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On the comparison of perturbation-iteration algorithm and residual power series method to solve fractional Zakharov-Kuznetsov equation

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ABSTRACT

In this paper, we present analytic-approximate solution of time-fractional Zakharov-Kuznetsov equation. This model demonstrates the behavior of weakly nonlinear ion acoustic waves in a plasma bearing cold ions and hot isothermal electrons in the presence of a uniform magnetic field. Basic definitions of fractional derivatives are described in the Caputo sense. Perturbation-iteration algorithm (PIA) and residual power series method (RPSM) are applied to solve this equation with success. The convergence analysis is also presented for both methods. Numerical results are given and then they are compared with the exact solutions. Comparison of the results reveal that both methods are competitive, powerful, reliable, simple to use and ready to apply to wide range of fractional partial differential equations.

Introduction

Fractional calculus has been studied by increasingly interest in recent years and has been extensively investigated and applied for many real problems which are modeled in various areas of science and engineering. Besides it has been the focus of many studies due to their frequent appearance in various applications such as fluid mechanics, thermodynamics, biology, electrical circuits and control theory. Therefore, a comprehensive attention has been noted to obtain numerical and analytic-approximate solutions of FDEs. These methods that have been used for different types of non-fractional and fractional differential equations include, Adomian decomposition method (ADM) [1,2] for fractional diffusion equations and for linear and nonlinear systems of fractional differential equations, homotopy perturbation method (HPM) [3–5] for Klein-Gordon equation, fractional IVPs and coupled system of partial fractional differential equations, homotopy analysis method [6–9] for fractional Hirota equations, time fractional PDEs, Sturm-Liouville problems and systems of nonlinear fractional differential equations, variational iteration method [10–12] for seepage flow problems, time-fractional partial differential equations and Zakharov-Kuznetsov equation, perturbation methods [13–15] for nonlinear problems in science and engineering and perturbation-iteration algorithm [16–22] for Bratu-type equations, nonlinear heat transfer equations, first-order differential equations, Fredholm and Volterra type integral equations and ordinary fractional differential equations.

In this paper, firstly, we proposed fractional Zakharov-Kuznetsov equation, then we described perturbation-iteration algorithm and in the next part, we investigated convergence analysis of PIA. Then we explained residual power series method (RPSM) in order to implement on Zakharov-Kuznetsov equation. After introducing RPSM, we described its convergence analysis and then we presented one example that shows reliability and efficiency of two methods in order to compare their numerical results. At last, we discussed about obtained results as a section for conclusion.

In this paper, the fractional Zakharov-Kuznetsov equation considered is of the form:

$$D_t^{\alpha} u + a(u^p)_x + b(u^q)_{xxx} + b(u^r)_{ttx} = 0 \tag{1}$$

where $u = u(x, y, t)$, α is order of the fractional derivative $(0 < \alpha \leqslant 1)$, a and b are arbitrary constants and p, q and r are integers and $p; q; r \neq 0$ that demonstrate the behavior of weakly nonlinear ion acoustic waves in a plasma comprising cold ions and hot isothermal electrons in the presence of a uniform magnetic field [23,24]. Firstly, the Zakharov-Kuznetsov equation was formulated for describing weakly nonlinear ion-acoustic waves in strongly

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magnetized lossless plasma in two dimensions studied in [25]. The FZK equations have been solved by means of VIM in [26] and HPM in [27].

Basic definitions

Definition 1. A real function $f(t)$, $t>0$ is said to be in the space C_μ, $(\mu>0)$ if there exists a real number $p(>\mu)$ such that $f(t)=t^p f_1(t)$ where $f_1\in C[0,\infty)$ and it is said to be in the space C_μ^m if $f^{(m)}\in C_\mu, m\in\aleph$ [28].

Definition 2. The Riemann-Liouville fractional integral operator (J^α) of order $\alpha\geqslant 0$ of a function $f\in C_\mu$, $\mu\geqslant -1$ is defined as [29]:

$$J^\alpha f(t)=\frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1}f(\tau)d\tau, \quad \alpha,t>0 \tag{2}$$

and $J^0 f(t)=f(t)$, where Γ is the well-known gamma function. Then the following properties hold for $f\in C_\mu$, $\mu\geqslant -1$, α, $\beta\geqslant 0$ and $\lambda>-1$.

- $J^\alpha J^\beta f(t)=J^{\alpha+\beta}f(t)$
- $J^\alpha J^\beta f(t)=J^\beta J^\alpha f(t)$
- $J^\alpha t^\lambda=\frac{\Gamma(\lambda+1)}{\Gamma(\lambda+1+\alpha)}t^{\lambda+\alpha}$.

Definition 3. The Caputo fractional derivative of a function f of order α, $f\in C_{-1}^m$, $m\in\aleph\cup\{0\}$, is defined as [30]:

$$\begin{aligned}D^\alpha f(t)&=J^{m-\alpha}f^{(m)}(t)\\&=\frac{1}{\Gamma(m-\alpha)}\int_0^t (t-\tau)^{m-\alpha-1}f^{(m)}(\tau)d\tau, \quad \alpha,t>0\end{aligned} \tag{3}$$

where $m-1<\alpha<m$ with the following properties;

- $D^\alpha(af(t)+bg(t))=aD^\alpha f(t)+bD^\alpha g(t)$, $a,b\in\Re$,
- $D^\alpha J^\alpha f(t)=f(t)$,
- $J^\alpha D^\alpha f(t)=f(t)-\sum_{j=0}^{k-1}f^{(j)}(0)\frac{t^j}{j!}$, $t>0$.

Overview and convergence of the PIA(1,1)

In this study, we first develop PIA approach in order to solve fractional Zakharov-Kuznetsov equation. To obtain the approximate solution of this model, we introduce and present perturbation-iteration algorithm PIA(m,n) by taking $m=1$ as one correction term in the perturbation expansion and $n=1$ as correction terms of just first derivatives in the Taylor series expansion.

Now, we consider the following initial value problem as follows:

$$F\left(u_t^{(\alpha)},u_x,u_{xxx},u_{ttx},\varepsilon\right)=0 \tag{4}$$

$$u(x,y,0)=c$$

where $u=u(x,y,t)$ and ε is a small perturbation parameter. The perturbation expansions with only one correction term is

$$u_{n+1}=u_n+\varepsilon(u_c)_n \tag{5}$$

Substituting Eq. (5) into Eq. (4) and writing in the Taylor Series expansion for first order derivatives in the neighborhood of $\varepsilon=0$ gives

$$\begin{aligned}&F\big((u_n^{(\alpha)})_t,(u_n)_x,(u_n)_{xxx},(u_n)_{ttx},0\big)\\&\quad+F_{u_t^{(\alpha)}}\big((u_n^{(\alpha)})_t,(u_n)_x,(u_n)_{xxx},(u_n)_{ttx},0\big)\varepsilon\big((u_c^{(\alpha)})_t\big)_n\\&\quad+F_{u_x}\big((u_n^{(\alpha)})_t,(u_n)_x,(u_n)_{xxx},(u_n)_{ttx},0\big)\varepsilon\big((u_c)_x\big)_n\\&\quad+F_{u_{xxx}}\big((u_n^{(\alpha)})_t,(u_n)_x,(u_n)_{xxx},(u_n)_{ttx},0\big)\varepsilon\big((u_c)_{xxx}\big)_n\\&\quad+F_{u_{ttx}}\big((u_n^{(\alpha)})_t,(u_n)_x,(u_n)_{xxx},(u_n)_{ttx},0\big)\varepsilon\big((u_c)_{ttx}\big)_n\\&\quad+F_\varepsilon\big((u_n^{(\alpha)})_t,(u_n)_x,(u_n)_{xxx},(u_n)_{ttx},0\big)\varepsilon=0\end{aligned} \tag{6}$$

or

$$\frac{F}{\varepsilon}+\big((u_c^{(\alpha)})_t\big)_nF_{u_t^{(\alpha)}}+\big((u_c)_x\big)_nF_{u_x}+\big((u_c)_{xxx}\big)_nF_{u_{xxx}}+\big((u_c)_{ttx}\big)_nF_{u_{ttx}}+F_\varepsilon=0 \tag{7}$$

where $F_{u_t^{(\alpha)}}=\frac{\partial F}{\partial u_t^{(\alpha)}}$, $F_{u_x}=\frac{\partial F}{\partial u_x}$, $F_{u_{xxx}}=\frac{\partial F}{\partial u_{xxx}}$, $F_{u_{ttx}}=\frac{\partial F}{\partial u_{xxx}}$ and $F_\varepsilon=\frac{\partial F}{\partial\varepsilon}$.

All derivatives in the expansion are evaluated at $\varepsilon=0$. For this reason in the calculation procedure, each term is obtained when ε tends to zero. Beginning with an initial function $u_0(x,y,t)$, first $(u_c)_0(x,y,t)$ is determined by the help of Eq. (7). Then using Eq. (5), $n+1$-th iteration solution is found. Iteration process is repeated using Eqs. (7) and (5) until obtaining an acceptable result. After presenting the convergence of PIA method, an example is given to show reliability and effectiveness of the method. The aim of this paper is to obtain approximate solutions of the fractional Zakharov-Kuznetsov equation by PIA and RPSM and to determine series solutions with high accuracy.

Regards the convergence of PIA, we present the following theorem.

Theorem 1. *PIA(m,n) converges to Eq. (1) for $m=1$ and $n=1$ when $\|u_{k+1}-u_k\|\leqslant\varepsilon'$ and $\varepsilon'\to 0$.*

Proof. The general iteration formula of PIA(m,n) for $m=1$ and $n=1$ according to do some computations on Eq. (7) in k-th and $k+1$-th step results as follows:

$$u_k'(t)+\frac{F_{u_k}}{F_{u_k'}}u_k(t)=-\frac{F_\varepsilon+\frac{F}{\varepsilon}}{F_{u_k'}} \tag{8}$$

$$u_{k+1}'(t)+\frac{F_{u_{k+1}}}{F_{u_{k+1}'}}u_{k+1}(t)=-\frac{F_\varepsilon+\frac{F}{\varepsilon}}{F_{u_{k+1}'}} \tag{9}$$

Then, by categorizing the elements F, F_u, F_{u_t} and F_ϵ in *PIA*(1,1) based on Eq. (1), we have:

$$F=u_t,\quad F_u=0,\quad F_{u_t}=1,\quad F_\varepsilon=-u_t+au_x^p+bu_{xxx}^q+bu_{ttx}^r+uD_t^\alpha$$

So, by substituting these elements in PIA formula, we have:

$$\begin{aligned}u_k'(x,y,t)=-\Big(&-u_k(x,y,t)+D_t^\alpha u_k(x,y,t)+a(u_k(x,y,t))_x^p\\&+b(u_k(x,y,t))_{xxx}^q+b(u_k(x,y,t))_{ttx}^r+\frac{u_k(x,y,t)}{\varepsilon}\Big)\end{aligned} \tag{10}$$

Thus, we can write it in k-th and $k+1$-th steps as follows:

$$\begin{aligned}u_k'(x,y,t)=\Big(&u_k(x,y,t)-D_t^\alpha u_k(x,y,t)-a(u_k(x,y,t))_x^p\\&-b(u_k(x,y,t))_{xxx}^q-b(u_k(x,y,t))_{ttx}^r-\frac{u_k(x,y,t)}{\varepsilon}\Big)\end{aligned} \tag{11}$$

and

$$\begin{aligned}u_{k+1}'(x,y,t)=\Big(&u_{k+1}(x,y,t)-D_t^\alpha u_{k+1}(x,y,t)-a(u_{k+1}(x,y,t))_x^p\\&-b(u_{k+1}(x,y,t))_{xxx}^q-b(u_{k+1}(x,y,t))_{ttx}^r-\frac{u_{k+1}(x,y,t)}{\varepsilon}\Big)\end{aligned} \tag{12}$$