). This model is described as Capacitively Shunted Junction – CSJ. The CSJ model presumes that the quasiparticle current in the junction is so small that can be neglected. The currents in the direction normal to the isolator layer are Josephson current of Cooper pairs and additionally displacement current due to capacitive effects.

The present paper aims in application of the CSJ model to description of the curvature effects in the long Josephson junction. We would like to indicate the condition which allows to recover the result obtained for the same system on the background of the Maxwell and London's equations with Landau-Ginzburg current.

Finally, let us underline that one of the promising areas of applications of the Josephson junction is the RSFQ (Rapid Single Flux Quantum) electronics [7]. The digital information in these instruments is carried by magnetic flux quanta identified with the kink solutions of the sine-Gordon model. An interesting role in this area can be played by the curved Josephson junctions. On the base of the modified sine-Gordon equation several geometries were

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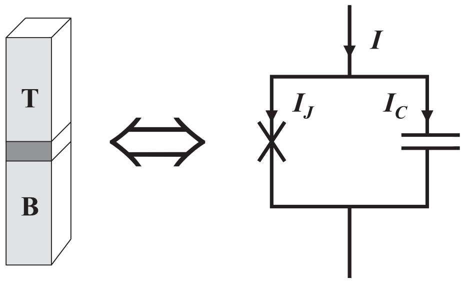

Fig. 1. In the CSJ model the point Josephson junction (left side of the figure) is equivalent to the circuit located on the right side of the picture. Here I_J and I_C denotes the Josephson and capacitive currents.

identified which can be used as electronic elements in RSFQ electronic devices [8].

The paper is organized as follows. For the sake of completeness, in the next Section we present Feynman approach for two-piece quantum system in order to obtain time changes of the phases of the macroscopic wave functions which describe behavior of the superconducting electrodes. Section "Curved junction in the CSJ model" contains construction of the field equation, in a curved system, on the base of the CSJ model. The last Section contains remarks.

Josephson relation

In the Feynman approach the Josephson junction can be viewed as quantum system that consists of two subsystems (superconducting electrodes) described by the many particle wave functions ψ_T and ψ_B. The indices T, B correspond to top and bottom electrode. The system of this type is described by the Schrödinger equation

$$\imath\hbar \frac{\partial}{\partial t}|\psi\rangle = \widehat{H}|\psi\rangle, \tag{1}$$

where the quantum state and the matrix elements of the hamiltonian are the following:

$$|\psi\rangle = \begin{bmatrix} \psi_T \\ \psi_B \end{bmatrix}, \qquad \begin{bmatrix} \langle\psi_T|\widehat{H}|\psi_T\rangle, & \langle\psi_T|\widehat{H}|\psi_B\rangle \\ \langle\psi_B|\widehat{H}|\psi_B\rangle, & \langle\psi_B|\widehat{H}|\psi_T\rangle \end{bmatrix} = \begin{bmatrix} E_T, & K \\ K, & E_B \end{bmatrix}. \tag{2}$$

The Schrödinger Eq. (1) can be transformed to the system of coupled equations

$$\imath\hbar \frac{\partial \psi_T}{\partial t} = E_T\psi_T + K\psi_B, \qquad \imath\hbar \frac{\partial \psi_B}{\partial t} = E_B\psi_B + K\psi_T, \tag{3}$$

where K describes interaction of the quantum subsystems. One can see that for $K = 0$ the subsystems are independent and are described by two independent Schrödinger equations. Next, we separate the modulus and the phase of the wave functions that describe the electrodes

$$\psi_T = |\psi_T|e^{\imath\varphi_T}, \qquad \psi_B = |\psi_B|e^{\imath\varphi_B}. \tag{4}$$

The real parts of Eq. (3) can be written in the form

$$-\hbar|\psi_T|\frac{\partial \varphi_T}{\partial t} = E_T|\psi_T| + K|\psi_B|\cos\phi,$$
$$-\hbar|\psi_B|\frac{\partial \varphi_B}{\partial t} = E_B|\psi_B| + K|\psi_T|\cos\phi. \tag{5}$$

where we denoted the phase difference as follows $\phi = \varphi_T - \varphi_B$. If we assume that modules of the wave functions are identical i.e. that the density of the Cooper pairs ρ in both electrodes are equal

$$|\psi_T|^2 = |\psi_B|^2 = \rho,$$

and also denote

$$E_T = qV_T, \qquad E_B = qV_B,$$

then Eq. (5) can be transformed as follows

$$\frac{\partial \varphi_i}{\partial t} = -\frac{q}{\hbar}V_i - \frac{K}{\hbar}\cos\phi. \tag{6}$$

In the above equation the index i denotes top and bottom electrode $i \in \{T, B\}$.

Curved junction in the CSJ model

In the literature the junction is studied in many ways [5]. In the analysis presented here we adopt the approach presented in papers [6]. The central part of the curved Josephson junction is depicted in Fig. 2. We consider the junction that has a form of a circle. The arc in the bottom electrode we parameterize by the parameter x. The arc in the top electrode is parameterized by the parameter y. The central curve of the isolator layer is parameterized by the parameter s. In the junction we chose the closed curve (with the corners **A**, **B**, **C**, **D**) and then add the phase differences between points **AB**, **BC**, **CD** and **DA**. Due to requirement of uniqueness of the wave function of the whole system, the sum of the phase differences in the considered loop is equal to multiplicity of 2π

$$[\varphi_T(y,t) - \varphi_T(y+\Delta y,t)] + [\varphi_T(y+\Delta y,t) - \varphi_B(x+\Delta x,t)]$$
$$+ [\varphi_B(x+\Delta x,t) - \varphi_B(x,t)] + [\varphi_B(x,t) - \varphi_T(y,t)] = 2\pi n. \tag{7}$$

Next, we introduce a new dynamical variable. Nontriviality of this variable follows from the fact that, from mathematical point of view, the system is not simply connected. Moreover, for appropriate choice of the electromagnetic vector potential the proposed variable coincides with the gauge invariant phase difference of the manyparticle wave functions of the superconducting electrodes. This variable is defined as the phase difference of the phases of the wave functions that describe the superconducting electrodes

$$\phi(s,t) \equiv \varphi_T(y,t) - \varphi_B(x,t). \tag{8}$$

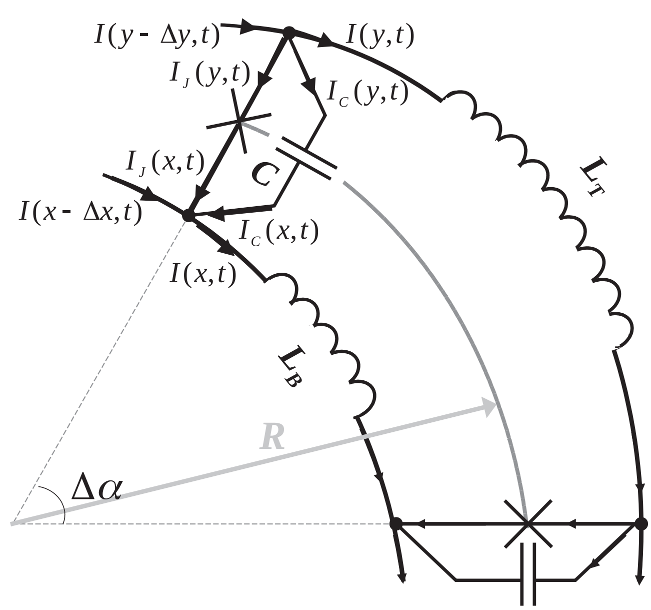

Fig. 2. The Josephson junction consists of the two superconducting electrodes (top and bottom). The electrodes are separated by the dielectric layer of thickness a. R_B is a radius of the circle located in the bottom electrode. This circle is parameterized by the parameter x. R_T is a radius of the the circle located in the top electrode. This arch is parameterized by the parameter y. R denotes the curvature radius of the central curve of the isolator which is parameterized by the parameter s. We consider long Josephson junction and therefore the transverse direction can be neglected.