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The development of the deterministic nonlinear PDEs in particle physics to stochastic case

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ABSTRACT

In the present work, accuracy method called, Riccati-Bernoulli Sub-ODE technique is used for solving the deterministic and stochastic case of the Phi-4 equation and the nonlinear Foam Drainage equation. Also, the control on the randomness input is studied for stability stochastic process solution.

Introduction

Many areas of science interested in the nonlinear problems, actually these models are reflected in interesting nonlinear deterministic (stochastic) PDEs. These solutions might be essential and significant for the explanation of some practical physical phenomena. Therefore investigating new technique for deterministic case to solve more complicated problems and also, we can develop these methods for random case as in [1–6]. Thus, many new techniques have been proposed, like as the tanh-sech technique [7–9], Jacobi elliptic function technique [10–12], exp-function technique [13–16], sine-cosine technique [17–19], homogeneous balance technique [20,21], F-expansion technique [22–24], extended tanh-technique [25–27], He's variational approach [28,29] $(\frac{G'}{G})$-expansion technique [30–32], the Laplace transformation technique [34,35] etc.

We will use the Riccati-Bernoulli sub-ODE technique [1] for extracting exact traveling wave solutions to NPDEs. The deterministic (stochastic) Phi-4 equation and deterministic (stochastic) Foam Drainage model, are taken to illustrate the validity of this technique. One very important feature, that this method gives new infinite sequence of solutions, using a Bäcklund transformation. As result, as we will see in this work, we implemented the Riccati-Bernoulli sub-ODE technique for finding the exact random solutions of the stochastic Phi-4 equation and the nonlinear stochastic Foam Drainage equation, when the problems have some disturbance in its coefficients, this mean the parameters are assigned random variables. To find the conditions for our method in random case we will state the stability conditions.

The Phi-4 model [36] plays an important role in particle and nuclear physics over the decades. So, exploration of exact traveling wave solutions to this equation turns into an essential task in the study of nonlinear physical phenomena. Thus, a lot of methods have been established, for example, modified simple equation method [37], the $(\frac{G'}{G})$ – [38] an so on. We will compare between our result given in sequel with these results. We also consider Foam Drainage model [39]. The drainage of liquid foams involves the interplay of surface tension, gravity and viscous forces. The study of this equation is very significant. Foam drainage is the flow of liquid through plateau borders and intersections of four channels between the bubbles driven by gravity and capillarity. There are so techniques have been presented, such as, the tanh method

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and Adomian decomposition method [33], the $\left(\frac{G'}{G}\right)$ – [34,39] an so on. We will compare between our result given in sequel with these results, in detail see Appendix B. Finally, We will also show that our method is efficacious, robust, general and much powerful than the other proposed method, in more detail see Appendices A and B.

The rest of the paper is given as follows: In Section "Description of the method" we describe the Riccati-Bernoulli sub-ODE method. We also give a Bäcklund transformation of the Riccati-Bernoulli equation. In Sections "Application", we apply the Riccati-Bernoulli sub-ODE technique to solve the Phi-4 model and the nonlinear Foam Drainage model. Additionally, in Section "Stochastic perturbation" we can discuss this technique for the randomness of solutions according to our problems. Finally, in Section "Conclusions" we give the summary of our contribution and two appendices.

Description of the method

Every nonlinear evolution equation can be expressed in following form:

$$P(\phi, \phi_t, \phi_x, \phi_{tt}, \phi_{xx}, \ldots) = 0, \tag{2.1}$$

where P is a polynomial in $\phi(x,t)$ and its partial derivatives. The basic steps of this technique [40] are:

Step 1. By consideration the wave transformation

$$\phi(x,t) = \phi(\xi), \quad \xi = k(x + \upsilon t), \tag{2.2}$$

Eq. (2.1) transforms into the following ODE:

$$H(\phi, \phi', \phi'', \phi''', \ldots..) = 0, \tag{2.3}$$

where H is a polynomial in $\phi(\xi)$ and, its total derivatives,while $\phi'(\xi) = \frac{d\phi}{d\xi}$, $\phi''(\xi) = \frac{d^2\phi}{d\xi^2}$ and so on.

Step 2. Let Eq. (2.3) has the following formal solution:

$$\phi' = a\phi^{2-n} + b\phi + c\phi^n, \tag{2.4}$$

where a, b, c and n are constants calculated in sequel. From Eq. (2.4), one get

$$\begin{aligned}\phi'' &= ab(3-n)\phi^{2-n} + a^2(2-n)\phi^{3-2n} + nc^2\phi^{2n-1} \\ &\quad + bc(n+1)\phi^n + (2ac + b^2)\phi,\end{aligned} \tag{2.5}$$

$$\begin{aligned}\phi''' &= (ab(3-n)(2-n)\phi^{1-n} + a^2(2-n)(3-2n)\phi^{2-2n} \\ &\quad + n(2n-1)c^2\phi^{2n-2} + bcn(n+1)\phi^{n-1} + (2ac + b^2))\phi',\end{aligned} \tag{2.6}$$

Remark 1. Eq. (2.4) called the Riccati-Bernoulli equation. At $ac \neq 0$ and $n = 0$, it's a Riccati equation. At $a \neq 0$, $c = 0$, and $n \neq 0$, it's a Bernoulli equation.

Solutions cases

The solutions of the Riccati-Bernoulli technique (2.4) given as follow:

Case 1. For $n = 1$, the solution is:

$$\phi(\xi) = \mu e^{(a+b+c)\xi}. \tag{2.7}$$

Case 2. For $n \neq 1$, $b = 0$ and $c = 0$, the solution is:

$$\phi(\xi) = (a(n-1)(\xi + \mu))^{\frac{1}{n-1}}. \tag{2.8}$$

Case 3. For $n \neq 1$, $b \neq 0$ and $c = 0$, the solution is:

$$\phi(\xi) = \left(\frac{-a}{b} + \mu e^{b(n-1)\xi}\right)^{\frac{r}{n-1}}. \tag{2.9}$$

Case 4. For $n \neq 1$, $a \neq 0$ and, $b^2 - 4ac < 0$, the solution is:

$$\phi(\xi) = \left(\frac{-b}{2a} + \frac{\sqrt{4ac - b^2}}{2a} tan\left(\frac{(1-n)\sqrt{4ac - b^2}}{2}(\xi + \mu)\right)\right)^{\frac{1}{1-n}} \tag{2.10}$$

and

$$\phi(\xi) = \left(\frac{-b}{2a} - \frac{\sqrt{4ac - b^2}}{2a} cot\left(\frac{(1-n)\sqrt{4ac - b^2}}{2}(\xi + \mu)\right)\right)^{\frac{1}{1-n}} \tag{2.11}$$

Case 5. For $n \neq 1$, $a \neq 0$ and, $b^2 - 4ac > 0$, the solution is:

$$\phi(\xi) = \left(\frac{-b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a} coth\left(\frac{(1-n)\sqrt{b^2 - 4ac}}{2}(\xi + \mu)\right)\right)^{\frac{1}{1-n}} \tag{2.12}$$

and

$$\phi(\xi) = \left(\frac{-b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a} tanh\left(\frac{(1-n)\sqrt{b^2 - 4ac}}{2}(\xi + \mu)\right)\right)^{\frac{1}{1-n}} \tag{2.13}$$

Case 6. For $n \neq 1$, $a \neq 0$ and, $b^2 - 4ac = 0$, the solution is:

$$\phi(\xi) = \left(\frac{1}{a(n-1)(\xi + \mu)} - \frac{b}{2a}\right)^{\frac{1}{1-n}}. \tag{2.14}$$

Here μ be an arbitrary constant.

Step 3. Subrogate the derivatives of ϕ into Eq. (2.3) gives an algebraic equation of ϕ, which uses to determining the value of n. Comparing the coefficients of ϕ^i one gets a set of algebraic equations for a, b, c, and υ. Hence we get the traveling wave solutions of Eq. (2.1), by solving these equations and superseding n, a, b, c, υ, and $\xi = k(x + \upsilon t)$ into Eqs. (2.7)–(2.14)).

Bäcklund transformation

When $\phi_{m-1}(\xi)$ and $\phi_m(\xi)(\phi_m(\xi) = \phi_m(\phi_{m-1}(\xi)))$ are the solutions of Eq. (2.4), we have

$$\frac{d\phi_m(\xi)}{d\xi} = \frac{d\phi_m(\xi)}{d\phi_{m-1}(\xi)}\frac{d\phi_{m-1}(\xi)}{d\xi} = \frac{d\phi_m(\xi)}{d\phi_{m-1}(\xi)}(a\phi_{m-1}^{2-n} + b\phi_{m-1} + c\phi_{m-1}^n),$$

namely

$$\frac{d\phi_m(\xi)}{a\phi_m^{2-n} + b\phi_m + c\phi_m^n} = \frac{d\phi_m(\xi)}{a\phi_{m-1}^{2-n} + b\phi_{m-1} + c\phi_{m-1}^n}. \tag{2.15}$$

Integrating Eq. (2.15) once with respect to ξ, we get a Bäcklund transformation of Eq. (2.4) in the form

$$\phi_m(\xi) = \left(\frac{-cA_1 + aA_2(\phi_{m-1}(\xi))^{1-n}}{bA_1 + aA_2 + aA_1(\phi_{m-1}(\xi))^{1-n}}\right)^{\frac{1}{1-n}}. \tag{2.16}$$